Math 295 - Fall 2020
Warm up 4.2
Due before class on Wednesday September 30

Please turn in this assignment on Gradescope.

Problem 1 : (Objectives C2, C3)

a) Show that the fractional linear transformation $f(z) = \dfrac{1+z}{1-z}$ maps the unit circle (minus the point $z = 1$) onto the imaginary axis.
Hint: $w \in \mathbb{C}$ is on the imaginary axis if and only if $\overline{w} = -w$.

b) What is $\lim\limits_{z \to 1} \dfrac{1+z}{1-z}$?

Problem 2 : (Objective C3)

a) Show that $\arg(1/\overline{z}) = \arg(z)$, i.e., if ϕ is any argument for z, then ϕ is also an argument for $1/\overline{z}$. What does this mean geometrically? In other words, what can you say about the positions of z and $1/\overline{z}$ in the complex plane from this equation?

b) What is the effect of the function $f(z) = 1/\overline{z}$ on the unit circle $|z| = 1$?

c) What is the effect of the function $f(z) = 1/\overline{z}$ on the points outside the unit circle $|z| = 1$?

d) What is the effect of the function $f(z) = 1/\overline{z}$ on the points inside the unit circle $|z| = 1$?

e) Show that $\arg(1/z) = -\arg(z)$, i.e., if ϕ is any argument for z, then $-\phi$ is an argument for $1/z$. What does this mean geometrically? In other words, what can you say about the positions of z and $1/z$ in the complex plane?

f) What is the effect of the function $g(z) = 1/z$ on the complex plane?

Hint: We have that $1/z = \overline{(1/\overline{z})}$, so you should use your work from the earlier parts of this problem.

Problem 3 : (Objective C2) Compare the rules of arithmetic for $\hat{\mathbb{C}}$ given by BMPS on p. 37 with those given by Bowman in Task 225 on page 51. How are they the same? How are they different? In the one aspect where they are different, which one do you think is correct?