MATH 213

Telescoping series answers.

a) $\displaystyle\sum_{i=1}^{\infty} \frac{1}{i(i+1)}$

We write the terms a_i so it looks like a telescoping series:

$$a_i = \frac{1}{i(i+1)} = \frac{1}{i} - \frac{1}{i+1}$$

i): $\displaystyle S_n = \sum_{i=1}^{n} \left(\frac{1}{i} - \frac{1}{i+1}\right)$

$$= 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{n-1} - \frac{1}{n} + \frac{1}{n} - \frac{1}{n+1}$$

$$= 1 - \frac{1}{n+1}$$

ii) $\displaystyle\sum_{i=1}^{\infty} \left(\frac{1}{i} - \frac{1}{i+1}\right) = \lim_{n\to\infty} S_n$

$$= \lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right) = 1$$

b) $\sum_{i=1}^{\infty}\left(\frac{1}{(i+2)^2}-\frac{1}{(i+3)^2}\right)$

i) $S_n=\sum_{i=1}^{n}\left(\frac{1}{(i+2)^2}-\frac{1}{(i+3)^2}\right)$

$=\frac{1}{9}-\frac{1}{16}+\frac{1}{16}-\frac{1}{25}+\ldots+\frac{1}{(n+2)^2}-\frac{1}{(n+3)^2}$

$=\frac{1}{9}-\frac{1}{(n+3)^2}$

ii) $\sum_{i=1}^{\infty}\left(\frac{1}{(i+2)^2}-\frac{1}{(i+3)^2}\right)$

$=\lim_{n\to\infty} S_n$

$=\lim_{n\to\infty}\left(\frac{1}{9}-\frac{1}{(n+3)^2}\right)=\frac{1}{9}$

c) $\sum_{i=2}^{\infty}\left(\frac{1}{i+1}-\frac{1}{i}\right)$

i) $S_n = \sum_{i=2}^{n}\left(\frac{1}{i+1}-\frac{1}{i}\right)$

$$= \frac{1}{3}-\frac{1}{2}+\frac{1}{4}-\frac{1}{3}+\frac{1}{5}-\frac{1}{4}+\dots$$

$$+\frac{1}{n}-\frac{1}{n-1}+\frac{1}{n+1}-\frac{1}{n}$$

$$= \frac{1}{n+1}-\frac{1}{2}$$

ii) $\sum_{i=2}^{\infty}\left(\frac{1}{i+1}-\frac{1}{i}\right)$

$$= \lim_{n\to\infty} S_n$$

$$= \lim_{n\to\infty}\left(\frac{1}{n+1}-\frac{1}{2}\right) = -\frac{1}{2}$$