1. We define the *transpose* of an $n \times m$ matrix with entries $a_{i,j}$ to be the $m \times n$ matrix with entries $a_{j,i}$. In others words, the transpose matrix is the matrix whose columns are the rows of the original matrix. Here is an example: If A is the 2×3 matrix
$$\begin{pmatrix} 1 & -2 & 4 \\ -3 & 0 & 1 \end{pmatrix},$$
then its transpose, denoted A^T, is the 3×2 matrix
$$\begin{pmatrix} 1 & -3 \\ -2 & 0 \\ 4 & 1 \end{pmatrix}.$$
Find the transpose of the following matrices.

   (a) $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$ (c) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

2. Give an equation for the plane through the points $(1, 1, 5, -1)$, $(2, 2, 2, 0)$ and $(3, 1, 0, 4)$.
Hint: If these three points are in the plane, then within the plane motion is allowed in any direction that goes from one of these points to another. This will give you some direction vectors.

3. Two vectors are *perpendicular* or *orthogonal* if the angle between them is $\pi/2$ or $-\pi/2$.

   (a) If the two vectors $\vec{u}$ and $\vec{v}$ are orthogonal, what is the value of $\vec{u} \cdot \vec{v}$?
   Hint: Use the formula for $\cos\theta$ that we developed in class.

   (b) Consider the set S of all vectors in $\mathbb{R}^3$ that are orthogonal to $\vec{v} = \left(\begin{smallmatrix} 1 \\ 2 \\ 0 \end{smallmatrix}\right)$.

      i. Show that S is a vector space.

      ii. Express S as a span.

4. Prove that the set of solutions of a homogeneous system of linear equations is a vector space. Can we show the same thing if the system is not homogeneous?