Name:

Problem 1: *Compute the inverse of the matrix*

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 1 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix}.$$

Solution: We follow the process outlined in class: we write

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right)$$

and do row operations (on both sides!) until the left matrix has become the identity matrix. Then the matrix on the right is the inverse of A.

$$\left(\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \underset{\rho_3-\rho_1}{\overset{\rho_2-\rho_1}{\sim}} \left(\begin{array}{ccc|ccc} 1 & -1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 3 & -2 & -1 & 0 & 1 \end{array}\right)$$

$$\underset{\rho_3-3\rho_2}{\overset{\rho_1+\rho_2}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -3 & 1 \end{array}\right)$$

$$\underset{\rho_2+\rho_3}{\overset{\rho_1-2\rho_3}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & 7 & -2 \\ 0 & 1 & 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 2 & -3 & 1 \end{array}\right)$$

Therefore we have

$$A^{-1} = \begin{pmatrix} -4 & 7 & -2 \\ 1 & -2 & 1 \\ 2 & -3 & 1 \end{pmatrix}.$$

Bonus: We can check that our answer is correct by doing the matrix multiplication

$$AA^{-1} = \begin{pmatrix} 1 & -1 & 3 \\ 1 & 0 & 2 \\ 1 & 2 & 1 \end{pmatrix} \begin{pmatrix} -4 & 7 & -2 \\ 1 & -2 & 1 \\ 2 & -3 & 1 \end{pmatrix}$$

and making sure that we get the identity matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$