

Name:

Problem 1: Consider the map

$$f: \mathbb{R}^3 \rightarrow \mathbb{R}^2$$

given by

$$f \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x + y \\ x + z \end{pmatrix}.$$

Give the matrix representation of this map.

Solution: We use the standard basis of \mathbb{R}^3 :

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\},$$

and compute the value of f on each:

$$f \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$f \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

and

$$f \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Now we put these three columns in a matrix, and we have the matrix representation of f :

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$