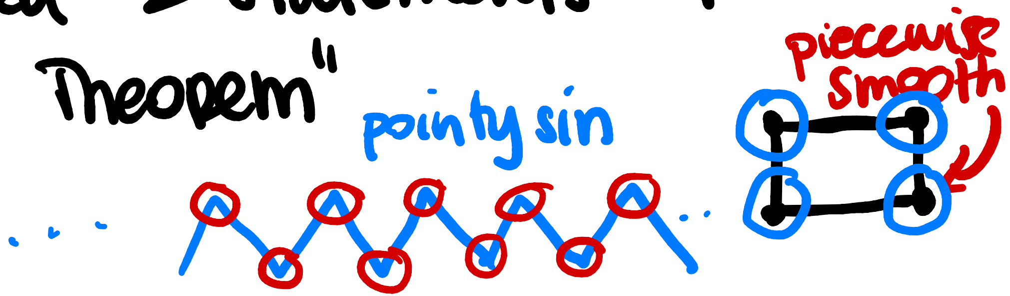
Complex Analysis

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

Last time we compared 2 statements of "Cauchy's Theorem" in BMPS.

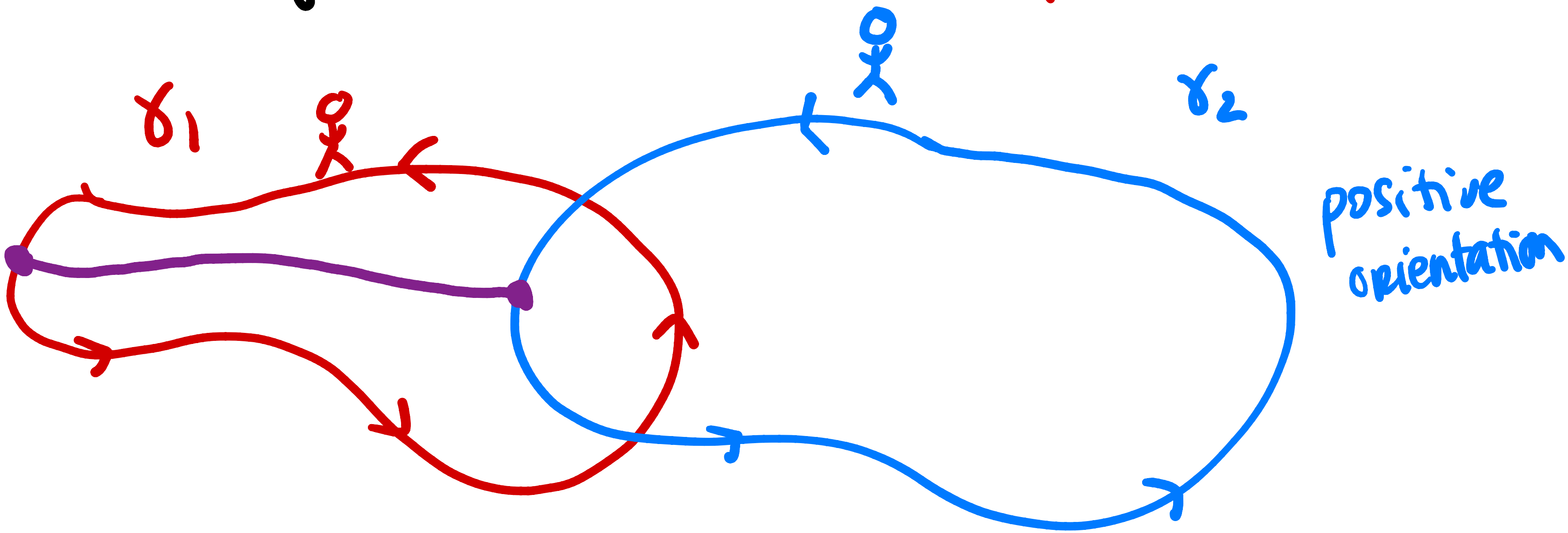


Let $U \subseteq \mathbb{C}$ be a region (open + connected), f holomorphic on U, γ piecewise smooth / γ_1, γ_2 piecewise smooth

Theorem 4.18 "easy" $\Rightarrow$ Corollary 4.20

If $\gamma_1 \sim_U \gamma_2$ $\int_{\gamma_1} f(z)dz = \int_{\gamma_2} f(z)dz$	If $\gamma \sim_U 0$ $\int_\gamma f(z)\,dz = 0$

