Math 019 C: Fall 2017
Exam 1

NAME: SOLUTIONS

Time: **50 minutes**

For each problem, you **must** write down all of your work carefully and legibly to receive full credit **and** use mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the UVM Code of Academic Integrity:

- You may not use any notes or book during the exam.
- You may not access your cell phone during the exam for any reason; if you think that you will want to check the time please wear a watch.
- The work you present must be your own.
- Finally, you will more generally be bound by the UVM Code of Academic Integrity, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: _____

Problem	Value	Score
1	4	
2	6	
3	9	
4	8	
5	3	
6	2	
7	2	
8	3	
9	6	
10	3	
11	4	
TOTAL	50	

Problem 1 : (4 points) For each of the following statement, decide if it is true or false. You do not need to justify your answer if you do not want to, but you are certainly welcome to write a few words to support your answer.

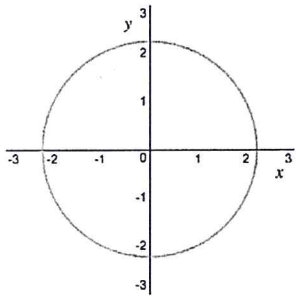
a) The notation $y = f(x)$ can be read out loud as "x is a function of y."

FALSE. It is "y is a function of x"

b) Suppose that f is an invertible function, and the point (a, b) is on the graph of f. Then the point (b, a) is on the graph of the inverse of f.

TRUE. Inverse functions reverse inputs and outputs.

c) The graph below is the graph of a function.



FALSE. It fails the Vertical Line Test.

d) Every line is a function.

FALSE, A vertical line is not a function.

Problem 2 : (6 points) For each of the following functions, give the domain. Please write your answer in interval notation.

a) $f(x) = \sqrt{5+x}$

We need $5+x \geq 0$

$x \geq -5$

$[-5, \infty)$

b) $f(x) = \log_2(-1-x)$

We need $-1-x > 0$

$-1 > x$

$(-\infty, -1)$

c) $f(x) = \dfrac{x^2+x+1}{2}$

There is no value of x that makes the denominator zero.

$(-\infty, \infty)$

Problem 3 : (9 points) Throughout this problem, consider the line passing through the points $(-2,-3)$ and $(2,-1)$.

a) (1 point) What is the slope of this line? If it is undefined, please answer "undefined."

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-(-3)}{2-(-2)} = \frac{-1+3}{2+2} = \frac{2}{4} = \frac{1}{2}$$

The slope is $\frac{1}{2}$

b) (2 points) Give an equation for this line.

Find b;

$y = mx + b$

$-3 = \frac{1}{2}(-2) + b$

$-3 = -1 + b$

$-2 = b$

The equation is

$y = \frac{1}{2}x - 2$

c) (2 points) Does this line have one or more x-intercepts? If so please give their coordinates.

$y = 0$

$0 = \frac{1}{2}x - 2$

$2 = \frac{1}{2}x$

$4 = x$

yes $(4,0)$

Recall that throughout this problem, we consider the line passing through the points $(-2, -3)$ and $(2, -1)$.

d) (2 points) Does this line have one or more y-intercepts? If so please give their coordinates.

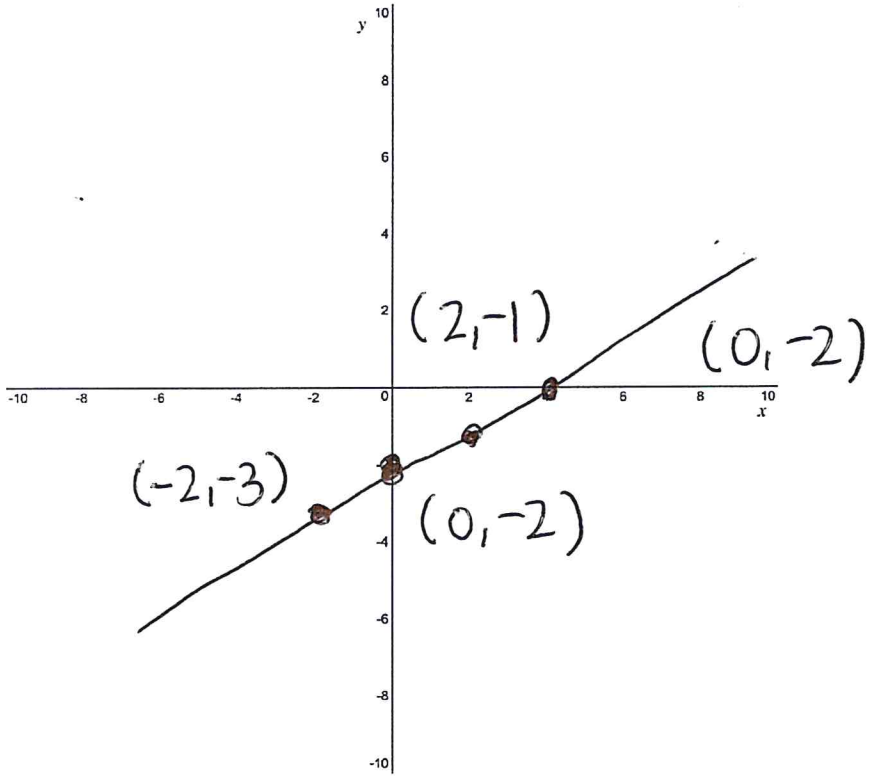
$x = 0$

$y = \frac{1}{2} \cdot 0 - 2$

$y = -2$

yes $(0, -2)$

e) (2 points) On the axes provided below, please sketch the graph of this line. On your graph, make sure to label the two points given above and the x-intercepts and y-intercepts (if any).



