Name:

Problem 1: *Consider the following vector space:*

$$\{a_0 + a_1 x : a_0 - 2a_1 = 0\},$$

with the usual polynomial addition and scalar multiplication operations.
List three elements of this vector space, and for each of the elements you list, give the additive inverse.

Solution: Any linear polynomial $a_0 + a_1 x$ with the property that $a_0 - 2a_1 = 0$ can be used as an element. Then the additive inverse can be found by taking the "negative" of each of the two coefficients. Here are some examples, among the infinitely many possible examples:

- The zero polynomial 0 is an element of this vector space (with the choice $a_0 = 0$ and $a_1 = 0$; then $a_0 - 2a_1$ is indeed 0). Its additive inverse is itself, the polynomial 0.

- The polynomial $2 + x$ is an element of this vector space (with $a_0 = 2$, $a_1 = 1$; then $a_0 - 2a_1$ is again 0). Its additive inverse is $-2 - x$.

- $-2 - x$ is also an element of this vector space (with $a_0 = -2$ and $a_1 = -1$; then $a_0 - 2a_1 = 0$). Its additive inverse is $2 + x$.

- $4\pi + 2\pi x$ is an element of the vector space, with $a_0 = 4\pi$ and $a_1 = 2\pi$. Its additive inverse is $-4\pi + 2\pi$.

One can notice that any multiple of $2 + x$ is an element of the space, and indeed the space is *spanned* by $2 + x$.