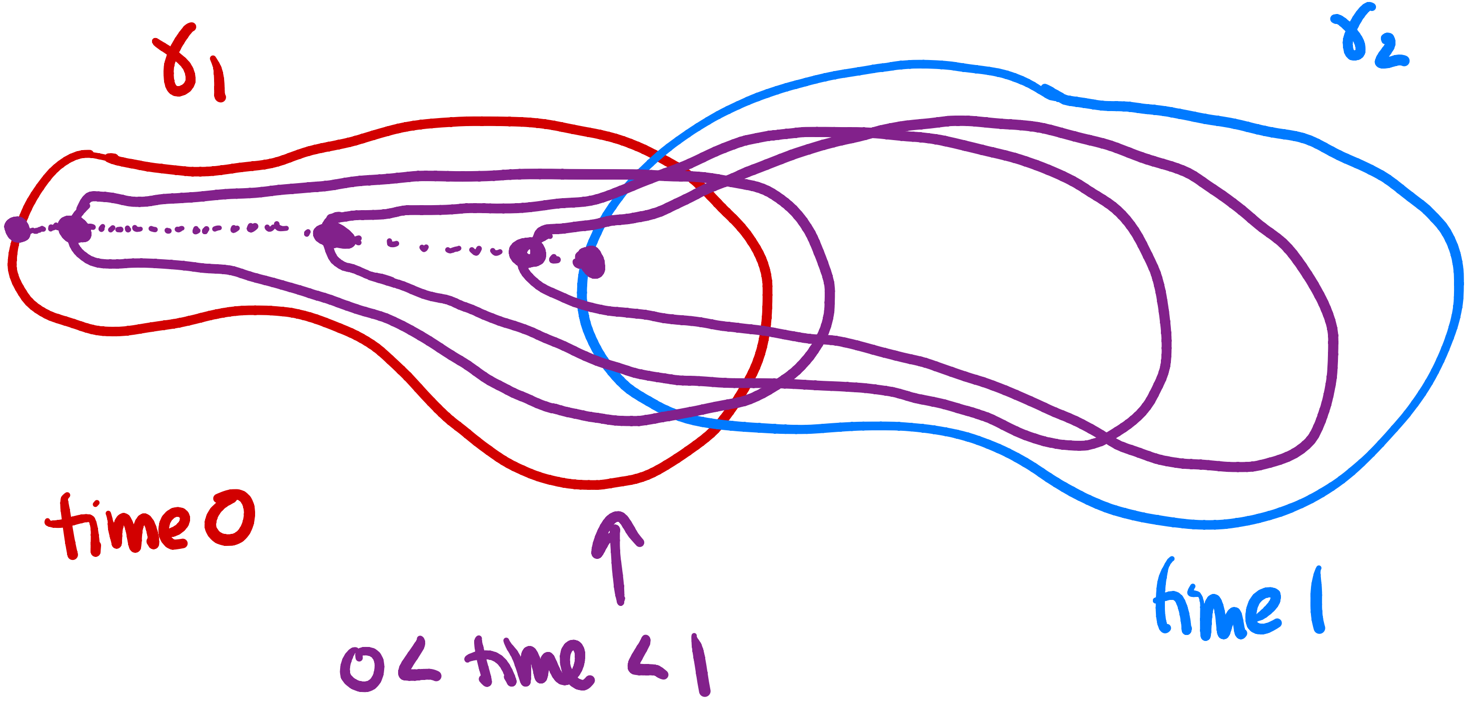
For other direction: Assume that if $\gamma \sim_u 0$ then $\int_\gamma f(z)\,dz = 0$. Now let $\gamma_1 \sim_u \gamma_2$



purple path is tracing the journey of this point from $t=0$ on γ_1 to $t=1$ on γ_2

$\gamma_1 \sim_U \gamma_2$ means that

all of the intermediate paths are inside U



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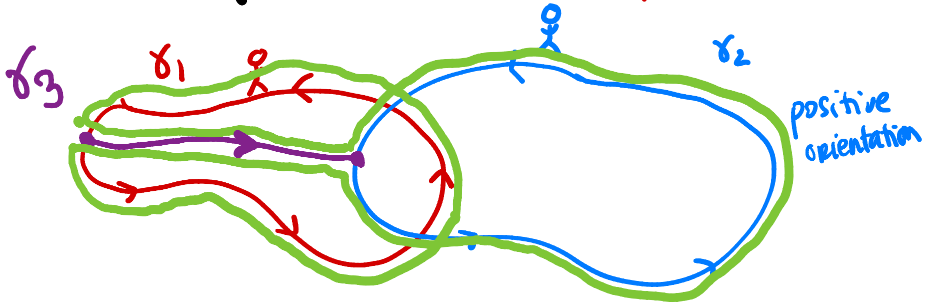

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2 facts

$\gamma \sim_u 0$

$\gamma = \gamma_1 + \gamma_3 - \gamma_2 - \gamma_3$

Bowman has 2 versions

U simply connected means that if $\gamma \subseteq U$ then $\gamma \sim_U 0$.

