HW4 #5 is a grad objective

# Complex Analysis

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

# Check in

Any questions or concerns? Anything unclear?

Plan: #2
#1

Warm up 4.1 #2f)

$f(z) = 2+4i-z$ is a rotation around $1+2i$

by angle $\cancel{180°}$ $\pi$ radians

The fixed point of $f$ will be the center of rotation

$f(z) = z$

$$f(z) = z$$

$$2+4i-z = z$$

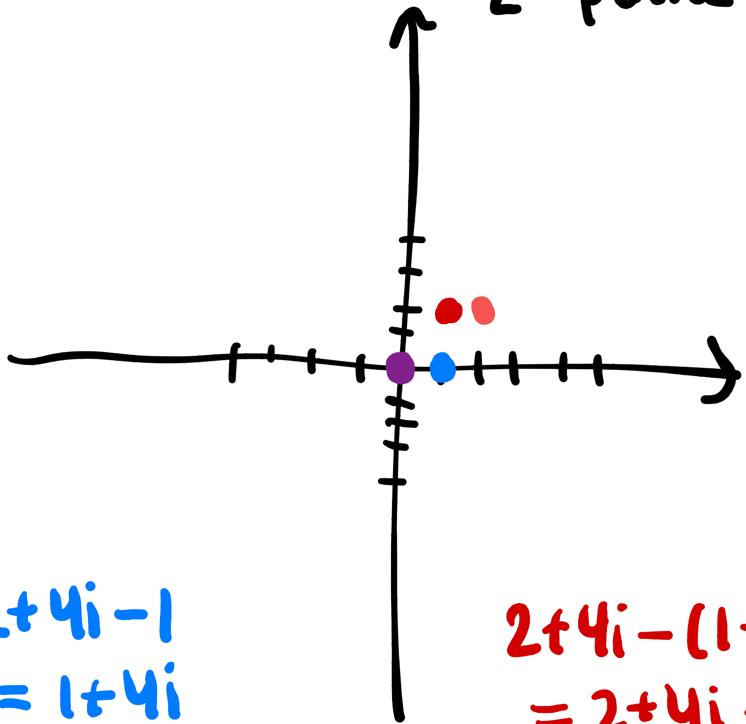
$$\frac{2+4i}{2} = \frac{2z}{2}$$

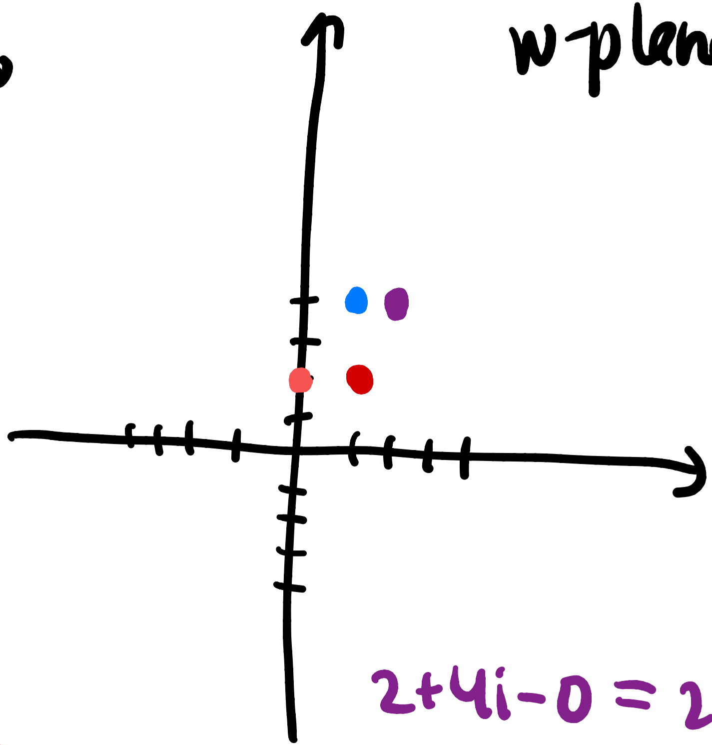
$$z = 1+2i$$

$$f(z) = 2+4i - z = w$$

$$2+4i-(2+2i) = 2+4i-2-2i = 2i$$

z-plane $\xrightarrow{f}$ w-plane





$$2+4i-1 = 1+4i$$

$$2+4i-(1+2i) = 2+4i-1-2i = 1+2i$$

$$2+4i-0 = 2+4i$$

Let's do this algebraically

- found center by solving $f(z) = z$ $\qquad z_0 = 1+2i$

- from Friday we know that a rotation around $z_0$ looks like $\quad z_0 + r(z - z_0)$

