Name:

Problem 1: *Consider $\mathcal{P}_2$, the set of polynomials of degree less than or equal to 2:*

$$\mathcal{P}_2 = \{a_0 + a_1x + a_2x^2 : a_0, a_1, a_2 \in \mathbb{R}\}.$$

Give a basis for this vector space.

Must any basis of $\mathcal{P}_2$ contain a constant polynomial, a polynomial of degree 1 and a polynomial of degree 2? Please give a one-sentence explanation of your answer.

Solution: There are infinitely many possible bases for $\mathcal{P}_2$. We give three to illustrate their variety.
The first and perhaps most obvious one is

$$\{1, x, x^2\}.$$

It is often considered the standard basis for $\mathcal{P}_2$.
A second basis is

$$\{x^2 - x + 1, 2x + 1, 2x - 1\}.$$

This is the basis that appeared on Quiz 13 on Wednesday.
A third basis is

$$\{x^2, x^2 + x, x^2 + x + 1\}.$$

The last two examples show that a basis **does not** need to have a constant polynomial, a polynomial of degree 1 and a polynomial of degree 2. In fact it can contain three polynomials of degree 2 as in the last example. The one thing that it **must** contain though, is at least one polynomial of degree 2.