# Abstract Algebra III

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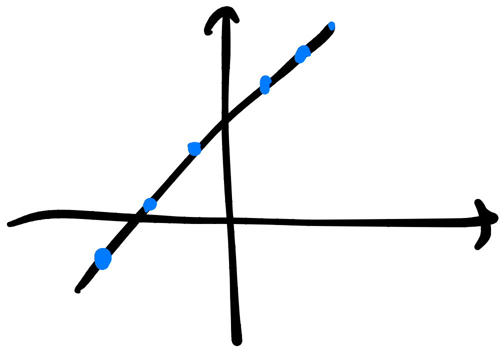

Q: Can we prove $(\mathbb{Z}/n\mathbb{Z})^\times = \{a : \gcd(a,n)=1\}$ ?

Yes. $\boxed{\text{The equation } ax+ny=c \text{ has integer solutions } (x,y) \text{ iff } \gcd(a,n)\mid c}$

The equation

$$ax + ny = c$$ has integer solutions $(x, y)$ iff $\gcd(a,n) \mid c$

If $\gcd(a,n) = 1$, then can solve $ax + \overset{0 \bmod n}{\overset{\shortparallel}{ny}} = 1$

$$\Rightarrow ax \equiv 1 \bmod n$$

$$\Rightarrow x \equiv a^{-1} \bmod n$$

For the other direction

if $ax \equiv 1 \mod n$

$\Rightarrow \exists y \in \mathbb{Z}$ with $ax = 1 + yn$

$\Rightarrow$ $ax - yn = 1 \quad x, y \in \mathbb{Z}$

so $\gcd(a, n) \mid 1$

Solve $4,6$ $4x+6y=1$

this has no sol. because $2 \mid (4x+6y)$

always if $x,y \in \mathbb{Z}$

$4x+6y=2$ $6=4+2$ gcd

$4=2\cdot 2+0$

$4(-1)+6(1)=2$

$$4\overset{-5}{x}+6\overset{5}{y}=10$$

# Chinese Remainder Theorem

$R = \mathbb{Z}$ $\quad n = p_1^{e_1} p_2^{e_2} \cdots p_r^{e_r}$ $\quad p_i$ prime, $p_i \neq p_j$ $i \neq j$

$e_i > 0$

$$(a_1, a_2, \ldots, a_r$$

$$\mathbb{Z}/n\mathbb{Z} \cong \mathbb{Z}/p_1^{e_1} \times \mathbb{Z}/p_2^{e_2} \times \cdots \times \mathbb{Z}/p_r^{e_r}$$

$$x \equiv a_1 \mod p_1^{e_1}$$
$$x \equiv a_2 \mod p_2^{e_2}$$
$$\vdots$$
$$x \equiv a_r \mod p_r^{e_r}$$

# Quiz 9

That's all for today!