Math 019 C: Fall 2017
Exam 1 Practice

NAME:

SOLUTIONS

Time: **50 minutes**

For each problem, you **must** write down all of your work carefully and legibly to receive full credit **and** use mathematical reasoning to support your answer, as appropriate.

Failure to follow these instructions will constitute a breach of the UVM Code of Academic Integrity:

- You may not use any notes or book during the exam.
- You may not access your cell phone during the exam for any reason; if you think that you will want to check the time please wear a watch.
- The work you present must be your own.
- Finally, you will more generally be bound by the UVM Code of Academic Integrity, which stipulates among other things that you may not communicate with anyone other than the instructor during the exam, or look at anyone else's solutions.

I understand and accept these instructions.

Signature: ______________________________

Problem	Value	Score
1	2	
2	3	
3	4	
4	4	
5	3	
6	4	
7	9	
8	2	
9	2	
10	4	
11	8	
12	5	
TOTAL	50	

Problem 1 : (2 points) In each part of this problem, a relation between u and v is given by a table or a graph. For each relation, answer "yes" if u is a function of v and "no" if u is not a function of v. You do not need to justify your answer if you do not want to, but you are certainly welcome to write a few words to support your answer.

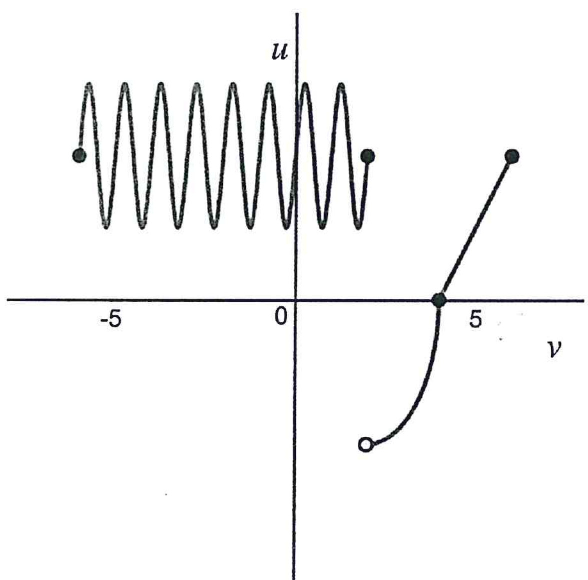
input is v
output is u

a)

u	2	5	3	-2	2
v	3	-3	1	0	6

Each input (v) has a single output, so yes.

b)



This graph passes the Vertical Line Test (even at x=2!) so yes.

Problem 2 : (3 points) Throughout this question, let

$$f(x) = x^2 - x - 6.$$

Find each of the following values. Simplify your answer completely.

a) $f(-2)$ $= (-2)^2 - (-2) - 6 = 4 + 2 - 6 = 0$

b) $f(b)$ $= b^2 - b - 6$

c) $\dfrac{f(x+h) - f(x)}{h}$

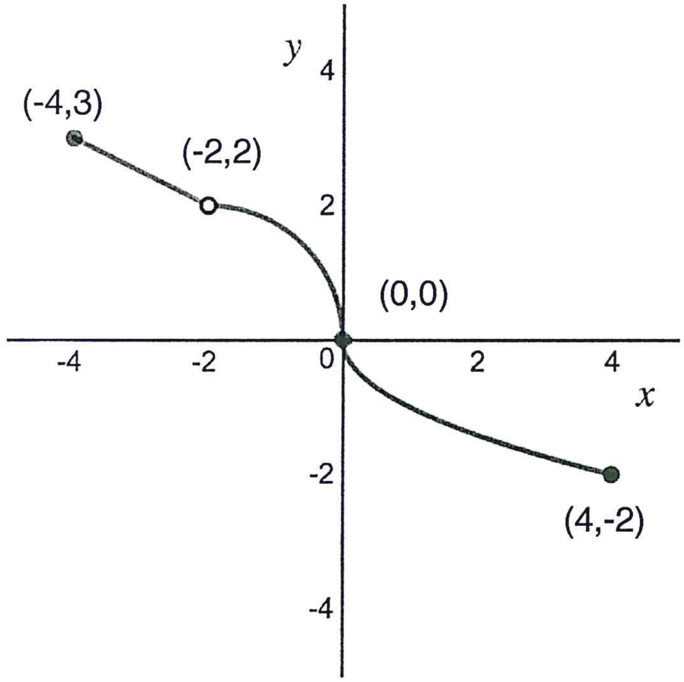
$$= \frac{[(x+h)^2 - (x+h) - 6] - [x^2 - x - 6]}{h}$$