can I write $f(z) = 2+4i - z \overset{?}{=} \underbrace{(1+2i)} + \underbrace{r(z-(1+2i))}$

$$= \underbrace{1+2i} + \underbrace{1+2i-z}$$

$$= 1+2i - (z-(1+2i))$$

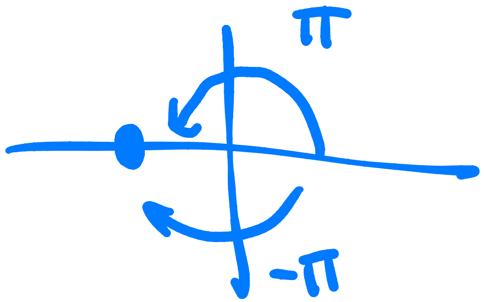
Because $f(z) = 2+4i - z = (1+2i) + (-1)(z-(1+2i))$

$f$ is a rotation by "$-1$" around $1+2i$

no scaling since $|-1| = 1$

rotation by $\arg(-1) = \pi$

$-1 = 1 \cdot e^{i\pi}$



#2 b) c) f) very similar

a) translation

e) scaling

↳ tricky?

d) rotation + scaling because

$|r| \neq 1$

$z_0 + r(z - z_0)$

This week

$$f(z) = \frac{az+b}{cz+d}$$

"fractional linear transformation"

if $ad-bc \neq 0$

#1 invertible if

$ad-bc=0$

aka Möbius

Mechanics

#1a) literally just check

$$\frac{a\left(\frac{dz-b}{-cz+a}\right)+b}{c\left(\frac{dz-b}{-cz+a}\right)+d} = z$$

&

$$\frac{d\left(\frac{az+b}{cz+d}\right)-b}{-c\left(\frac{az+b}{cz+d}\right)+a} = z$$

$f \circ f^{-1} = id$

$f^{-1} \circ f = id$

# INVERSES AND BIJECTIONS

A function is ~~invertible~~ a bijection if and only if it has a two-sided inverse.

$f: A \to B$

$$\left.\begin{aligned} f \circ g &= id \\ h \circ f &= id \end{aligned}\right\} g = h = f^{-1}$$

bijection:

