NAME:

To get the most of this quiz, allow yourself no more than 15 minutes to completely answer it, and do not use any notes or outside help. I will grade it if you hand it in during discussion on February 9 or 11.
Note: This ended up being more tricky than I thought, because of all the fractions. Computing the inverse takes a while (longer than 15 minutes).

Problem 1 (10 points): *Consider the matrix*

$$\boldsymbol{A} = \begin{bmatrix} -3 & -2 & 3 \\ 0 & 3 & 2 \\ 2 & 3 & -5 \end{bmatrix}$$

a) *Compute the determinant of* $\boldsymbol{A}$.

b) *Compute the inverse of* $\boldsymbol{A}$.

c) *Use the inverse to compute the solution* $\boldsymbol{x}$ *to the system* $\boldsymbol{Ax}=\boldsymbol{b}$ *if*

$$\boldsymbol{b} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Solution:

a) We have:

$$\begin{bmatrix} -3 & -2 & 3 \\ 0 & 3 & 2 \\ 2 & 3 & -5 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 & -2 \\ 0 & 3 & 2 \\ 2 & 3 & -5 \end{bmatrix} \sim \begin{bmatrix} -1 & 1 & -2 \\ 0 & 3 & 2 \\ 0 & 5 & -9 \end{bmatrix}$$

where all of the row operations performed were adding a multiple of a row to another row, so that the determinant was not changed. I can now do cofactor expansion (I could have done it earlier, of course, I just chose to do it this way):

$$\det \mathbf{A} = -1 \cdot \begin{vmatrix} 3 & 2 \\ 5 & -9 \end{vmatrix} = 37$$

b) Because the determinant is 37, which is a little bit large prime number, we expect this to be a little bit painful. But it's still doable!

$$\left[\begin{array}{ccc|ccc} -3 & -2 & 3 & 1 & 0 & 0 \\ 0 & 3 & 2 & 0 & 1 & 0 \\ 2 & 3 & -5 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} -1 & 1 & -2 & 1 & 0 & 1 \\ 0 & 3 & 2 & 0 & 1 & 0 \\ 2 & 3 & -5 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} -1 & 1 & -2 & 1 & 0 & 1 \\ 0 & 3 & 2 & 0 & 1 & 0 \\ 0 & 5 & -9 & 2 & 0 & 3 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 2 & -1 & 0 & -1 \\ 0 & 1 & 2/3 & 0 & 1/3 & 0 \\ 0 & 5 & -9 & 2 & 0 & 3 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 8/3 & -1 & 1/3 & -1 \\ 0 & 1 & 2/3 & 0 & 1/3 & 0 \\ 0 & 0 & -37/3 & 2 & -5/3 & 3 \end{array}\right]$$

At this point we realize that carrying on will be awful. This is where we will get tricky: instead of aiming to get 1's on the diagonal, we will aim to get 37's. This works because we can just divide all rows by 37 at the end. So here we go:

$$\left[\begin{array}{ccc|ccc} 37 & 0 & 37\cdot 8/3 & -37 & 37/3 & -37 \\ 0 & 37 & 37\cdot 2/3 & 0 & 37/3 & 0 \\ 0 & 0 & 37 & -6 & 5 & -9 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 37 & 0 & 0 & -21 & -1 & -13 \\ 0 & 37 & 0 & 4 & 9 & 6 \\ 0 & 0 & 37 & -6 & 5 & -9 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -21/37 & -1/37 & -13/37 \\ 0 & 1 & 0 & 4/37 & 9/37 & 6/37 \\ 0 & 0 & 1 & -6/37 & 5/37 & -9/37 \end{array}\right]$$

So we have

$$\mathbf{A}^{-1} = \frac{1}{37}\begin{bmatrix} -21 & -1 & -13 \\ 4 & 9 & 6 \\ -6 & 5 & -9 \end{bmatrix}$$

c) If $\mathbf{Ax}$=$\mathbf{b}$ and $\mathbf{A}$ is invertible, then we have $\mathbf{x}$=$\mathbf{A}^{-1}\mathbf{b}$. So

$$\mathbf{x} = \frac{1}{37}\begin{bmatrix} -21 & -1 & -13 \\ 4 & 9 & 6 \\ -6 & 5 & -9 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{37}\begin{bmatrix} 15 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 15/37 \\ 2/37 \\ -3/37 \end{bmatrix}$$