Complex Analysis

This lecture will be recorded. If you do not want your face in the recording, please turn off your camera. If you do not want your voice in the recording, please participate using the chat.

Check in

Any questions or concerns? Anything unclear?

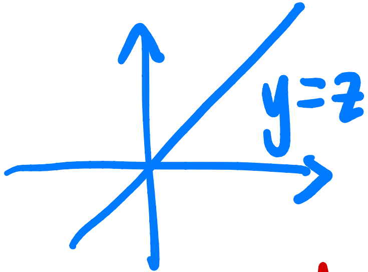
Warm up 3.1 #2

Fact $\boxed{\begin{array}{l} f: \mathbb{C} \to \mathbb{C} \\ \quad z \mapsto z \end{array}}$ is entire

entire → holomorphic on all of $\mathbb{C}$

given by

$f(z) = z$



name of function → $f: \mathbb{C} \longrightarrow \mathbb{C}$

$\mathbb{C}$ ← domain (input values)

$\mathbb{C}$ ← codomain (output values) not the range!

f is holomorphic means that f is differentiable on an open set

$f: \mathbb{C} \to \mathbb{C}$, $z \mapsto \frac{1}{z}$ not holomor...

$g: \mathbb{C} - \{0\} \to \mathbb{C}$, $z \mapsto \frac{1}{z}$ is holomorphic

$f: \mathbb{C} \to \mathbb{C}$ is entire.
$z \mapsto z$

$f'(z_0)$ exists $\forall z_0 \in \mathbb{C}$ because

Show that every polynomial function is entire

$$f'(z_0) = \lim_{h \to 0} \frac{f(z_0+h) - f(z_0)}{h}$$

$$= \lim_{h \to 0} \frac{(z_0+h) - z_0}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= \lim_{h \to 0} 1 = 1$$

$P: \mathbb{C} \to \mathbb{C}$ polynomial function

$$z \mapsto \sum_{i=0}^{n} a_i z^i = a_0 + a_1 z + \dots + a_n z^n \qquad a_i \in \mathbb{C}$$

Derivative rules:

- if f is differentiable at z_0 then $c \cdot f$ is differentiable at z_0
- a constant function is entire $\quad \lim_{h \to 0} \frac{c - c}{h} = 0$
- product of 2 differentiable functions is differentiable
- sum of " " " "

$f: \mathbb{C} \to \mathbb{C}$ is entire (given)
$z \mapsto z$

$p: \mathbb{C} \to \mathbb{C}$ is entire $\leftarrow$ show this
$z \mapsto \sum_{i=0}^{n} a_i z^i$

proof goes: $g: \mathbb{C} \to \mathbb{C}$ is entire since constant
$z \mapsto a_0$

f is entire so diff @ z_0
so $a_i z$ diff @ z_0

$h: \mathbb{C} \to \mathbb{C}$ is entire since constant times entire function
$z \mapsto a_1 z$

So $g+h$ is entire i.e. $p_1 : \mathbb{C} \to \mathbb{C}$

$z \mapsto a_0 + a_1 z$

squaring is a product of entire functions

$\mathbb{C} \to \mathbb{C}$

$z \mapsto z \cdot z$

cubing is entire because product

$\mathbb{C} \to \mathbb{C}$ is entire if $n \geq 0$

$z \mapsto z^n$ $n \in \mathbb{Z}$

Then $p : \mathbb{C} \to \mathbb{C}$

$$z \mapsto \sum_{j=0}^{n} a_j z^j$$

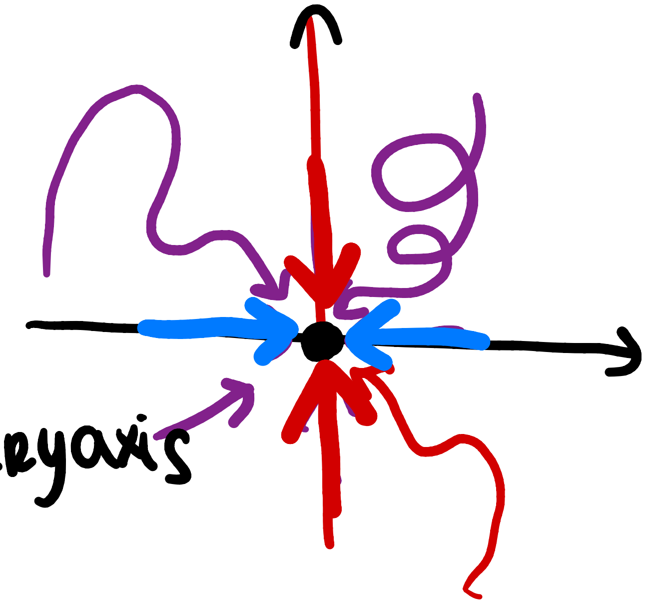
is a sum of entire functions because for each j, $z \mapsto a_j z^j$ is entire

$$a_j z^j = a_j \cdot \underbrace{z \cdot z \dots z}_{j \text{ times}}$$

(product of entire functions)

Warm up 3.1 #3

a) $\lim_{z \to 0} \frac{\bar{z}}{z}$ does not exist!



if $z \to 0$ on the y-axis/imaginary axis

$z = iy$ $\quad \bar{z} = -iy$ $\quad \lim_{z \to 0} \frac{\bar{z}}{z} = \lim_{z \to 0} \frac{-iy}{iy} = -1$

if $z \to 0$ on the real axis

$z = x$ $\quad \bar{z} = x$ $\quad \lim_{z \to 0} \frac{\bar{z}}{z} = \lim_{z \to 0} \frac{x}{x} = 1$