Problem 4 : (8 points) Throughout this question we will consider the following function:

$$f(x) = \frac{x^2 + 3x - 10}{x + 5}.$$

a) (2 points) What is the domain of this function? Please write your answer in interval notation.

we must avoid $x+5=0$

$x=-5$

$(-\infty, -5) \cup (-5, \infty)$

b) (1 point) Does this function have one or more vertical asymptotes? If so please give their equations.

$x^2+3x-10 = x^2+5x-2x-10 = x(x+5)-2(x+5)$

$= (x+5)(x-2)$

sum $= +3 = 5-2$

product $= -10$

$f(x) = \frac{(x+5)(x-2)}{x+5}$

No vertical asymptote

c) (1 point) Does this function have one or more holes? If so please give the x-coordinate of each hole.

yes, there is a hole at $x=-5$

Recall that throughout this question, we consider the following function:

$$f(x) = \frac{x^2 + 3x - 10}{x + 5}.$$

d) (2 points) Does this function have a horizontal asymptote on the right? If so please give its equation.

$$\lim_{x\to\infty} \frac{x^2+3x-10}{x+5} = \lim_{x\to\infty} \frac{\frac{x^2}{x^2} + \frac{3x}{x^2} - \frac{10}{x^2}}{\frac{x}{x^2} + \frac{5}{x^2}}$$

$$= \lim_{x\to\infty} \frac{1 + \frac{3}{x} - \frac{10}{x^2}}{\frac{1}{x} + \frac{5}{x^2}} = \frac{1+0-0}{0+0} = \frac{1}{0}$$

This is undefined so there is no horizontal asymptote on the right.

e) (2 points) Does this function have a horizontal asymptote on the left? If so please give its equation.

We do the same thing but with x approaching $-\infty$ instead of ∞:

$$\lim_{x\to-\infty} \frac{x^2+3x-10}{x+5} = \lim_{x\to-\infty} \frac{1 + \frac{3}{x} - \frac{10}{x^2}}{\frac{1}{x} + \frac{5}{x^2}} = \frac{1}{0}$$

(the steps in between are the same as in part d)

There is also no horizontal asymptote on the left.

Problem 5 : (3 points) Throughout this question, let

$$f(x) = -5x + 2$$

Find each of the following values. Simplify your answer completely.

a) $f(-2)$ $= -5(-2)+2 = 10+2 = 12$

b) $\dfrac{f(x+h)-f(x)}{h}$

$$= \frac{-5(x+h)+2-(-5x+2)}{h}$$

$$= \frac{-5x-5h+2+5x-2}{h}$$

$$= \frac{-5h}{h} = -5$$

Problem 6 : (2 points) Write this exponential expression as a logarithmic expression:

$$27^{1/3} = 3.$$

$\log_{27} 3 = \frac{1}{3}$

Problem 7 : (2 points) Write this logarithmic expression as an exponential expression:

$$\log_{16}\left(\frac{1}{4}\right) = -\frac{1}{2}.$$

$16^{-1/2} = \frac{1}{4}$

Problem 8 : (3 points) Simplify the following logarithmic expression completely:

$$\log_2\left(\frac{7\sqrt[4]{x^3}}{3y^2}\right)$$

$\log_2\left(\frac{7x^{3/4}}{3y^2}\right) = \log_2 7 + \log_2 x^{3/4} - \log_2 3 - \log_2 y^2$

$= \log_2 7 + \frac{3}{4}\log_2 x - \log_2 3 - 2\log_2 y$

Problem 9 : (6 points) Solve the following exponential and logarithmic equations.

a) $10^{x^2+x} = 1$

$10^{x^2+x} = 10^0$

$\log_{10}(10^{x^2+x}) = \log_{10}(10^0)$

$x^2 + x = 0$

$x(x+1) = 0$

$x = 0$ OR $x+1 = 0$

$x = -1$

$\boxed{x=0,\ x=-1}$

b) $\log_2 x + \log_2(x-1) = 1$

$\log_2(x(x-1)) = 1$

$2^{\log_2(x(x-1))} = 2^1$

$x(x-1) = 2$

$x^2 - x - 2 = 0$

sum $= -1 = 1 - 2$

product $= -2$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x+1)(x-2) = 0$

so $x+1=0$ OR $x-2=0$

$x = -1$ $\quad$ $x = 2$

We reject $x=-1$ since $\log_2(-1)$ is not defined.

We keep $x=2$ since $\log_2 2$ and $\log_2 1$ are both defined

$\boxed{x=2}$

Problem 10 : (3 points) Do the points $(-3,1)$, $(0,0)$ and $(2,-2)$ lie on a line?

We check if we get the same slope

First slope: $(x_1,y_1)=(-3,1)$ $(x_2,y_2)=(0,0)$

$$m=\frac{0-1}{0-(-3)}=\frac{-1}{3}$$

Second slope $(x_1,y_1)=(0,0)$ $(x_2,y_2)=(2,-2)$

$$m=\frac{-2-0}{2-0}=\frac{-2}{2}=-1$$

Since the slope is not constant, these points do not lie on a line.

Problem 11 : (4 points) Suppose that $\log_b 2 = a$ and $\log_b 3 = c$. Use the properties of logarithms to compute the value of

$$\log_b 64.$$

We have that $64=2\cdot 32=2\cdot(2\cdot 16)=2^2\cdot(2\cdot 8)$
$=2^3\cdot(2\cdot 4)=2^4\cdot(2\cdot 2)=2^6$

so $\log_b 64=\log_b 2^6=6\log_b 2=6a$