

Name:

**Problem 1:** Solve the following system of linear equations

$$\begin{aligned}x - z &= 1 \\y + 2z - w &= 3 \\x + 2y + 3z - w &= 7\end{aligned}$$

Please give your answer in vector form.

**Solution:** We first put the system in echelon form:

$$\left(\begin{array}{cccc|c}1 & 0 & -1 & 0 & 1 \\0 & 1 & 2 & -1 & 3 \\1 & 2 & 3 & -1 & 7\end{array}\right) \xrightarrow{\rho_3 - \rho_1} \left(\begin{array}{cccc|c}1 & 0 & -1 & 0 & 1 \\0 & 1 & 2 & -1 & 3 \\0 & 2 & 4 & -1 & 6\end{array}\right) \xrightarrow{\rho_3 - 2\rho_2} \left(\begin{array}{cccc|c}1 & 0 & -1 & 0 & 1 \\0 & 1 & 2 & -1 & 3 \\0 & 0 & 0 & 1 & 0\end{array}\right)$$

We see that there is no contradiction and  $z$  is a free variable, so there are infinitely many solutions.

Now we walk up the stairs to give them: The last equation tells us that  $w = 0$ .

The middle equation tells us that  $y + 2z - w = 3$ . Plugging in  $w = 0$  and solving for  $y$ , we get  $y = 3 - 2z$ .

Finally, the first equation says that  $x - z = 1$ . This is already only in terms of  $z$ , so we solve for  $x$ :  $x = 1 + z$ .

In vector form, this becomes

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} z.$$