Name:

Problem 1: *Consider the rational function given by the following rule:*

$$f(x) = \frac{x+1}{x^2 - x - 2}.$$

a) Does this function have a horizontal asymptote on the right*? If so, please give its equation. To receive credit for this question, you must use the steps outlined in class or a similar technique.*

The horizontal asymptote on the right is computed by considering the limit

$$\lim_{x\to\infty} \frac{x+1}{x^2 - x - 2}.$$

We first find the highest power of x that appears in the expression. For this function, the highest power of x is x^2.

Now we factor out x^2 from each term in the rational function and simplify:

$$\lim_{x\to\infty} \frac{x^2\left(\frac{x}{x^2} + \frac{1}{x^2}\right)}{x^2\left(\frac{x^2}{x^2} - \frac{x}{x^2} - \frac{2}{x^2}\right)} = \lim_{x\to\infty} \frac{\frac{1}{x} + \frac{1}{x^2}}{1 - \frac{1}{x} - \frac{2}{x^2}}.$$

Next, each term that has an x on the bottom approaches zero as $x \to \infty$, so we substitute that in:

$$\lim_{x\to\infty} \frac{0+0}{1-0-0} = \lim_{x\to\infty} \frac{0}{1} = \lim_{x\to\infty} 0 = 0$$

After we have simplified everything, we see that we end up with the number 0.

The equation of the asymptote on the right is $y = 0$.

b) What limit would you write to compute the asymptote on the left*? Please write the limit but do not compute the asymptote.*

The first step would be to write the limit

$$\lim_{x\to-\infty} \frac{x+1}{x^2 - x - 2}.$$

Notice that the difference is that x goes to $-\infty$ instead of ∞.