Complex Analysis

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

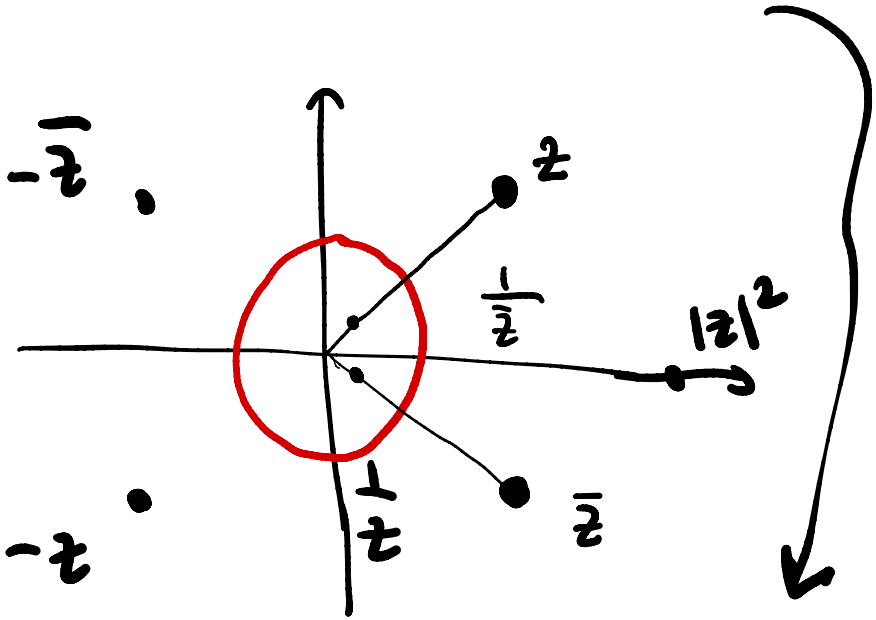
HW 1+2 graded, 3 is coming soon

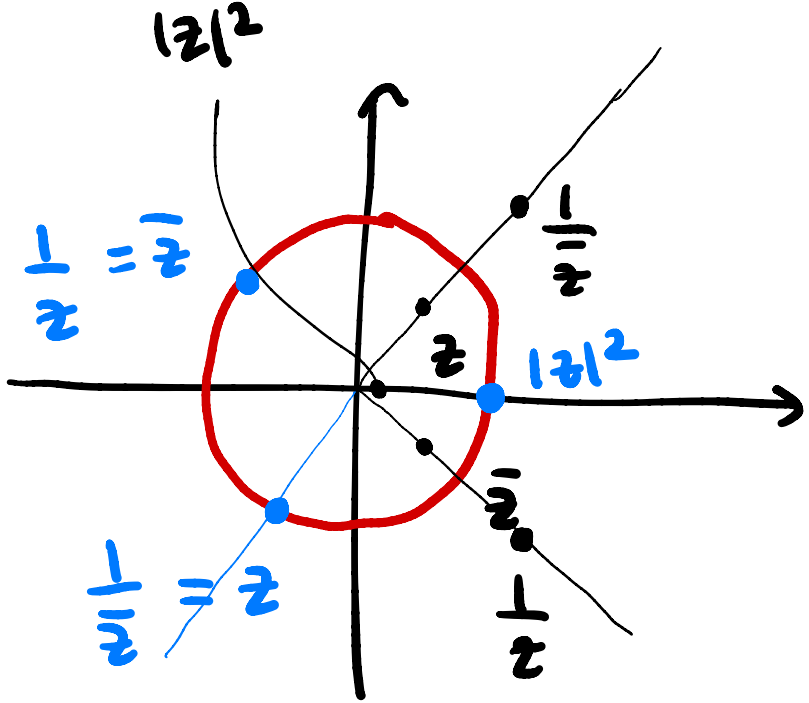
→ see your scores

Week 5 material coming today

