Name:

**Problem 1:** *Compute the rank of the matrix*

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \\ -2 & 2 & -4 \end{pmatrix}.$$

*Since the rank is a dimension, justify your answer by giving a basis, and arguing that the set you give is a basis.*

**Solution:** To compute the rank of a matrix, we can compute either its row rank or its column rank. We will compute its column rank since we can do it directly, without taking transposes. The column rank will be the number of linearly independent columns. We will compute this number by shrinking the set of columns to a linearly independent set.

We do this by row-reducing the matrix $A$:

$$\begin{pmatrix} 1 & -1 & 2 \\ 3 & -3 & 6 \\ -2 & 2 & -4 \end{pmatrix} \underset{\rho_3+2\rho_1}{\overset{\rho_2-3\rho_1}{\sim}} \begin{pmatrix} 1 & -1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Since the variables in the second and third column are free, the second and third column vectors are superfluous.

Therefore a basis for the column space is

$$\left\{ \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \right\}.$$

This set is linearly independent, and we can justify this in two ways:

1. The process we have to shrink a spanning set to a basis is guaranteed to give us a linearly independent subset; therefore this set is linearly independent and spanning and it is a basis.

2. One vector, as long as it is not the zero vector, is always linearly independent.

Since a basis for the column space has one element, the column rank is one, and the rank is one.