1st version: 2nd version:

Let f be holomorphic ~~with continuous derivative~~ on a simply connected open set U and γ parametrizes a closed contour then

$$\int_\gamma f(z)\,dz = 0$$

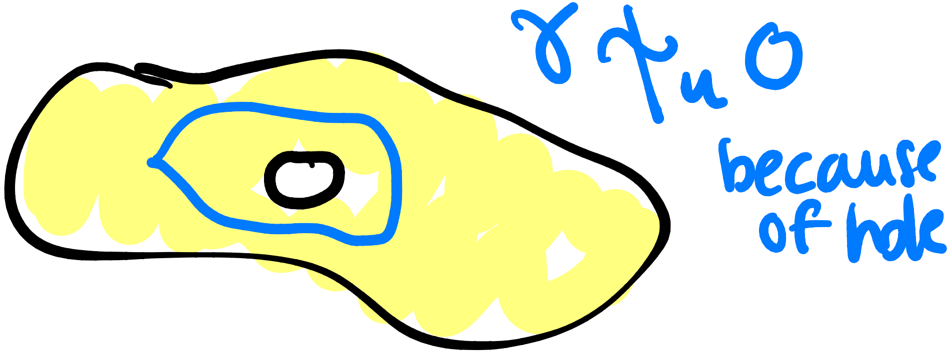
BMPS would have $\gamma \sim_U 0$

Take away: Cauchy's Theorem says that if f is holomorphic everywhere (no holes) inside a path, then $\int_\gamma f(z)\,dz = 0$

<u>**or**</u> if f is holomorphic "between 2 paths" then $\int_{\gamma_1} f(z)\,dz = \int_{\gamma_2} f(z)\,dz$

Being contractible is almost as much a property of the contour γ as a property of U.

Why? It's features of $\underline{\underline{U}}$ that make a path not contractible



Questions about warm up 7.2

→ none for now

Questions about the homework?

Problem 1 a)
b)
c)

} choose to solve 2 of them for full credit

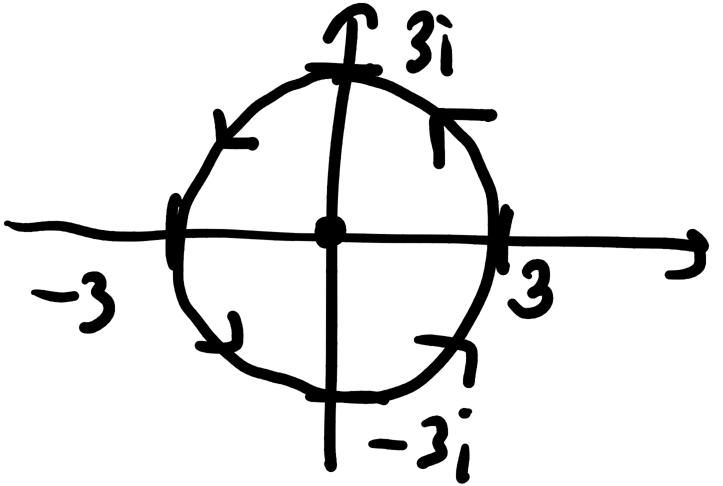
Problem 2 a)
b)
c)

} choose to solve 2

One trick for Problem 2: Partial fraction decomposition

#2b) $\int_\gamma \frac{1}{z^2-2z-8}\,dz$

γ circle of radius 3 centered at 0



$f(w)=\int_\gamma \frac{f(z)}{z-w}\,dz$

w is in interior of γ

To apply CIF (Cauchy Integral Formula) need to rewrite $\dfrac{1}{z^2-2z-8}$ so it looks like the formula

Technique is partial fraction decomposition

Idea: $$\frac{1}{z-1}\cdot\frac{z+1}{z+1} + \frac{1}{z+1}\cdot\frac{z-1}{z-1} = \frac{z+1}{(z-1)(z+1)} + \frac{z-1}{(z-1)(z+1)}$$

$$= \frac{2z}{(z-1)(z+1)}$$

$$\frac{1}{z^2-2z-8} = \frac{?}{?} + \frac{?}{?}$$

denominators are the factors of the original denominator

$$= \frac{A}{z-4} + \frac{B}{z+2}$$

$$z^2-2z-8 = z^2+2z-4z-8$$
$$= z(z+2)-4(z+2)$$
$$= (z-4)(z+2)$$

now I want the numerators and I will solve for them

sum $= -2 = 2-4$

product $= -8 = 2\cdot(-4)$

$$\frac{1}{z^2-2z-8} = \frac{A}{z-4}\cdot\frac{z+2}{z+2} + \frac{B}{z+2}\cdot\frac{z-4}{z-4}$$

to solve, just add the fractions

$$= \frac{A(z+2)}{(z-4)(z+2)} + \frac{B(z-4)}{(z+2)(z-4)}$$

Everything has the same denominator

$$\frac{1}{(z-4)(z+2)} = \frac{A(z+2)+B(z-4)}{(z-4)(z+2)}$$

2 fractions with the same denominator are equal iff their numerators are equal

What should A and B be so that

$$1 = A(z+2) + B(z-4)$$

$$1 = Az + 2A + Bz - 4B$$

$$0z + 1 = (A+B)z + (2A - 4B)$$

2 polynomials are equal iff the coefficients in front of z are the same and the constant terms are the same.

want: $0 = A + B$ # of zs

$2A - 4B = 1$ constant

Solve $\quad 0 = A + B$

$\qquad\quad 1 = 2A - 4B$

then get ....

$$\frac{1}{z^2 - 2z - 8} = \frac{A}{z-4} + \frac{B}{z+2}$$

$w = -2$

$z - (-2)$

That's all for today!