$$= \frac{x^2 + 2xh + h^2 - x - h - 6 + x^2 + x + 6}{h}$$

$$= \frac{2xh + h^2 - h}{h} = \frac{h(2x + h - 1)}{h}$$

$$= 2x + h - 1$$

Problem 3 : **(4 points)** Throughout this question, consider the function given by the following graph:



a) Give the domain of this function. Please write your answer in interval notation.

$[-4,-2) \cup (-2,4]$

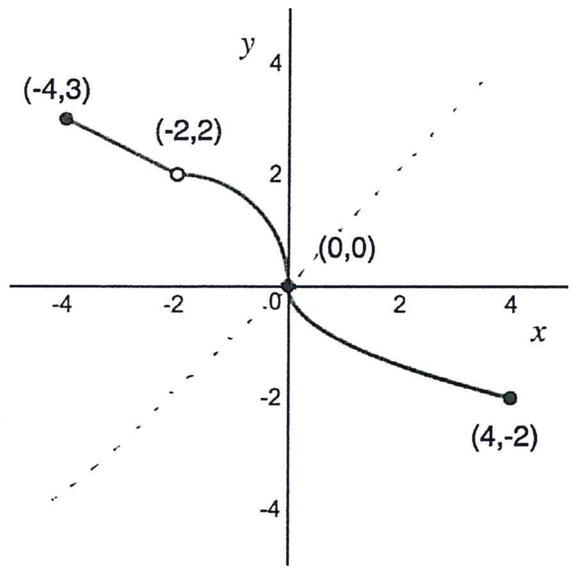
b) Give the range of this function. Please write your answer in interval notation.

$[-2,2) \cup (2,3]$

c) This function is invertible. Explain using **one sentence** why that is the case. (Hint: You should be able to give a complete answer using only seven words.)

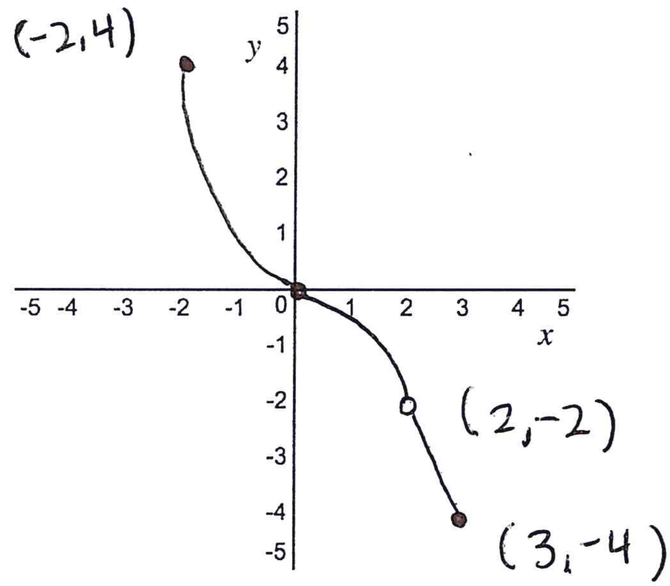
The graph passes the Horizontal Line Test.

Recall that throughout this question, we consider the function given by the following graph:



Reflect the graph about the line x=y to flip x and y.

d) On the axes below, please sketch the graph of the inverse of this function.



