S-parameters

MUSE
Material developed by Prof. L. Dunleavy, USF
2-Port Parameters

- Recall Z-Parameters:

\[
\begin{align*}
V_1 &= Z_{11} I_1 + Z_{12} I_2 \\
V_2 &= Z_{21} I_1 + Z_{22} I_2
\end{align*}
\]

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]
2-Port Parameters

- Recall Z-Parameters:

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V_2
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} \\
Z_{21} & Z_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]
2-PORT Parameters (cont’d)

- **Y-Parameters:**

\[
\begin{align*}
I_1 &= Y_{11} V_1 + Y_{12} V_2 \\
I_2 &= Y_{21} V_1 + Y_{22} V_2
\end{align*}
\]

\[
\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
Y_{11} & Y_{12} \\
Y_{21} & Y_{22}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix}
\]

- **h-Parameters:**

\[
\begin{align*}
V_1 &= h_{11} I_1 + h_{12} V_2 \\
I_2 &= h_{21} I_1 + h_{22} V_2
\end{align*}
\]

\[
\begin{bmatrix}
V_1 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
h_{11} & h_{12} \\
h_{21} & h_{22}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
V_2
\end{bmatrix}
\]
2-Port Parameters (cont’d)

- 2-port Parameter Determination:

\[ h_{11} = \frac{V_1}{I_1} I_2 | V_2 = 0 \]  
(Put a Short Circuit at Port #2)

\[ h_{21} = \frac{I_2}{I_1} | V_2 = 0 \]

\[ h_{12} = \frac{V_1}{V_2} I_1 = 0 \]  
(Put an Open Circuit at Port #1)

\[ h_{22} = \frac{I_2}{V_2} | I_1 = 0 \]
S-Parameters

At high RF and Microwave frequencies direct measurement of Y-, Z-, or H- parameters is difficult due to:

• Unavailability of equipment to measure RF/MW total current and voltage.
• Difficulty of obtaining perfect opens/shorts
• Active devices may be unstable under open/short conditions.
S-Parameters

- For a two-port device there are four S-parameters $S_{11}$, $S_{21}$, $S_{12}$, and $S_{22}$
- $S_{11}$, and $S_{22}$ are simply the forward and reverse reflection coefficients, with the opposite port terminated in $Z_0$ (usually 50 ohms.)
- $S_{21}$ and $S_{12}$ are simply the forward and reverse gains assuming a $Z_0$ source and load (again usually 50 ohms).
S-Parameters (cont’d)

- S-Parameters:

\[
b_1 = S_{11}a_1 + S_{12}a_2 \\
b_2 = S_{21}a_1 + S_{22}a_2
\]

\[
\begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}
\]
S-Parameters (cont’d)

• Q. So what’s the deal with the a’s and b’s?

• A. a1 and a2 are incident waves; b1 and b2 are reflected waves
 Incident & Reflected Waves:
Simplified Case: $Z_1 = Z_s = Z_2 = Z_L = Z_0$ (real)

\[ a_1 = \frac{V_1 + Z_0 I_1}{2 \sqrt{Z_0}} = \frac{\text{Incident port 1 voltage}}{\sqrt{Z_0}} = \frac{E_{i1}}{\sqrt{Z_0}} \]

\[ a_2 = \frac{V_2 + Z_0 I_2}{2 \sqrt{Z_0}} = \frac{\text{Incident port 2 voltage}}{\sqrt{Z_0}} = \frac{E_{i2}}{\sqrt{Z_0}} \]

\[ b_1 = \frac{V_1 - Z_0 I_1}{2 \sqrt{Z_0}} = \frac{\text{reflected port 1 voltage}}{\sqrt{Z_0}} = \frac{E_{r1}}{\sqrt{Z_0}} \]

\[ b_2 = \frac{V_2 - Z_0 I_2}{2 \sqrt{Z_0}} = \frac{\text{reflected port 2 voltage}}{\sqrt{Z_0}} = \frac{E_{r2}}{\sqrt{Z_0}} \]
$$s_{11} = \frac{b_1}{a_1} \bigg|_{a_2 = 0} = \text{Input reflection coefficient } \Gamma_{in} \text{ for case of } Z_L = Z_0$$

$$s_{21} = \frac{b_2}{a_1} \bigg|_{a_2 = 0} = \text{Forward transmission (insertion) gain} \text{ for case of } Z_L = Z_0$$

$$s_{12} = \frac{b_1}{a_2} \bigg|_{a_1 = 0} = \text{Reverse transmission (insertion) gain} \text{ for case of } Z_s = Z_0$$

$$s_{22} = \frac{b_2}{a_2} \bigg|_{a_1 = 0} = \text{Output reflection coefficient } \Gamma_{out} \text{ for case of } Z_s = Z_0$$
Desired Measurement Conditions

\[ Z_s = Z_0 \quad a_1 \quad b_1 \quad b_2 \quad a_2 = 0 \]

2-port

\[ V_g \quad Z_0 \quad 2-port \quad Z_0 \quad V_2 \]

\[ Z_L = Z_0 \]

Note...the input and output are terminated in \( Z_0 \)

\[ Z_s = Z_0 \quad a_1 \quad b_1 \]

1-port

\[ V_g \quad Z_0 \quad 1-port \quad \Gamma \quad S_{11} = \Gamma \]
GRAPHICAL VIEW OF S-PARAMETERS

Device Under Test

S12

Reverse Gain
Insertion Loss,
Transmission Phase

S11, Input Refl. Coeff. $\Gamma_{\text{in}}$, Return Loss, VSWR

S21, Forward Gain
Insertion Loss,
Transmission Phase

S22, Output Refl. Coeff. $\Gamma_{\text{out}}$, Return Loss, VSWR
# S-Parameters in Decibels

<table>
<thead>
<tr>
<th>dB</th>
<th>Meaning or interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{11}$</td>
<td>$20 \log_{10}</td>
</tr>
<tr>
<td>$S_{12}$</td>
<td>$20 \log_{10}</td>
</tr>
<tr>
<td>$S_{21}$</td>
<td>$20 \log_{10}</td>
</tr>
<tr>
<td>$S_{22}$</td>
<td>$20 \log_{10}</td>
</tr>
</tbody>
</table>
WHAT TO EXPECT

Ideal Lossless T-line
\[ \theta = \beta d \]

\[ Z_0, \varepsilon_r \]

S11 = S22 = 0
S21 = S12 = 1e^{-j\theta}
S21DB = 0

Ideal “X” dB Attenuator

S11 = S22 = 0
S21 = S12 = xe^{-j\theta_{pad}}
S21DB = X = 20\log(|S21|)
x = 10^{-X/20}

Ideal “G” dB Gain Amp

S11 = S22 = 0 = S12
S21 = g_v e^{-j\theta_{amp}}
S21DB = G = 20\log(S21)

\[ \frac{G}{g_v} = 10^{20} \]

\[ G \]

\[ g_v \]
WHAT TO EXPECT: Ideal Filters

Ideal Band Pass

S21

$S_{21} = S_{12} = 1e^{-j\theta(f)}$

$S_{21}(dB) = S_{12}(dB) = 0\text{dB}$

Ideal Low Pass

S21

$f_c$

$S_{21}(dB) = 0\text{dB}$

Ideal High Pass

S21

$f_c$

$S_{21}(dB) = 0\text{dB}$

GENERAL “IN-BAND”

$S_{11} = S_{22} = 0$

S21

GENERAL “OUT-OF-BAND”

$|S_{11}| = |S_{22}| = 1$

$S_{21} = S_{12} = 0$

$S_{21}(dB) = S_{12}(dB) = 0\text{dB}$