

Name:

Problem 1: For the system

$$\begin{aligned}2x - y - w &= 3 \\ y + z + 2w &= 2 \\ x - 2y - z &= -1\end{aligned}$$

can the vector

$$\begin{pmatrix} 0 \\ -3 \\ 5 \\ 0 \end{pmatrix}$$

be used as the particular solution part of some general solution?

Solution: To be eligible to be the particular solution part of a general solution, a vector must be a solution of the system of linear equations. Therefore, we must check that plugging in $x = 0$, $y = -3$, $z = 5$, and $w = 0$ satisfies all three equations of the system. We do each in order.

First equation:

$$2(0) - (-3) - (0) = 3,$$

as the equation claims. The tuple $(0, -3, 5, 0)$ satisfies the first equation.

Second equation:

$$(-3) + (5) + 2(0) = 2,$$

again as the equation claims. The tuple $(0, -3, 5, 0)$ also satisfies the second equation.

Third equation:

$$(0) - 2(-3) - (5) = 6 - 5 = 1,$$

which is **not** what the equation claims. The tuple $(0, -3, 5, 0)$ does not satisfy the third equation.

Since the vector

$$\begin{pmatrix} 0 \\ -3 \\ 5 \\ 0 \end{pmatrix}$$

does not satisfy the third equation, it does not satisfy the system and **cannot** be used as a particular solution. (A vector must satisfy *all* of the equations to be a solution.)