Math 295 - Fall 2020
Warm up 9.1
Due before class on Monday November 9

Please turn in this assignment on Gradescope.

Problem 1 : (Objective E4) In this problem we will study the complex function given by the rule

$$f(z) = \frac{-z}{1-z},$$

and connect it to a series given below.

a) Give the power series expansion of this function centered at $z_0 = 0$, and give its radius of convergence.

Now shift your attention for a bit and consider the series

$$\sum_{k=0}^{\infty} \left(\frac{1}{z}\right)^k.$$

b) Use the Ratio Test to find the values of z for which this series converges.

c) Use the formula

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r},$$

which is valid when $|r| < 1$, to find the sum of the series. Do you recognize f?

We now bring all parts of the problem together.

d) Draw the complex plane and label the region where the power series expansion of f converges. Label the region where f is equal to the series given in this problem.