Complex Analysis

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

No warm up due on Monday - free warm up

Significantly less HW for rest of week

#8 a) We have $\exp(z) := e^x(\cos y + i \sin y)$

$z = x + iy$

defined to be

We also have $e \in \mathbb{R}$ $\quad e := \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

"Euler's number"

$$e^z := \exp(z \operatorname{Log} e)$$

complex raising to an exponent

You can show that with these definitions

$$\exp(z) = e^z$$

BMPS

#8a) $(e^z)^w$

or can do it with exp notation

$$(\exp(z))^w$$

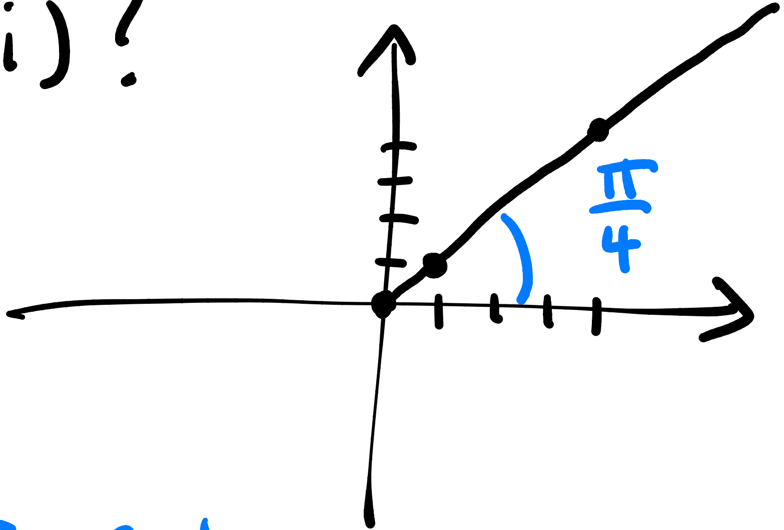
$e^{zw} = \exp(zw)$

Is $\arg(1+i) = \arg(4+4i)$?

Yes, as sets

Is $\mathrm{Arg}(1+i) = \mathrm{Arg}(4+4i)$?

Yes



$\mathrm{Arg}(1+i) = \mathrm{Arg}(4+4i) = \frac{\pi}{4}$

$\arg(1+i) = \arg(4+4i) = \frac{\pi}{4} + 2\pi k, \quad k \in \mathbb{Z}$

#4 $f''(z)=0$ on a region $\mathcal{U} \subseteq \mathbb{C}$

↳ open connected set

Show that $f(z)=az+b$ for $a,b \in \mathbb{C}$

So far, we had: Let $g(z)=f'(z)$. Then g is holomorphic with $g'(z)=f''(z)=0$ on $\mathcal{U}$

So g is constant, $g(z)=a$, $a \in \mathbb{C}$.

$\Rightarrow f'(z)=a$

$$f'(z) = a.$$

Let $h(z) = f(z) - az$.

Then $h'(z) = f'(z) - a$

because $\frac{d}{dz}(-az) = -a$

So $h'(z) = a - a = 0$

and h is holomorphic on U

$h(z) = b$, $b \in \mathbb{C}$ (Solve for f)

Thinking:

<u>Want</u> $f(z) = az + b$

Can detect: functions that are constant

But f is not constant

You know what is though?

$h(z) = f(z) - az$

if right, then $h(z) = b$

Theorem

If f, g are holomorphic on U a region and $f'(z) = g'(z)$ $\forall z \in U$ then $f(z) = g(z) + c$ for $c \in \mathbb{C}$.

proof: Let $h(z) = f(z) - g(z)$

Then $h'(z) = f'(z) - g'(z) = 0$, h holomorphic on U

$\Rightarrow h(z) = C$, $C \in \mathbb{C}$ so $f(z) - g(z) = C$.

