

Name:

Problem 1: *Simplify the following rational expression completely:*

$$\frac{3x^2 - 2x - 1}{3x^2 - 14x - 5}$$

In other words, factor the numerator and denominator and cancel as many factors as you can.

Solution: We will take the following steps:

1. We first factor the numerator.
2. Then we factor the denominator.
3. Finally, we cancel common factors.

To factor the two quadratic expressions, we use the *ac*-method, or the “splitting the middle term” method, or the “factoring by grouping” method (these are all different ways to talk about the same method).

We start first with the numerator. To factor it, we must find two numbers that add up to -2 and multiply out to -3 . Those numbers are 1 and -3 . Then we have:

$$\begin{aligned} 3x^2 - 2x - 1 &= 3x^2 + x - 3x - 1 \\ &= x(3x + 1) - 1(3x + 1) \\ &= (x - 1)(3x + 1). \end{aligned}$$

We now turn our attention to the denominator. This time, we must find two numbers that add up to -14 and multiply out to -15 . Those numbers are 1 and -15 . Then we have:

$$\begin{aligned} 3x^2 - 14x - 5 &= 3x^2 + x - 15x - 5 \\ &= x(3x + 1) - 5(3x + 1) \\ &= (x - 5)(3x + 1). \end{aligned}$$

Now, the fruit of our work tells us that

$$\frac{3x^2 - 2x - 1}{3x^2 - 14x - 5} = \frac{(x - 1)(3x + 1)}{(x - 5)(3x + 1)}$$

We see that the numerator and the denominator have the factor $3x + 1$ in common, which we can cancel. Therefore the simplified expression is

$$\frac{x - 1}{x - 5}$$