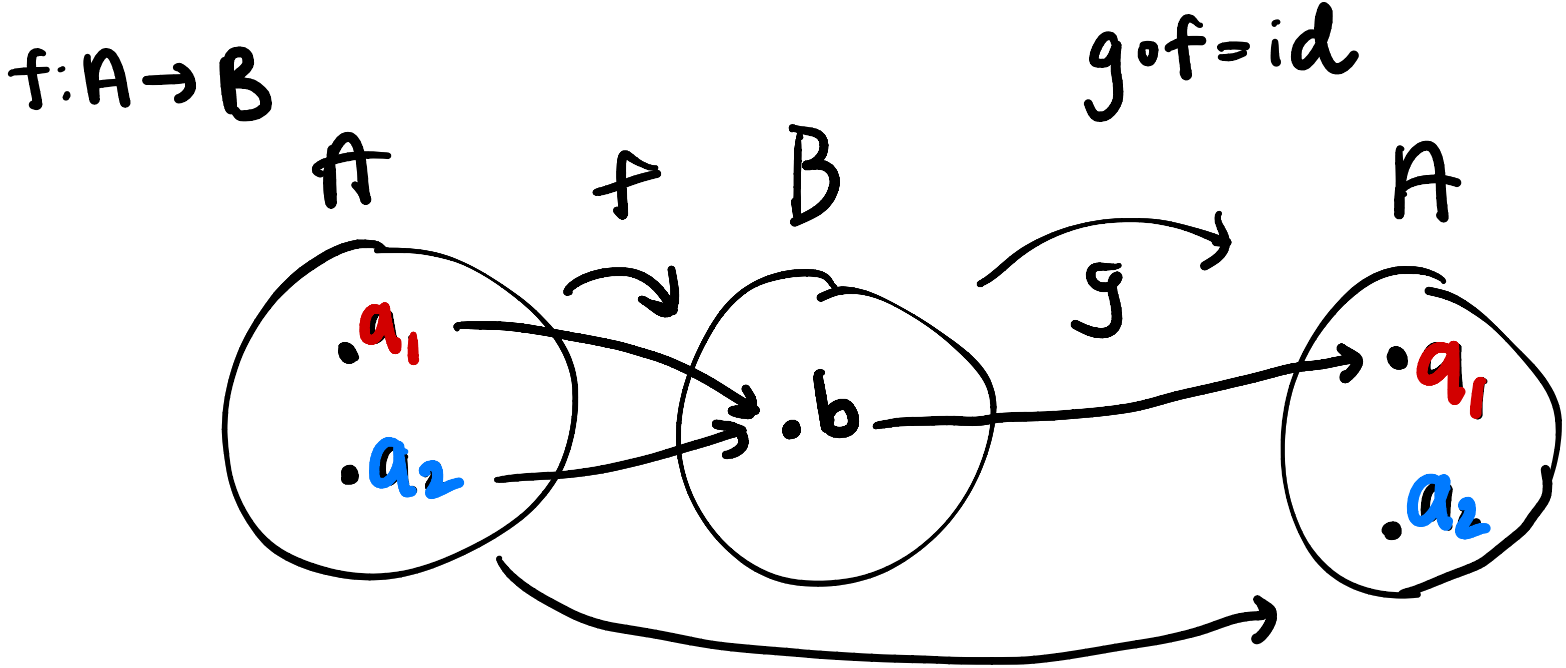
onto

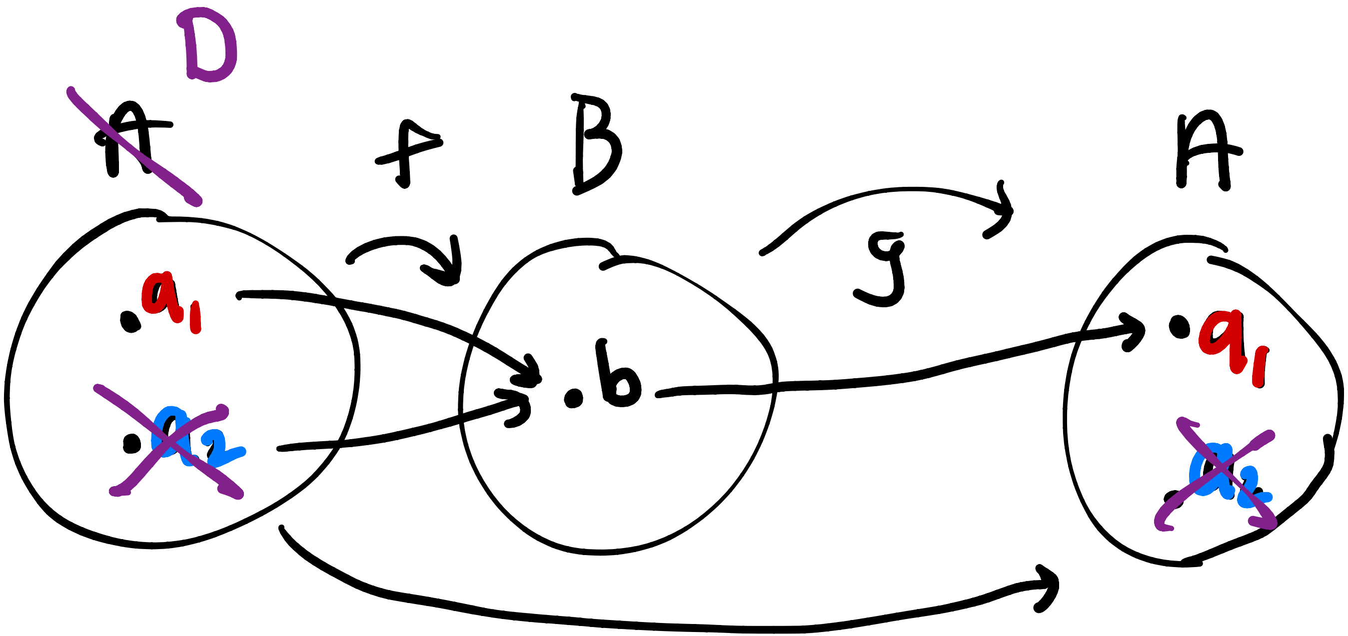
- surjective: $\forall b \in B \ \exists a \in A$ with $f(a) = b$
- injective: if $a_1 \neq a_2 \in A$ then $f(a_1) \neq f(a_2) \in B$

one-to-one

# INJECTIVITY

f is injective if and only if it has a left inverse.





