On Sense and Nominatum *

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The idea of Sameness challenges reflection. It raises questions which are not quite easily answered. Is Sameness a relation? A relation between objects? Or between names or signs of objects? I assumed the latter alternative in my Begriffsschrift. The reasons that speak in its favor are the following: "a = a" and "a = b" are sentences of obviously different cognitive significance: "a = a" is valid a priori and according to Kant is to be called analytic, whereas sentences of the form "a = b" often contain very valuable extensions of our knowledge and cannot always be justified in an a priori manner. The discovery that it is not a different and novel sun which rises every morning, but that it is the very same, certainly was one of the most consequential ones in astronomy. Even nowadays the re-cognition (identification) of a planetoid or a comet is not always a matter of self-evidence. If we wished to view identity as a relation between the objects designated by the names 'a' and 'b' then "a = b" and "a = a" would not seem different if "a = b" is true. This would express a relation of a thing to itself, namely, a relation such that it holds between every thing and itself but never between one thing and another. What one wishes to express with "a = b" seems to be that the signs or names 'a' and 'b' name the same thing; and in that case we would be dealing with those signs, a relation between them would be asserted. But this relation could hold only inasmuch as they name or designate somethings. The relation, as it were, is mediated through the connection of each sign with the same nominatum. This connection, however, is arbitrary. You cannot forbid the use of an arbitrarily produced process or object as a sign for something else. Hence, a sentence like "a = b" would no longer refer to a matter of fact but rather to our manner of designation; no genuine knowledge would be expressed by it. But this is just what we do want to express in many cases. If the sign 'a' differs from the sign 'b' only as an object (here by its shape) but not by its rôle as a sign, that is to say, not in the manner in which it designates anything, then the cognitive significance of "a = a" would be essentially the same as that of


1 I use this word in the sense of identity and understand "a = b" in the sense of "a is the same as b" or "a and b coincide".

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sense. It can perhaps be granted that an expression has a sense if it is formed in a grammatically correct manner and stands for a proper name. But as to whether there is a denotation corresponding to the connotation is hereby not decided. The words 'the heavenly body which has the greatest distance from the earth' have a sense; but it is very doubtful as to whether they have a nominatun. The expression 'the series with the least convergence' has a sense; but it can be proved that it has no nominatun, since for any given convergent series, one can find another one that is less convergent. Therefore the grasping of a sense does not with certainty warrant a corresponding nominatun.

When words are used in the customary manner then what is talked about are their nominatun. But it may happen that one wishes to speak about the words themselves or about their senses. The first case occurs when one quotes someone else's words in direct (ordinary) discourse. In this case one's own words immediately name (denote) the words of the other person and only the latter words have the usual nominatun. We thus have signs of signs. In writing we may make use of quotes enclosing the word-icms. A word-icm in quotes must therefore not be taken in the customary manner.

If we wish to speak of the sense of an expression 'A' we can do this simply through the locution 'the sense of the expression 'A' '. In indirect (oblique) discourse we speak of the sense, e.g., of the words of someone else. From this it becomes clear that also in indirect discourse words do not have their customary nominatun; they here name what customarily would be their sense. In order to formulate this succinctly we shall say: words in indirect discourse are used indirectly, or have indirect nominatun. Thus we distinguish the customary from the indirect nominatun of a word; and similarly, its customary sense from its indirect sense. The indirect nominatun of a word is therefore its customary sense. Such exceptions must be kept in mind if one wishes correctly to comprehend the manner of connection between signs, senses and nominatun in any given case.

Both the nominatun and the sense of a sign must be distinguished from the associated image. If the nominatun of a sign is an object of sense perception, my image of the latter is an inner picture, born of memories of sense impressions and activities of mine, internal or external. Frequently this image is suffused with feelings; the definiteness of its various parts may vary and fluctuate. Even with the same person the same sense is not always accompanied by the same image. The image is subjective; the image of one person is not that of another. Hence, the
various differences between the images connected with one and the same sense. A painter, a rider, a zoologist probably connect very different images with the name ‘Bucephalus’. The image thereby differs essentially from the connotation of a sign, which latter may well be common property of many and is therefore not a part or mode of the single person’s mind; for it cannot well be denied that mankind possesses a common treasure of thoughts which is transmitted from generation to generation.4

While, accordingly, there is no objection to speak without qualification of the sense in regard to images, we must, to be precise, add whose images they are and at what time they occur. One might say: just as words are connected with different images in two different persons, the same holds of the senses also. Yet this difference would consist merely in the manner of association. It does not prevent both from apprehending the same sense, but they cannot have the same image. *Si duo idem faciunt, non est idem.* When two persons imagine the same thing, each still has his own image. It is true, occasionally we can detect differences in the images or even in the sensations of different persons. But an accurate comparison is impossible because these images cannot be had together in one consciousness.