HW 4 #4

HW4 #4 a) $z\bar{z} = |z|^2$

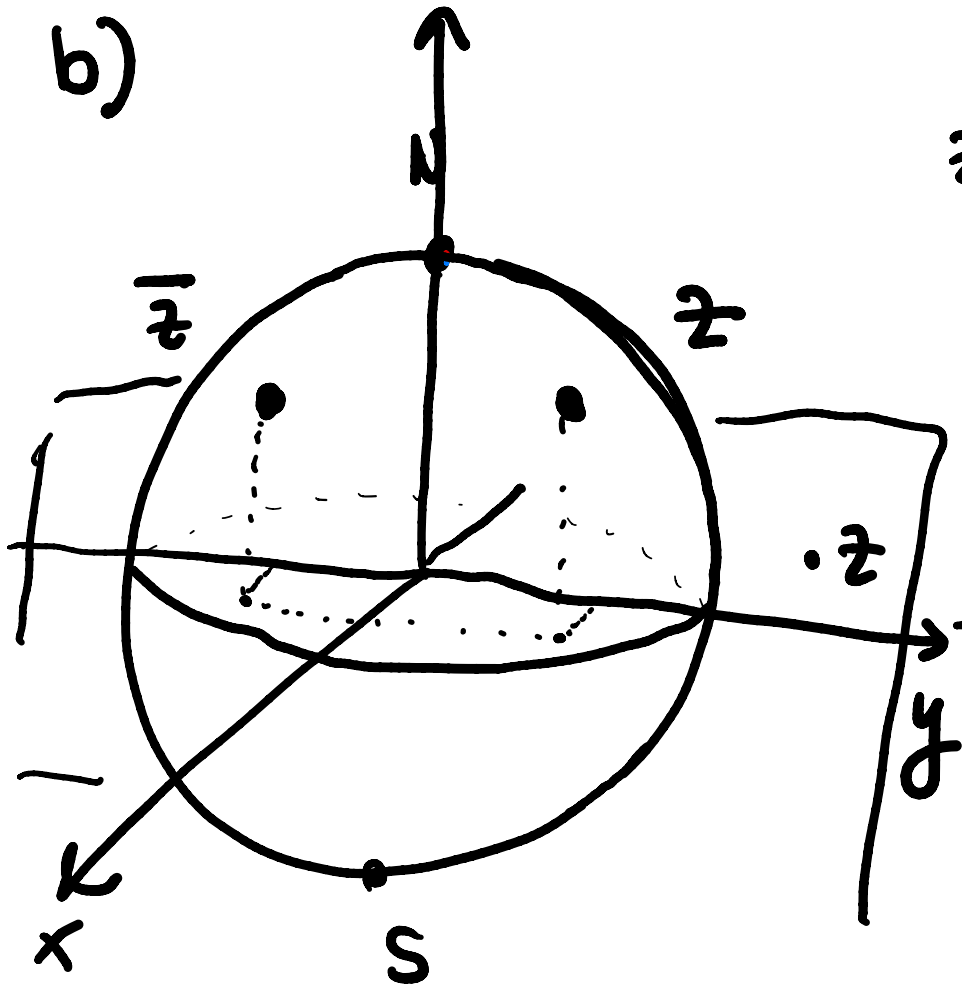




$$\frac{1}{|z|^2} \cdot z = \frac{z}{|z|^2} = \frac{1}{\bar{z}}$$

$$\overline{\left(\frac{1}{z}\right)} = \frac{1}{z}$$

b)