Problem 4 : (4 points) For each of the following functions, give the domain. Please write your answer in interval notation.

a) $f(x) = \dfrac{x^2 + x + 1}{6x^2 + x - 1}$

denominator $\neq 0$

sum $= 1 = 3 - 2$
product $= -6$

$6x^2 + x - 1 = 6x^2 + 3x - 2x - 1$
$= 3x(2x+1) - (2x+1)$
$= (2x+1)(3x-1)$

$\boxed{(-\infty, -\tfrac{1}{2}) \cup (-\tfrac{1}{2}, \tfrac{1}{3}) \cup (\tfrac{1}{3}, \infty)}$

so $2x + 1 = 0$
$2x = -1$
$x = -\frac{1}{2}$

$3x - 1 = 0$
$3x = 1$
$x = \frac{1}{3}$

b) $f(x) = \sqrt[4]{5 - 3x}$

under the (even) root must be ≥ 0

$5 - 3x \geq 0$
$-3x \geq -5$
$x \leq \frac{5}{3}$

$\boxed{(-\infty, \tfrac{5}{3}]}$

c) $f(x) = \log(x + 3)$

inside the log must be > 0

$x + 3 > 0$

$x > -3$

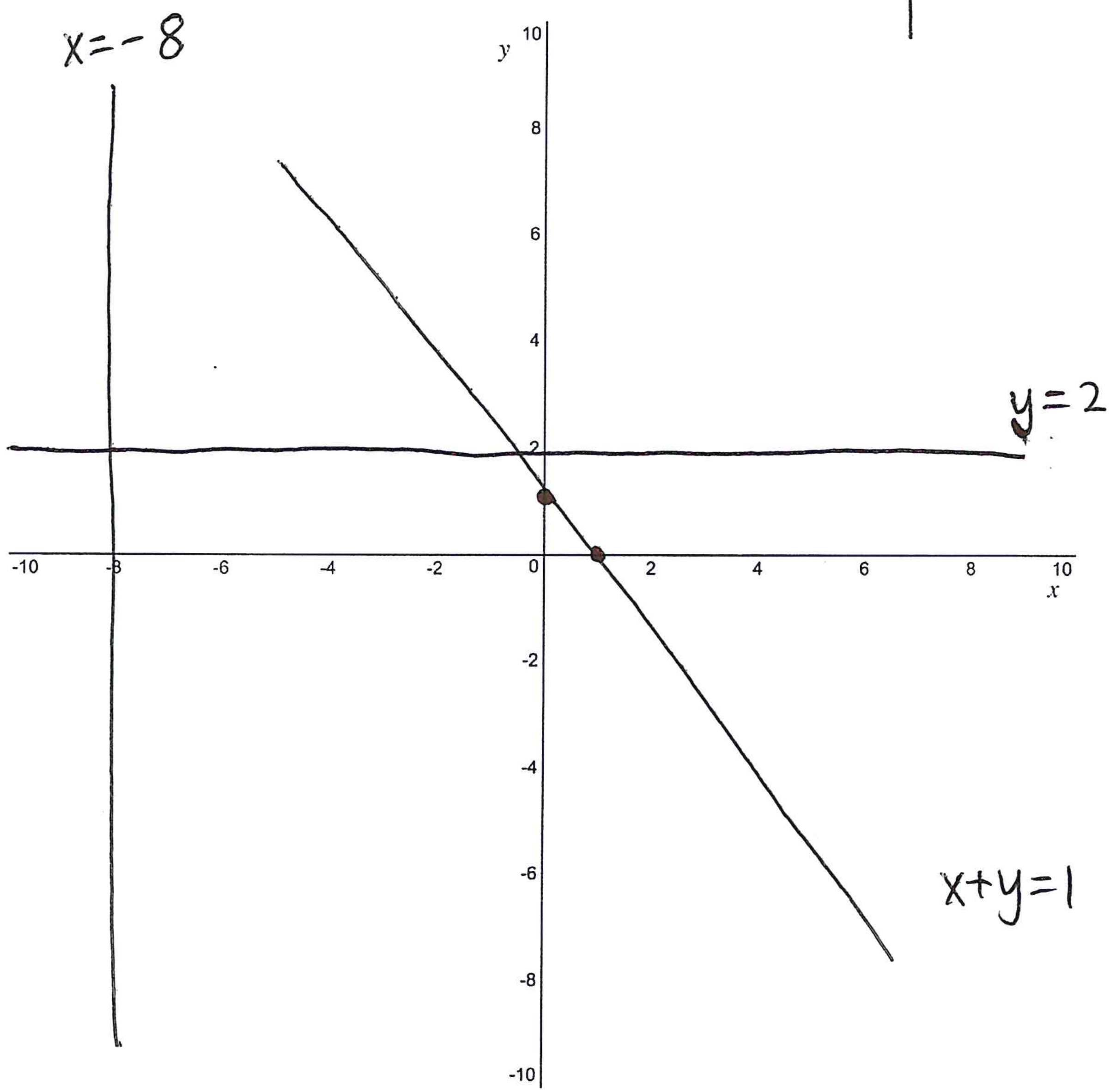
