Complex Analysis

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

Graded HW 1–4

Add problems to Redo HW

Questions on Redo HW

#3a)

#5

#6

#3 a) $f(z) = \bar{z}^2 = (x - iy)^2 = x^2 - ixy - ixy + i^2y^2$

$= (x^2 - y^2) + i(-2xy)$

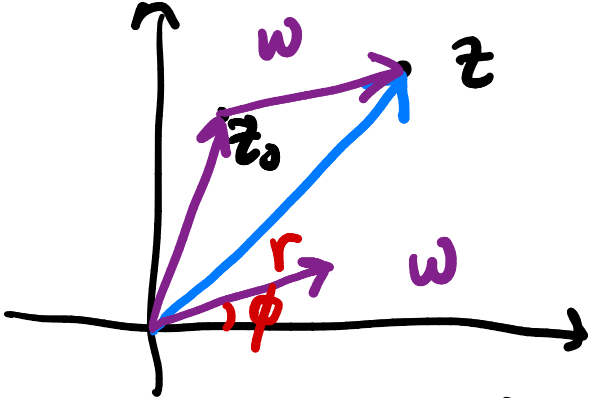
$u(x,y) = x^2 - y^2 \qquad v(x,y) = -2xy$

$$f'(z_0) = \lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0}$$

$$= \lim_{z \to z_0} \frac{\bar{z}^2 - \bar{z}_0^2}{z - z_0}$$

$$= \lim_{r \to 0} \frac{\overline{(z_0 + re^{i\phi})}^2 - \bar{z}_0^2}{z_0 + re^{i\phi} - z_0}$$

$z = z_0 + re^{i\phi}$



$z = z_0 + w$

$w = re^{i\phi}$

Next: simplify this

as $z \to z_0$
$w \to 0$
$r \to 0$

#5 $f(z) = \frac{az+b}{cz+d}$ $a, b, c, d \in \mathbb{R}$

$ad-bc > 0$

Show that if $\text{Im}(z) > 0$ then $\text{Im}(f(z)) > 0$

2 ways to solve this problem

① "Hands on" let $z = x+iy$ $y > 0$

plug into f $\frac{a(x+iy)+b}{c(x+iy)+d}$

② "More thought"

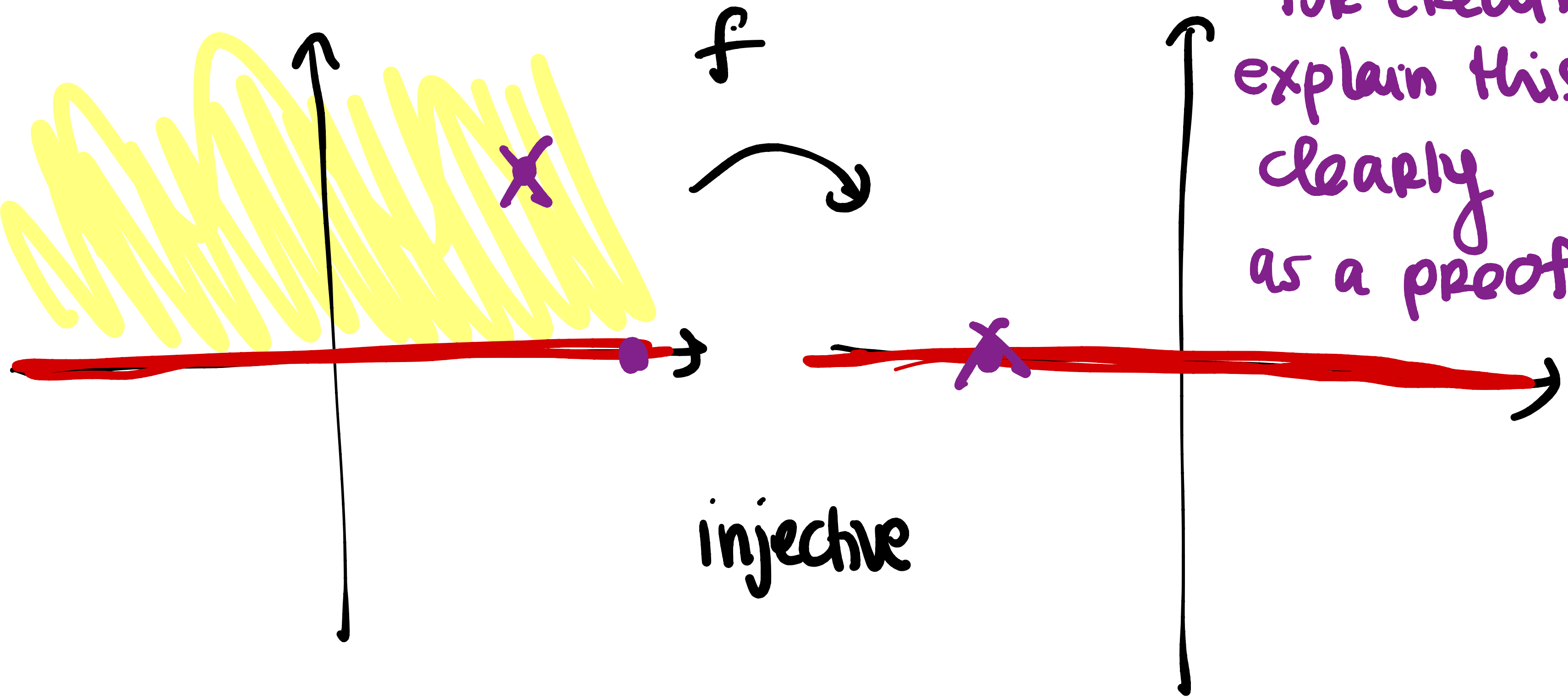
A) Show that f sends $\mathbb{R}$ to $\mathbb{R}$