BMPS p. 41

$z = x + iy$ image on sphere is

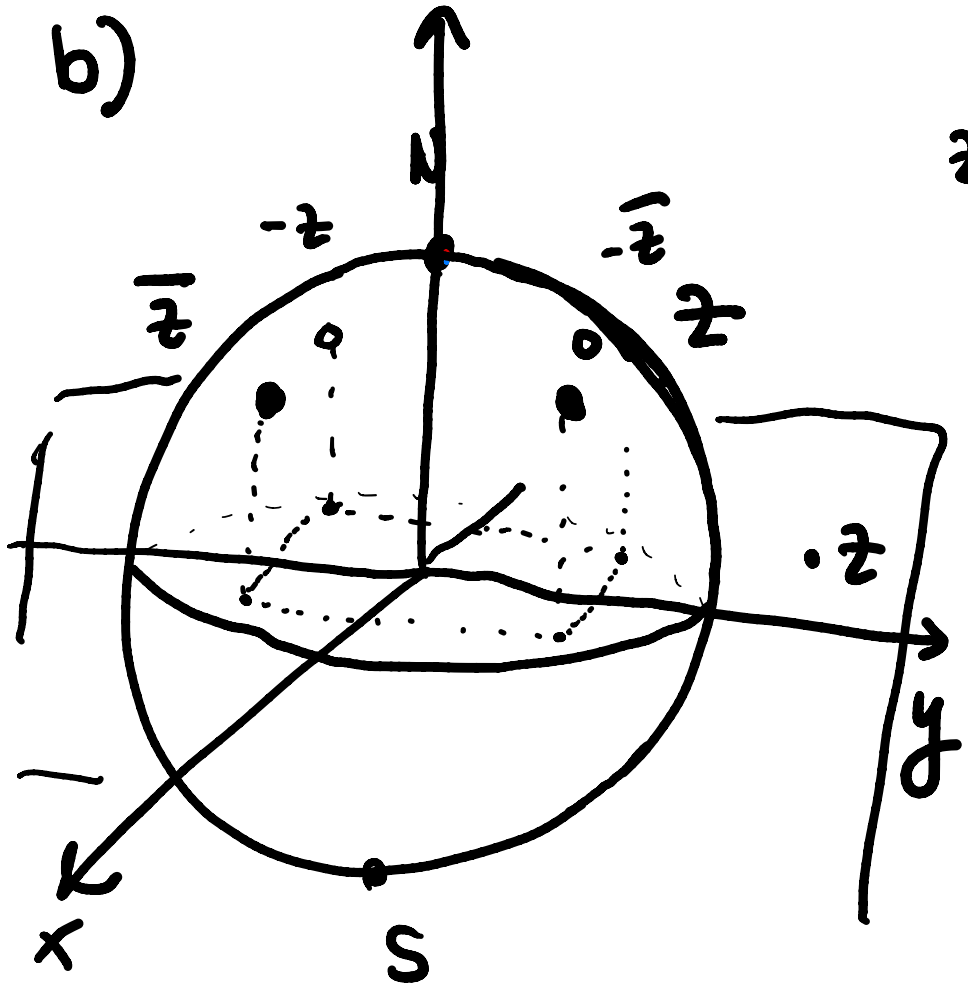
$$\left(\frac{2x}{|z|^2+1}, \frac{2y}{|z|^2+1}, \frac{|z|^2-1}{|z|^2+1} \right)$$

$\bar{z} = x - iy$

$c|z|^2$

$$\left(\frac{2x}{|\bar{z}|^2+1}, \frac{-2y}{|\bar{z}|^2+1}, \frac{|\bar{z}|^2-1}{|z|^2+1} \right)$$

b)



BMPS p. 41

$z = x + iy$ image on sphere is

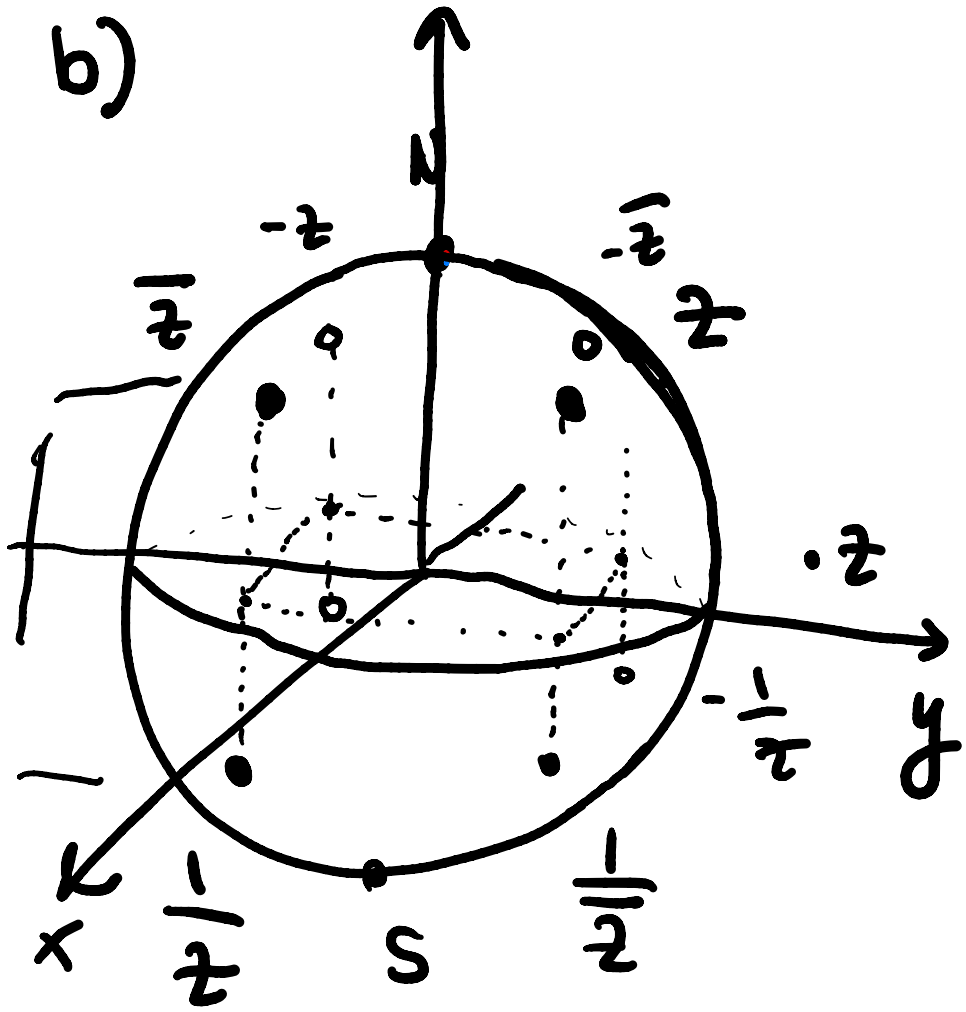
$$\left(\frac{2x}{|z|^2+1}, \frac{2y}{|z|^2+1}, \frac{|z|^2-1}{|z|^2+1} \right)$$

