MATH 320 QUIZ 4 DUE DATE: APRIL 20 OR 22

NAME:

To get the most of this quiz, allow yourself no more than 20 minutes to completely answer it, and do not use any notes or outside help. I will grade it if you hand it in during discussion on April 20 or 22.

Problem 1 (8 points): *For each of the following two equations,*

a) Write down the general solution of the equation.

b) Using the initial value, write down the equations for c_1 and c_2 in matrix-vector form.

c) Find the particular solution of the differential equation with initial value and the range of validity of your solution.

i. $3x^2y'' - 3xy' + 6y = 0 \quad y(1) = 2 \quad y'(1) = 5$

ii. $x^2y'' + 3xy' + y = 0 \quad y(e) = 0 \quad y'(e) = e^{-1}$

Solution:

i. This differential equation has characteristic equation $3r(r-1) - 3r + 6 = 3r^2 - 6r + 6 = 0$, which has two complex conjugate roots $1+i$ and $1-i$.

a) The general solution is $y(x) = c_1 x\cos(\ln x) + c_2 x\sin(\ln x)$.

b) We have $y'(x) = c_1\cos(\ln x) - c_1\sin(\ln x) + c_2\sin(\ln x) + c_2\cos(\ln x)$, so that the initial values give us the two equations

$$y(1) = 2 = c_1$$

$$y'(1) = 5 = c_1 + c_2$$

In matrix-vector form, this is

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

c) We have that $c_1 = 2$, and so $c_2 = 5 - c_1 = 5 - 2 = 3$, so the particular solution is $y(x) = 2x\cos(\ln x) + 3x\sin(\ln x)$, and it is valid for $x > 0$.

ii. This differential equation has characteristic equation $r(r-1)+3r+1 = r^2+2r+1 = 0$, which has the repeated real root -1.

a) The general solution is $y(x) = \frac{c_1}{x} + \frac{c_2}{x}\ln x$.

b) We have $y'(x) = -\frac{c_1}{x^2} - \frac{c_2}{x^2}\ln x + \frac{c_2}{x^2}$, so that the initial values give us the two equations

$$y(e) = 0 = \frac{c_1}{e} + \frac{c_2}{e}$$

$$y'(e) = e^{-1} = -\frac{c_1}{e^2} - \frac{c_2}{e^2} + \frac{c_2}{e^2} = -\frac{c_1}{e^2}$$

In matrix-vector form, this is

$$\begin{bmatrix} e^{-1} & e^{-1} \\ -e^{-2} & 0 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ e^{-1} \end{bmatrix}$$

c) We have $c_1 = -e$, and $c_2 = e$, so the particular solution is $y(x) = -\frac{e}{x} + \frac{e}{x}\ln x$, and is valid for $x > 0$.

Problem 2 (2 points): *Go on wikipedia.org, and search for "invertible matrix". List the ten properties that are equivalent to being invertible that we have talked about in class. (There are 16 properties listed there in total, but we have only talked about 10 of them so far, and we'll talk about an eleventh one soon.)*

Solution: $\mathbf{A}$ is invertible.
$\mathbf{A}$ is row-equivalent to the n by n identity matrix.
$\mathbf{A}$ has n pivot positions.
det $\mathbf{A} \neq 0$.
rank $\mathbf{A} = n$.
The equation $\mathbf{Ax}= 0$ has only the trivial solution $\mathbf{x}$=0.
The equation $\mathbf{Ax}$=$\mathbf{b}$ has exactly one solution for each $\mathbf{b}$.
The columns of $\mathbf{A}$ are linearly independent.
The columns of $\mathbf{A}$ span $\mathbb{R}^n$.
The columns of $\mathbf{A}$ form a basis of $\mathbb{R}^n$.
Soon we will talk about eigenvalues, and when we know about them, 0 is not an eigenvalue of $\mathbf{A}$ is also equivalent to being invertible.