Name:

Problem 1: *Find all of the eigenvalues of this matrix:*

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Solution: We compute the characteristic polynomial:

$$\begin{aligned}
p(\lambda) &= \begin{vmatrix} 1-\lambda & 1 & 1 \\ 0 & -\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} \\
&= (1-\lambda) \begin{vmatrix} -\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} \\
&= (1-\lambda)(-\lambda)(1-\lambda) \\
&= -\lambda(1-\lambda)^2.
\end{aligned}$$

The two eigenvalues are $\lambda = 0$ and $\lambda = 1$.

Bonus: The eigenspace associated with $\lambda = 0$ has dimension 1, and the eigenspace associated with $\lambda = 1$ has dimension 1 or 2.