Name:

Problem 1: *Compute the determinant of the following matrix:*

$$\begin{pmatrix} 4 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 1 & 3 & -1 & 0 \\ 2 & 0 & 0 & 2 \end{pmatrix}$$

Solution: Both the second column and the second row have three zeroes, so they are great choices for cofactor expansion. We use the second row here.

$$\begin{aligned} \begin{vmatrix} 4 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 \\ 1 & 3 & -1 & 0 \\ 2 & 0 & 0 & 2 \end{vmatrix} &= (-1)^{2+1} \cdot 0 + (-1)^{2+2} \cdot 0 + (-1)^{2+3} \cdot 1 \cdot \begin{vmatrix} 4 & 0 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & 2 \end{vmatrix} + (-1)^{2+4} \cdot 0 \\ &= -\left((-1)^{1+2} \cdot 0 + (-1)^{2+2} \cdot 3 \cdot \begin{vmatrix} 4 & 2 \\ 2 & 2 \end{vmatrix} + (-1)^{3+2} \cdot 0 \right) \\ &= -3(4 \cdot 2 - 2 \cdot 2) \\ &= -3(8 - 4) = -12 \end{aligned}$$

Here to compute the 3×3 determinant we used the middle column because it had two zeroes, and to compute the 2×2 determinant we used the formula

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc.$$