$\boxed{(-3, \infty)}$

Problem 5 : (3 points) On the axes below, graph each of the following lines:

$$x = -8$$
$$y = 2$$
$$x + y = 1$$

x	y
0	1
1	0

Label each line with its equation.



Problem 6 : (4 points) Throughout this problem, consider the line passing through the points $(-1, 1)$ and $(2, 0)$.

a) Give an equation for this line.

First find the slope $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{2-(-1)} = \frac{-1}{3}$

Now find the y-intercept:

$y = mx + b$

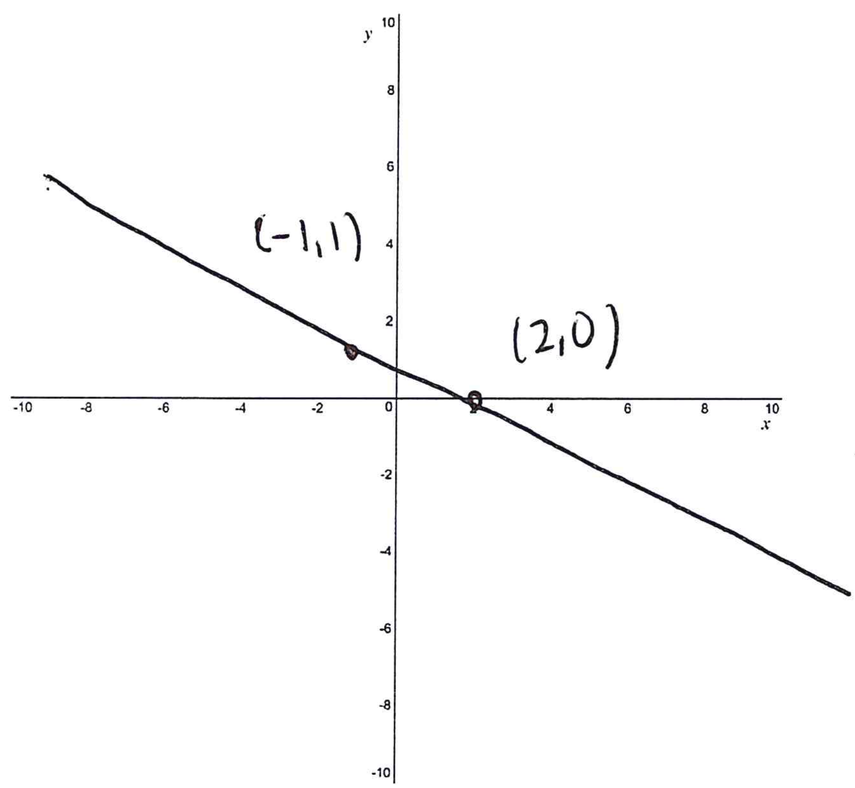
$0 = \frac{-1}{3} \cdot 2 + b$

$0 = -\frac{2}{3} + b$

$b = \frac{2}{3}$

$$\boxed{y = -\frac{1}{3}x + \frac{2}{3}}$$

b) On the axes provided below, please sketch the graph of this line. On your graph, make sure to label the two points given above.



