

Math 295 - Fall 2020  
Warm up 5.1  
Due before class on Monday October 5

Please turn in this assignment on Gradescope.

**Problem 1 : (Objective C5)** For this problem,  $f(z) = z^2$ .

- a) Express the real and imaginary parts of  $z^2$  in terms of the real and imaginary parts of  $z$ .
- b) What does  $f$  do to vertical lines?
- c) What does  $f$  do to horizontal lines?
- d) Express the argument and modulus of  $z^2$  in terms of the argument and modulus of  $z$ .
- e) What does  $f$  do to circles centered at the origin?
- f) What does  $f$  do to lines through the origin?
- g) Let  $T$  be the figure formed by the horizontal line segment from 0 to 2, the circular arc from 2 to  $2i$ , and then the vertical segment from  $2i$  to 0. Draw  $T$  and  $f(T)$ .
- h) Is the right angle at the origin in part c) preserved? Is something wrong here?
- i) Let  $R$  be the right half-plane,  $\{z \in \mathbb{C} : \operatorname{Re}(z) > 0\}$ . Show that the image of  $R$  under  $f$  is all of  $\mathbb{C}$  except 0 and negative real axis.

**Problem 2 : (Objective C6)** Use the definition of the exponential function (on page 43 of BMPS) to prove some familiar and unfamiliar properties of the exponential function:

- a)  $\frac{1}{\exp(z)} = \exp(-z)$  (hint: rationalize the denominator)
- b)  $\exp(z) \neq 0$  for any  $z \in \mathbb{C}$
- c)  $\exp(z + 2\pi i) = \exp(z)$

**Problem 3 : (Objective C6)** Describe the image of the following sets under the exponential function  $\exp(z)$ :

- a) the line segment defined by  $z = iy, 0 \leq y \leq 2\pi$
- b) the line segment defined by  $z = 1 + iy, 0 \leq y \leq 2\pi$
- c) the rectangle  $\{z = x + iy \in \mathbb{C} : 0 \leq x \leq 1, 0 \leq y \leq 2\pi\}$
- d) express the modulus  $r$  and argument  $\phi$  of  $\exp(z)$  in terms of the real and imaginary parts  $x$  and  $y$  of  $z$ .