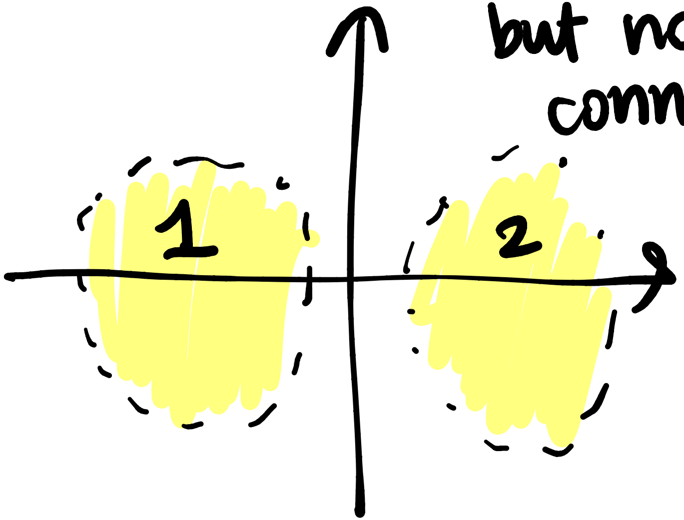
Why is the fact that $\mathcal{U}$ is connected so important?

$$f(z) = \begin{cases} z & \text{on circle 1} \\ 3z & \text{on circle 2} \end{cases}$$

then $f''(z) = 0$ on $\mathcal{U}$

but f is not of the form $az+b$

$\mathcal{U}$ open but not connected



#8e) Seen $\text{Log } z \cancel{=} \ln|z| + i \overset{\text{Arg}}{\arctan}\left(\frac{y}{x}\right)$

$z = x + iy$

false

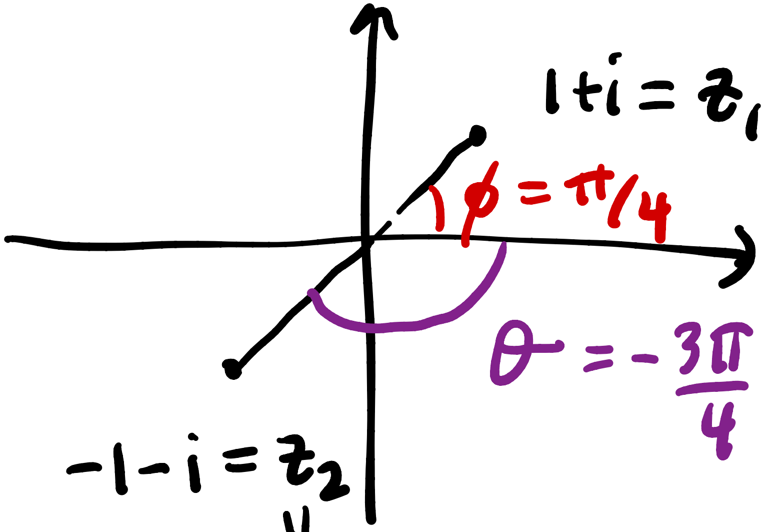
1. $\text{Arg}(z) \in (-\pi, \pi]$

but $\arctan\left(\frac{y}{x}\right) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$

true that $\text{Arg}(z) = \arctan\left(\frac{y}{x}\right)$ if $\text{Arg}(z) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$

② by picture

$$\tan\phi = \frac{\sin\phi}{\cos\phi}$$



$$\mathrm{Arg}(z_1) \neq \mathrm{Arg}(z_2)$$

But $\arctan\left(\frac{1}{1}\right) = \arctan\left(\frac{-1}{-1}\right) = \arctan 1 = \frac{\pi}{4}$

$\uparrow\ y_1/x_1$ $\quad\quad \leftarrow y_2/x_2$

$$\mathrm{Arg}(z) = \arctan\left(\frac{y}{x}\right) \text{ if } \mathrm{Arg}(z) \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$z = 1 + \sqrt{3}i$