Problem 7 : (9 points) Throughout this question we will consider the following function:

$$f(x) = \frac{x-3}{x^2 - x - 6}.$$

a) What is the domain of this function? Please write your answer in interval notation.

sum=-1
product=-6
2-3=-1

denominator is not zero

$\boxed{(-\infty,-2)\cup(-2,3)\cup(3,\infty)}$

$x^2-x-6=0$

$x^2-3x+2x-6=0$

$x(x-3)+2(x-3)=0$

$(x-3)(x+2)=0$

$x-3=0 \quad x+2=0$

$x=3 \quad x=-2$

b) Does this function have one or more x-intercepts? If so please give their coordinates.

$y=0$

$0=\dfrac{x-3}{x^2-x-6}$

$x=3$

$\boxed{\text{yes, } (3,0)}$

$0=x-3$

c) Does this function have one or more y-intercepts? If so please give their coordinates.

$x=0$

$f(0)=\dfrac{0-3}{0^2-0-6}=\dfrac{-3}{-6}=\dfrac{1}{2}$

$\boxed{\text{yes } (0, \tfrac{1}{2})}$

d) Does this function have one or more vertical asymptotes? If so please give their equations.

$\dfrac{x-3}{x^2-x-6}=\dfrac{\cancel{x-3}}{\cancel{(x-3)}(x+2)}$

$\boxed{\text{yes } x=-2}$

Recall throughout this question, we consider the following function:

$$f(x) = \frac{x-3}{x^2-x-6}.$$

e) Does this function have one or more holes? If so please give the x-coordinate of each hole.

see work for d)

$\boxed{\text{yes } x=3}$

f) Does this function have a horizontal asymptote on the right? If so please give its equation.

$$\lim_{x\to\infty} \frac{x-3}{x^2-x-6} = \lim_{x\to\infty} \frac{\frac{x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}-\frac{6}{x^2}} = \lim_{x\to\infty} \frac{\frac{1}{x}-\frac{3}{x^2}}{1-\frac{1}{x}-\frac{6}{x^2}} = \frac{0-0}{1-0-0}$$

$$= \frac{0}{1} = 0$$

$\boxed{\text{yes, } y=0}$

g) Does this function have a horizontal asymptote on the left? If so please give its equation.

$$\lim_{x\to-\infty} \frac{x-3}{x^2-x-6} = \lim_{x\to-\infty} \frac{\frac{x}{x^2}-\frac{3}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}-\frac{6}{x^2}} = \lim_{x\to-\infty} \frac{\frac{1}{x}-\frac{3}{x^2}}{1-\frac{1}{x}-\frac{6}{x^2}}$$

$$= \frac{0-0}{1-0-0} = \frac{0}{1} = 0$$

$\boxed{\text{yes } y=0}$

Problem 8 : (2 points) Write each exponential expression as a logarithmic expression.

a) $2^3 = 8$

$\log_2 8 = 3$

b) $10^{-2} = \frac{1}{100}$

$\log_{10} \frac{1}{100} = -2$

Problem 9 : (2 points) Write each logarithmic expression as an exponential expression.

a) $\log_5 125 = 3$

$5^3 = 125$

b) $\log_8 2 = \frac{1}{3}$

$8^{\frac{1}{3}} = 2$

Problem 10 : (4 points) Simplify the following logarithmic expressions completely.

