Math 295 - Fall 2020
Warm up 3.1
Due before class on Monday September 21

Please turn in this assignment on Gradescope.

Problem 1 : (Objectives B1,B2) Consider the function $f\colon \mathbb{C} \to \mathbb{C}$ given by $f(z) = z^2$. Apply the limit definition of the derivative to give a direct proof that $f'(z) = 2z$. Where is f differentiable in $\mathbb{C}$? On what region is it holomorphic?

Problem 2 : (Objectives B1,B2) Use the derivative rules and the fact that the identity function $f\colon \mathbb{C} \to \mathbb{C}$ given by $f(z) = z$ is entire to show that every polynomial function $g\colon \mathbb{C} \to \mathbb{C}$ is entire.

Problem 3 : (Objective B1)

a) Read Example 2.3 of BMPS, and make sure that you understand it. What is $\overline{z}$ when z is restricted to the real axis? What is $\overline{z}$ when z is restricted to the imaginary axis?

b) Read Examples 2.8 and 2.9 of BMPS, and make sure that you understand them. Is $f\colon \mathbb{C} \to \mathbb{C}$ given by $f(z) = (\overline{z})^2$ differentiable anywhere on $\mathbb{C}$? Is it holomorphic in any region? What about the function given by $f(z) = \overline{z}$? Is it differentiable anywhere? Is it holomorphic in any region of $\mathbb{C}$?

c) In your opinion, does the complex conjugate behave well insofar as the complex derivative is concerned?