Remember that $\mathbb{R}$ is a line (a circle through ∞)

B) Show that f sends one specific point with $\operatorname{Im}>0$ to a point with $\operatorname{Im}>0$.

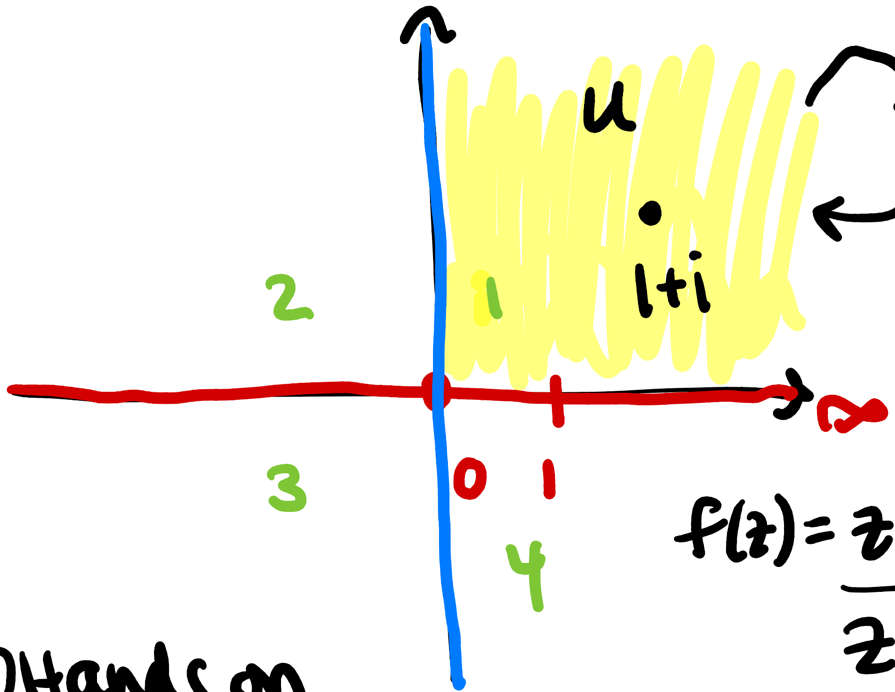
c) Since f sends $\mathbb{R}$ to $\mathbb{R}$ and f is continuous (because it's differentiable), the image of the upper half plane must be contained either in the upper half plane or in the lower half plane because the image of a connected set must be a connected set.

for credit, explain this clearly as a proof

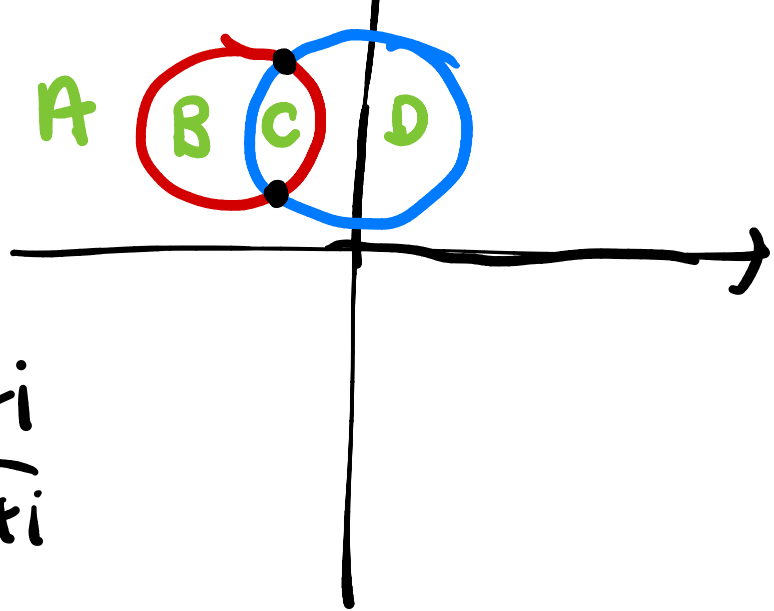


no yellow point can be sent to the real line because f is injective

#6 a) $U = \{ z \in \mathbb{C} : \operatorname{Re}(z) > 0, \ \operatorname{Im}(z) > 0 \}$



NOT TRUE ??



$f(z) = \dfrac{z-i}{z+i}$

① Hands on way

$z = x + iy \qquad x, y > 0$

b) image is open , not closed

c) connected

THAT'S ALL FOR TODAY!

OH today canceled

OH Wednesday 4pm-5pm

contact me for appt @ different