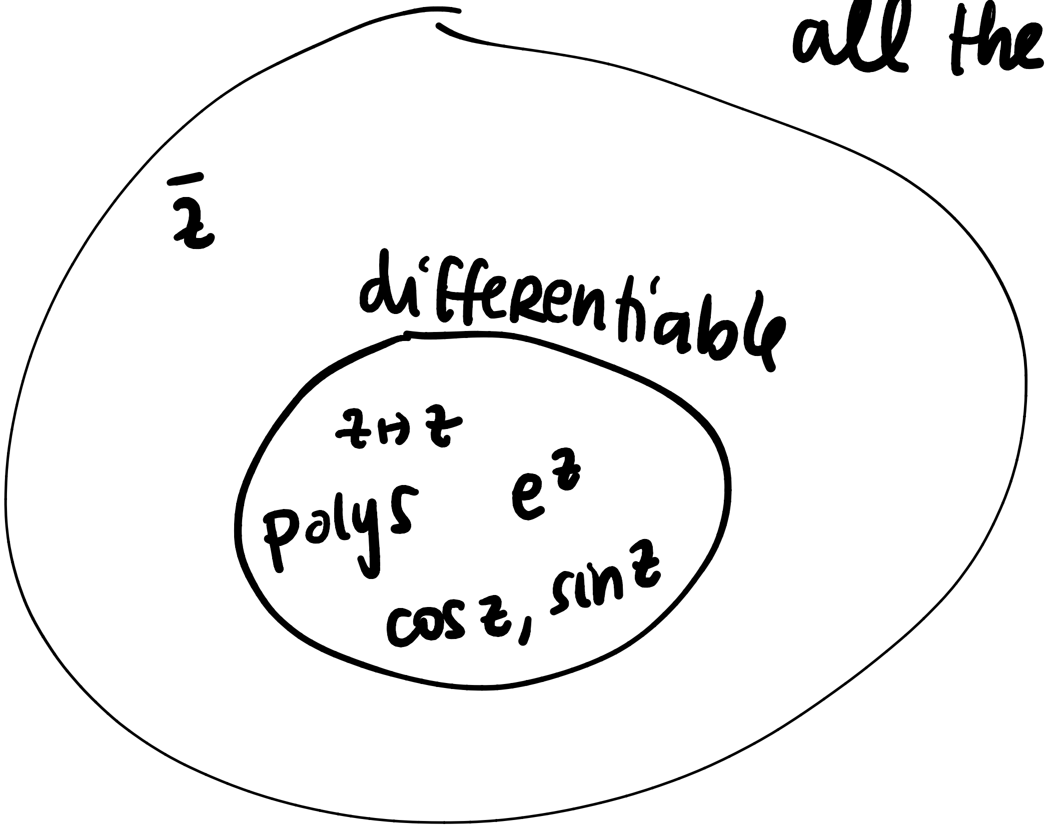
b) if $f(z) = (\bar{z})^2$ is differentiable? no

..... work out the limit doesn't exist

$f: \mathbb{C} \to \mathbb{C}$ not differentiable
$z \to \bar{z}$

c) basically any function with $\bar{z}$ in it will not be differentiable.

all the functions



$$z \mapsto \bar{z}$$
$$\begin{array}{ccc} z & \mapsto & \bar{z} \\ \shortparallel & & \shortparallel \\ x+iy & & x-iy \end{array}$$

On your mind

1. limit definition of differentiability
2. derivative rules
3. feeling about which functions are differentiable | holomorphic

yes

$\boxed{\text{polynomials}}$

no

$\boxed{\text{things with } \bar{z}}$

Quick qs 5

vs

Holomorphic / conformal 3

Holomorphic is basically conformal

Suppose that f is holomorphic in a neighborhood of a point.

$$f(z) \approx z_0 + f'(z_0)(z - z_0)$$

$$f(z) \approx z_0 + f'(z_0)(z - z_0)$$

LIGHTNING ROUND 1

The definition of holomorphic in complex analysis is essentially the same as the definition of derivative from calculus.

same limit
same derivative rules

A. True (confident)

B. True (not confident)

C. False (not confident)

D. False (confident)

consequences are not the same
holomorphic $\Rightarrow$ ∞ly differentiable
analytic

Lightning round 2

If f(z) and g(z) are entire, then f(z)g(z) is entire.

A. True (confident)
B. True (not confident)
C. False (not confident)
D. False (confident)

LIGHTNING ROUND 3

If f(z) and g(z) are entire, then f(z)/g(z) is entire.

A. True (confident)

B. True (not confident)

C. False (not confident)

D. False (confident)

in English: it depends

if $g(z) \neq 0$ then yes

if $g(z) = 0$ somewhere then

$\frac{f}{g}$ is not holomorphic

when $g = 0$

Lightning Round 4

If $f(z)$ and $g(z)$ are entire, then $if(z)-3g(z)$ is entire.

A. True (confident)
B. True (not confident)
C. False (not confident)
D. False (confident

LIGHTNING ROUND 5

If f(z) is entire, then f(1/z) is entire.

"g(z)"

then g is not defined when $z=0$

A. True (confident)

B. True (not confident)

C. False (not confident)

D. False (confident)

Lightning Round 6

If f(z) and g(z) are entire, then g(f(z)) is entire.

A. True (confident)
B. True (not confident)
C. False (not confident)
D. False (confident)

That's all for today!