Math 295 - Fall 2020
Warm up 3.2
Due before class on Wednesday September 23

Please turn in this assignment on Gradescope.

Problem 1 : (Objective B3) Of great importance are the *Cauchy-Riemann equations*. They are given in equations (2.2) and (2.3) of BMPS (there are two version of the Cauchy-Riemann equations, which are equivalent to each other). Copy down the two equivalent versions of the Cauchy-Riemann equations, and the proof that they are equivalent.

Problem 2 : (Objective B3) Let $f\colon \mathbb{C} \to \mathbb{C}$ be given by $f(z) = z^2$. Write $f(z)$ in the form $f(z) = u(x, y) + iv(x, y)$, where u and v are real-valued. Show that u and v satisfy the Cauchy-Riemann equations.

Problem 3 : (Objective B3) We will often use the *converse to the Cauchy-Riemann equations* to prove differentiability of functions. This result is given in BMPS in Theorem 2.13(b), in Bowman in Theorem 4 of Section 5.3, and in Knopp it is Theorem 4 in Part 1, Section 7. Please copy down the converse to the Cauchy-Riemann equations, which gives an easy check to see if a function is differentiable. Take the time to compare the statements in the different books you have access to.

Problem 4 : (Objective B3) Use the converse to the Cauchy Riemann equations to find where the function $f(x + iy) = x^3 + 3xy^2 - 3x + i(y^3 + 3x^2y - 3y)$ is differentiable. Where is it holomorphic?

Problem 5 : (Objective B4)

a) Please go on Wikipedia, look up the definition of *conformal map*, and copy it down. If you read far enough, you will see that when the domain is the complex plane (or basically $\mathbb{R}^2$), conformal maps are exactly the maps that are complex differentiable with nowhere zero derivative. If you continue to read even further you will see that in higher dimensions ($\mathbb{R}^3$ and up) conformal maps are rare.

b) Please go on Wikipedia, look up the definition of *harmonic function*, and copy it down. If you go to the section on the connection with complex function theory, you will see that, roughly speaking (or more precisely "locally"), every harmonic function on $\mathbb{R}^2$ is the real part of a complex holomorphic function.