Gauss's Method

Linear systems

1.1 *Definition* A *linear combination* of $x_1, \ldots, x_n$ has the form

$$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n$$

where the numbers $a_1, \ldots, a_n \in \mathbb{R}$ are the combination's *coefficients*.

Example This is a linear combination of x, y, and z.

$$(1/4)x + y - z$$

1.1 *Definition* A *linear equation* in the variables $x_1, \ldots, x_n$ has the form $a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = d$ where $d \in \mathbb{R}$ is the *constant*.

An n-tuple $(s_1, s_2, \ldots, s_n) \in \mathbb{R}^n$ is a *solution* of, or *satisfies*, that equation if substituting the numbers $s_1, \ldots, s_n$ for the variables gives a true statement: $a_1s_1 + a_2s_2 + \cdots + a_ns_n = d$. A *system of linear equations*

$$\begin{aligned}
a_{1,1}x_1 + \; a_{1,2}x_2 + \cdots + \; a_{1,n}x_n &= d_1 \\
a_{2,1}x_1 + \; a_{2,2}x_2 + \cdots + \; a_{2,n}x_n &= d_2 \\
&\vdots \\
a_{m,1}x_1 + a_{m,2}x_2 + \cdots + a_{m,n}x_n &= d_m
\end{aligned}$$

has the solution $(s_1, s_2, \ldots, s_n)$ if that n-tuple is a solution of all of the equations.

Example There are three linear equations in this linear system.

$$\begin{aligned}
(1/4)x + \; y - \; z &= 0 \\
x + 4y + 2z &= 12 \\
2x - 3y - \; z &= 3
\end{aligned}$$