$x = 1$

$y = \sqrt{3}$

$\mathrm{Arg}(z) = \frac{\pi}{3}$

$\arctan\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$ ✓

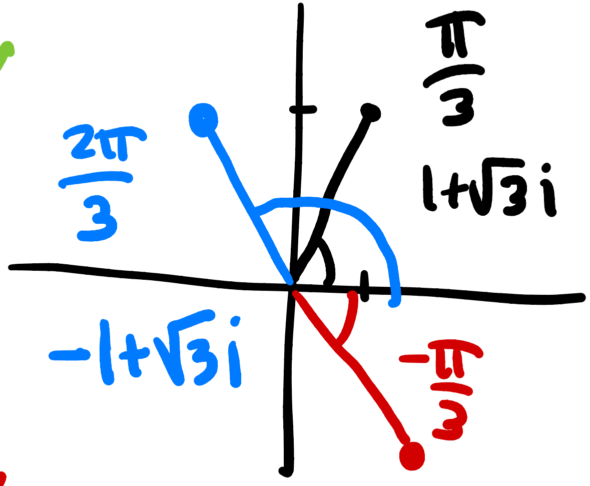
$z = -1 + \sqrt{3}i$

$x = -1$

$y = \sqrt{3}$

$\mathrm{Arg}(z) = \frac{2\pi}{3}$

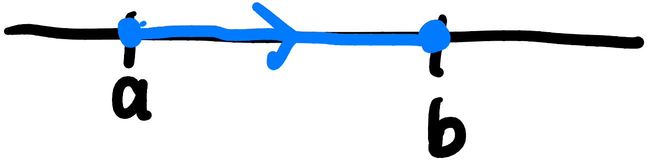
$\arctan(-\sqrt{3}) = -\frac{\pi}{3}$ ✗



$\mathbb{R}$

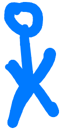
$$\int_a^b f(x)\,dx$$

 $a \to b$

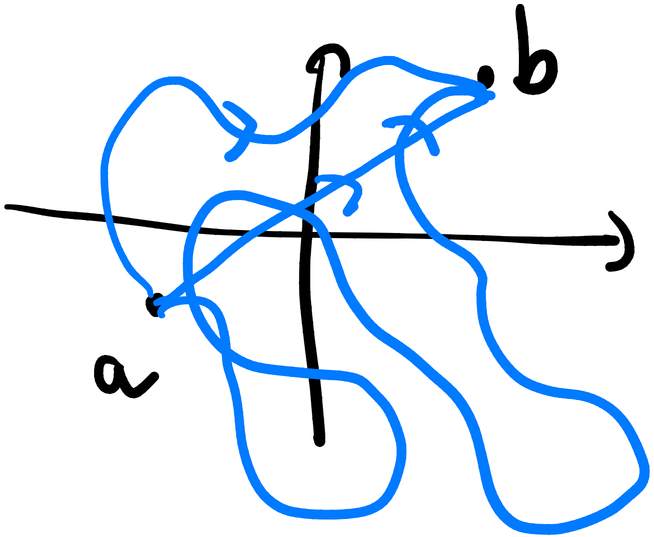


$\mathbb{C}$

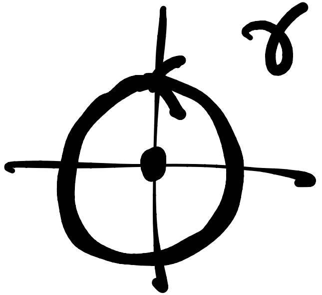
$$\int_a^b f(z)\,dz$$

 ???

does it matter?



$$\int_\gamma \frac{1}{\sqrt{x^2+y^2}}\, d\gamma \neq 0$$



$\gamma(t) = e^{it}$

$0 \le t \le 2\pi$

$$\int_\gamma 2xy \, d\gamma = 0$$

THAT'S ALL FOR TODAY!

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