a) $\log\left(\sqrt{\frac{2y^3}{x}}\right) = \frac{1}{2}\log\left(\frac{2y^3}{x}\right) = \frac{1}{2}\left(\log 2 + \log y^3 - \log x\right)$

$= \frac{1}{2}\left(\log 2 + 3\log y - \log x\right)$

$= \frac{1}{2}\log 2 + \frac{3}{2}\log y - \frac{1}{2}\log x$

OR $= \log\left(\frac{2^{1/2}y^{3/2}}{x^{1/2}}\right) = \log 2^{1/2} + \log y^{3/2} - \log x^{1/2}$

$= \frac{1}{2}\log 2 + \frac{3}{2}\log y - \frac{1}{2}\log x$

b) $\log_a\left(\frac{64x^2}{\sqrt[3]{y}}\right) = \log_a\left(\frac{2^6 x^2}{y^{1/3}}\right)$

$= \log_a 2^6 + \log_a x^2 - \log_a y^{1/3}$

$= 6\log_a 2 + 2\log_a x - \frac{1}{3}\log_a y$

Problem 11 : (8 points) Solve the following exponential and logarithmic equations.

a) $7^{2x+1} = 1$

$$7^{2x+1} = 7^0$$

$$\log_7(7^{2x+1}) = \log_7(7^0)$$

$$2x+1 = 0$$

$$2x = -1$$

$$\boxed{x = -\frac{1}{2}}$$

b) $2^{x^2+3x} - 16 = 0$

$$2^{x^2+3x} = 16$$

$$2^{x^2+3x} = 2^4$$

$$\log_2(2^{x^2+3x}) = \log_2(2^4)$$

$$x^2+3x = 4$$

$$x^2+3x-4 = 0$$

$$x^2+4x-x-4 = 0$$

$$x(x+4)-(x+4) = 0$$

$$(x+4)(x-1) = 0$$

sum $= 3 = 4 - 1$
product $= -4$

so $x+4=0 \quad x-1=0$

$$\boxed{x=-4 \quad x=1}$$

c) $\log_3(2x+5) = 2$

$$3^{\log_3(2x+5)} = 3^2$$

$$2x+5=9$$

$$2x=4$$

$$x = \frac{4}{2} = 2$$

and

$$2 \cdot 2 + 5 = 4 + 5 = 9 > 0$$

So this is in the domain of $\log_3$

$$\boxed{x=2}$$

d) $\log_5 x + \log_5(x+24) = 2$

$$\log_5(x(x+24)) = 2$$

$$5^{\log_5(x(x+24))} = 5^2$$

$$x^2 + 24x = 25$$

$$x^2+24x-25=0$$

sum $=24=25-1$, product $=-25$

$$x^2+25x-x-25=0$$

$$x(x+25)-(x+25)=0$$

$$(x+25)(x-1)=0$$

$x+25=0$ $\quad$ $x-1=0$

$x=-25$ $\quad$ $x=1$

but!

$x=-25<0$ is not in the domain of $\log_5$, so we reject the solution

$x=1$ and $1+24=25$ are both positive so we keep this solution

$$\boxed{x=1}$$

Problem 12 : (5 points) In this problem we will prove that

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y.$$

Throughout this question, we will write

$$m = \log_a x$$

and

$$n = \log_a y$$

a) Convert the logarithmic equation $m = \log_a x$ into an exponential equation.

$$a^m = x$$

b) Convert the logarithmic equation $n = \log_a y$ into an exponential equation.

$$a^n = y$$

c) Use your work from parts a) and b) to write $\frac{x}{y}$. Simplify the expression completely.

$$\frac{x}{y} = \frac{a^m}{a^n} = a^{m-n}$$

d) Use your work from part c) to compute $\log_a\left(\frac{x}{y}\right)$. Simplify the expression completely.

$$\log_a\left(\frac{x}{y}\right) = \log_a\left(a^{m-n}\right) = m-n$$

e) In your answer for part d), replace m by $\log_a x$ and n by $\log_a y$.

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$