Math 295 - Fall 2020
Warm up 7.2
Due before class on Wednesday October 28

Please turn in this assignment on Gradescope.

Problem 1 : (Objective D4) The *Cauchy Integral Formula* is the following theorem:

Theorem. *Let U be an open set and f be holomorphic on U. Let γ parametrize a positively oriented, simple, closed contour in U whose interior is completely contained in U, and let z_0 be an interior point of this contour. Then*

$$f(z_0) = \frac{1}{2\pi i} \int_\gamma \frac{f(z)}{z - z_0} dz.$$

(This is Task 194 of Bowman, or Theorem 4.27 in BMPS.)

For each of the following expressions of the form $\int_\gamma \frac{f(z)}{z-z_0} dz$, identify f and z_0. Then draw a quick sketch to make sure that z_0 is inside γ. Finally use the Cauchy Integral formula to compute the integral.

a) $\displaystyle\int_\gamma \frac{z^2 - 1}{z - i} dz$, γ is the circle of radius 2 centered at 0, oriented positively

b) $\displaystyle\int_\gamma \frac{z^3 + 2z}{z + i} dz$, γ is the circle of radius 2 centered at 0, oriented positively

c) $\displaystyle\int_\gamma \frac{1}{z} dz$, γ is the unit circle oriented positively