MATH 213

Quiz 10

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Recall: The Taylor series of a function $f(x)$ is given by

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

Example: Let $f(x) = e^x$.

Then $f'(x) = e^x$ $f'(0) = 1$

$f''(x) = e^x$ $f''(0) = 1$

$\vdots$ $\vdots$

$f^{(n)}(x) = e^x$ $f^{(n)}(0) = 1$

Also $f(0) = 1$.

So the Taylor series of $f(x) = e^x$ is

$$\sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

We can use the definition of a Taylor series to get information about the derivatives of $f(x)$ evaluated at 0:

Example 1: Let $f(x) = \ln(1+x)$. What is $f^{(100)}(0)$?

We know that

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}$$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

So for each n, we have

$$\frac{f^{(n)}(0)}{n!} = \frac{(-1)^{n-1}}{n}$$

(They are what is in front of x^n in each sum.)

So $$\frac{f^{(100)}(0)}{100!} = \frac{(-1)^{99}}{100}$$

$$f^{(100)}(0) = -\frac{100!}{100} = -99!$$

Example 2 (Quiz question)

Let $f(x) = \sum_{n=0}^{\infty} nx^n$. What is $f^{(12)}(0)$?

we still have that

$$\frac{f^{(n)}(0)}{n!} = a_n$$

where a_n is what is in front of x^n

i.e. $a_n = n$ here.

So $$\frac{f^{(12)}(0)}{12!} = 12$$

$$f^{(12)}(0) = 12 \cdot 12!$$