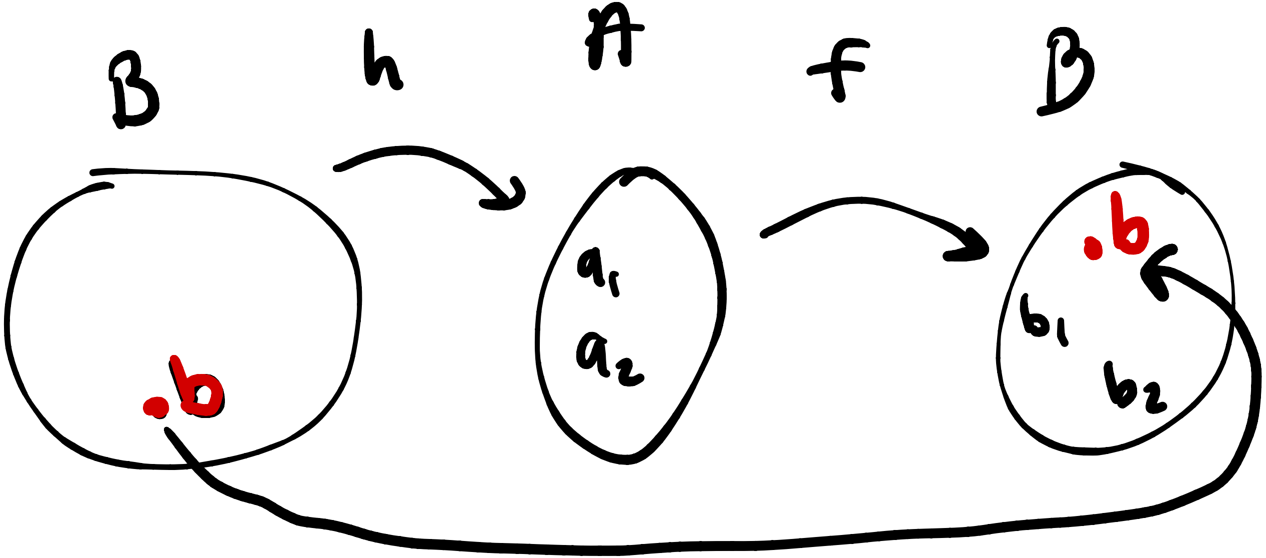
D
A
f
B
g
A
a1
a2
b
a1
a2

# SURJECTIVITY

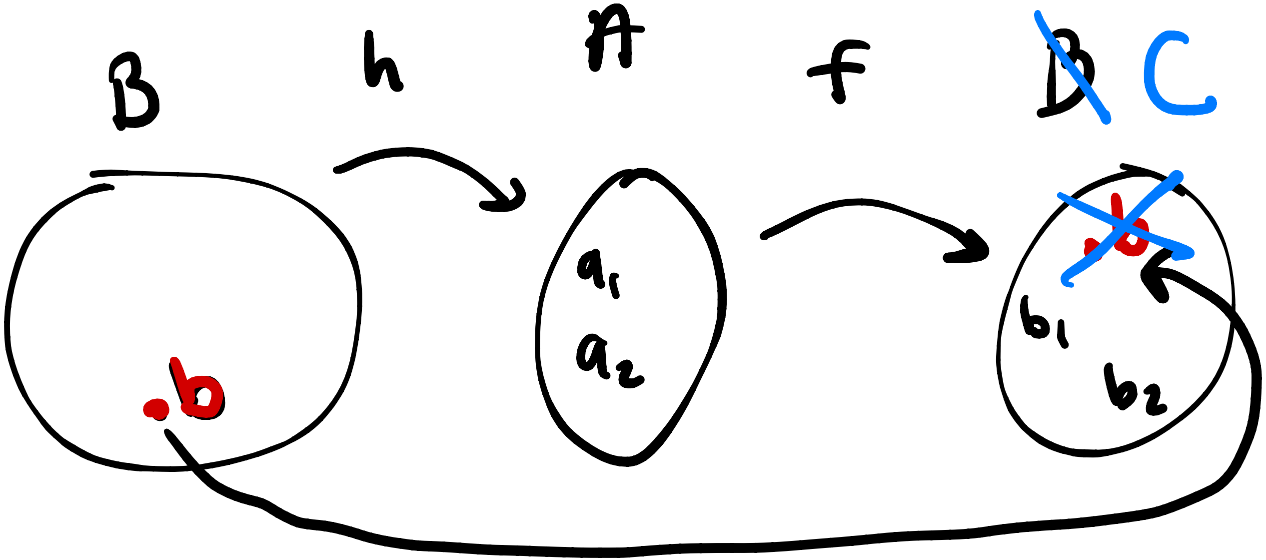
f is surjective if and only if it has a right inverse.