$-\bar{z} = -x + iy$

$$\left(\frac{-2x}{|-\bar{z}|^2+1}, \frac{2y}{|-\bar{z}|^2+1}, \frac{|-\bar{z}|^2-1}{|-\bar{z}|^2+1} \right)$$

BMPS p. 41

b)



$z = x + iy$ image on sphere is $\left(\frac{2x}{|z|^2+1}, \frac{2y}{|z|^2+1}, \frac{|z|^2-1}{|z|^2+1}\right)$

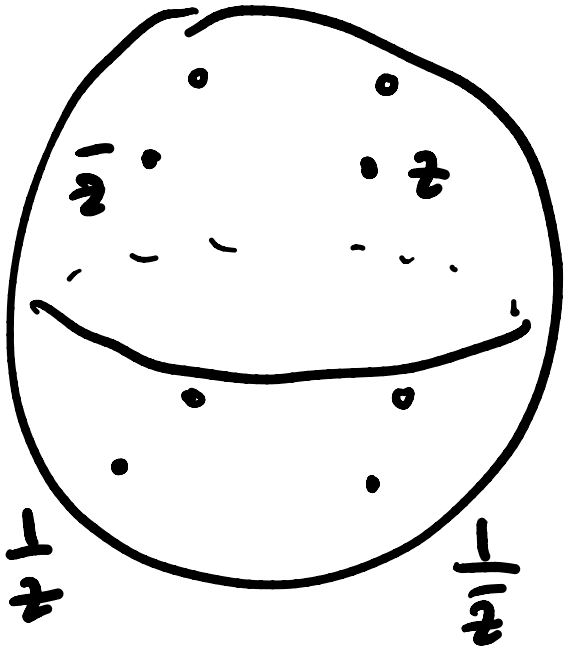
$\left|\frac{1}{z}\right| = \frac{1}{|z|}$

$\frac{1}{z} = \frac{1}{z}\cdot\frac{\bar{z}}{\bar{z}} = \frac{\bar{z}}{|z|^2} = \frac{x}{|z|^2} - i\frac{y}{|z|^2}$

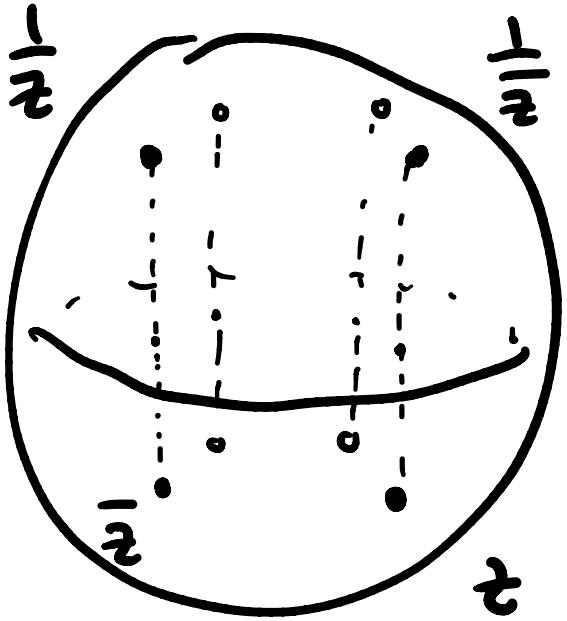
$$\left(\frac{2x}{|z|^2+1}, \frac{-2y}{|z|^2+1}, \frac{-(|z|^2-1)}{|z|^2+1}\right)$$