$f: A \to B$

$f \circ h = id$



# Make f surjective



# "Make" a function bijective

If f is not bijective, but we want an inverse, we can:

take out what isn't in the image

1. Restrict the codomain so the function is surjective.
2. Restrict the domain so the function is injective.

$$f(z) = \frac{az+b}{cz+d}$$

b) domain: allowable inputs

- division by 0
- ~~$\sqrt{\ }$ of a negative number~~
- ln of a negative number

is complicated

When is $cz+d=0$

when $cz=-d$

$$z=-\frac{d}{c}$$

$$\mathbb{C}-\{-d/c\}$$

c) $f^{-1}(z) = \dfrac{dz-b}{-cz+a}$

same thing, no division by 0

avoid $-cz+a=0$

$-cz=-a$

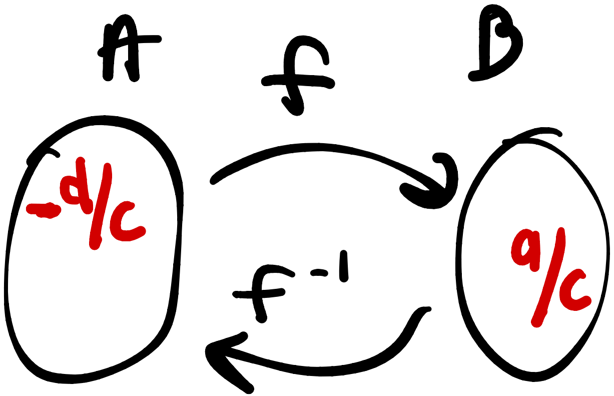
$z=a/c$

$\mathbb{C}-\left\{\frac{a}{c}\right\}$

d) $f(z)=\dfrac{az+b}{cz+d}$ $\qquad f: \mathbb{C}-\{-d/c\} \to \mathbb{C}-\left\{\dfrac{a}{c}\right\}$

$c \neq 0$

$f^{-1}(z)=\dfrac{dz-b}{-cz+a}$ $\qquad f^{-1}: \mathbb{C}-\left\{\dfrac{a}{c}\right\} \to \mathbb{C}-\left\{\dfrac{-d}{c}\right\}$



When you have a bijection

domain of $f^{-1}$ is the image of $f$

A bijection between two sets

$$A \longrightarrow B$$

$A$ is the domain of $f$

$B$ is the image of $f$ $=$ domain of $f^{-1}$

if $c=0$ $f(z)=\dfrac{az+b}{d}$ domain $\mathbb{C}$

$ad-bc\neq0$ so if $c=0$ $d\neq0$

$$f^{-1}(z)=\frac{dz-b}{a} \quad \text{domain } \mathbb{C}$$

# Inverses and derivatives

Proposition 2.12 of BMPS:

If f : A -> B is a bijection with inverse g, and

1. f is differentiable at z_0
2. f'(g(z_0)) is nonzero
3. g is continuous at z_0

then g is differentiable at z_0 with derivative

$$g'(z_0) = \frac{1}{f'(g(z_0))}.$$

THAT'S ALL FOR TODAY!

See you on Campuswire!