$$\left(\frac{2\cdot\frac{x}{|z|^2}}{\frac{1}{|z|^2}+1},\ \frac{2\frac{-y}{|z|^2}}{\frac{1}{|z|^2}+1},\ \frac{\frac{1}{|z|^2}-1}{\frac{1}{|z|^2}+1}\right)=\left(\frac{2x}{|z|^2+1},\ \frac{-2y}{|z|^2+1},\ \frac{-(|z|^2-1)}{|z|^2+1}\right)$$

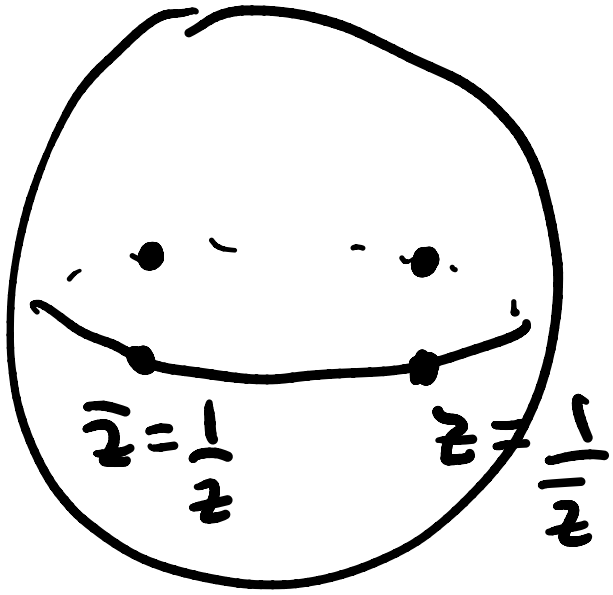
$\cdot|z|^2$ $\cdot|z|^2$ $\cdot|z|^2$ $\cdot|z|^2$ $\cdot|z|^2$ $\cdot|z|^2$


$\bar{z}$
z
$\frac{1}{z}$
$\frac{1}{\bar{z}}$

$|z|>1$


$\frac{1}{z}$
$\frac{1}{\bar{z}}$
$\bar{z}$
z

$|z|<1$


$\bar{z}=\frac{1}{z}$
$z=\frac{1}{\bar{z}}$

$|z|=1$

#3 $f(z) = \frac{iz+1}{z+i}$ sends any circle through $-i$ to a line in $\mathbb{C}$

Think about $\hat{\mathbb{C}}$

- f sends a circle to a circle on $\hat{\mathbb{C}}$
  (including circles through ∞ which are lines in $\mathbb{C}$)

- f sends $-i$ to ∞

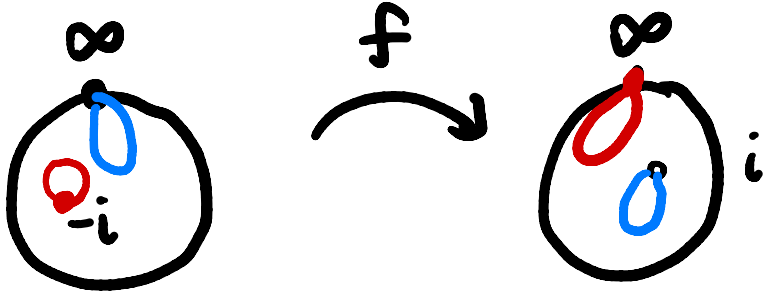
$\Rightarrow$ f sends a circle through $-i$ to a ~~circle through ∞~~ line in $\mathbb{C}$ (in $\hat{\mathbb{C}}$)

$f(z) = \dfrac{iz+1}{z+i}$ what does it do to lines?

a circle through ∞

it sends them to circles through $f(\infty)$

$$f(\infty) = \lim_{z\to\infty} \frac{iz+1}{z+i} = \lim_{z\to\infty} \frac{i+\frac{1}{z}}{1+\frac{i}{z}} = \lim_{z\to\infty} i = i$$



That's all for today!

See you on Campuswire