The nominatum of a proper name is the object itself which is designated thereby; the image which we may have along with it is quite subjective; the sense lies in between, not subjective as is the image, but not the object either. The following simile may help in elucidating these relationships. Someone observes the moon through a telescope. The moon is comparable with the nominatum; it is the object of the observation which is mediated through the real image projected by the object lens into the interior of the telescope, and through the retinal image of the observer. The first may be compared with the sense, the second with the presentation (or image in the psychological sense). The real image inside the telescope, however, is relative; it depends upon the standpoint; yet, it is objective in that it can serve several observers. Arrangements could be made such that several observers could utilize it. But every one of them would have only his own retinal image. Because of the different structures of the eyes not even geometrical congruence could be attained; a real coincidence would in any case be impossible. One could elaborate the simile by assuming that the retinal image of *A* could be made visible to *B*; or *A* could see his own retinal image in a mirror. In this manner one could possibly show how a presentation itself can be made into an object; but even so, it would never be to the (outside) observer what it is to the one who possesses the image. However, these lines of thought lead too far afield.

We can now recognize three levels of differences of words, expressions and complete sentences. The difference may concern at most the imagery, or else the sense but not the nominatum, or finally also the nominatum. In regard to the first level, we must note that, owing to the uncertain correlation of images with words, a difference may exist for one person that another does not discover. The difference of a translation from the original should properly not go beyond the first level. Among the differences possible in this connection we mention the shadings and colorings which poetry seeks to impart to the senses. These shadings and colorings are not objective. Every listener or reader has to add them in accordance with the hints of the poet or speaker. Surely, art would be impossible without some kinship among human imaginations; but just how far the intentions of the poet are realized can never be exactly ascertained.

We shall henceforth no longer refer to the images and picturizations; they were discussed only lest the image evoked by a word be confused with its sense or its nominatum. In order to facilitate brief and precise expression we may lay down the following formulations:

A proper name (word, sign, sign-compound, expression) expresses its sense, and designates or signifies its nominatum. We let a sign *express* its sense and *designate* its nominatum.

Perhaps the following objection, coming from idealistic or skeptical quarters, has been kept in abeyance for some time: ‘You have been speaking without hesitation of the moon as an object; but how do you know that the name ‘the moon’ has in fact a nominatum? How do you know that anything at all has a nominatum?’ I reply that it is not our intention to speak of the image of the moon, nor would we be satisfied with the sense when we say ‘the moon’; instead, we presuppose a nominatum here. We should miss the meaning altogether if we assumed we had reference to images in the sentence “the moon is smaller than the earth”. Were this intended we would use some such location as ‘my image of the moon’. Of course, we may be in error as regards that assumption, and such errors have occurred on occasion. However, the question whether we could possibly always be mistaken in this respect may here remain unanswered; it will suffice for the moment to refer to our intention in speaking and thinking in order to justify our reference to the nominatum of a sign; even if we have to make the proviso: if there is such a nominatum.

Thus far we have considered sense and nominatum only of such expressions, words and signs which we called proper names. We are now going to inquire into the sense and the nominatum of a whole declarative sentence. Such a sentence contains a proposition.5 Is this thought to be regarded as the sense or the nominatum of the sentence? Let us for the

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4 It is therefore inexpedient to designate fundamentally different things by the one word ‘image’ (or ‘idea’).

5 By ‘proposition’ I do not refer to the subjective activity of thinking but rather to its objective content which is capable of being the common property of many.
as a work of art. Thus, it is the striving for truth which urges us to penetrate beyond the sense to the nominatum.

We have realized that we are to look for the nominatum of a sentence whenever the nominata of the sentence-components are the thing that matters; and that is the case whenever and only when we ask for the truth value.

Thus we find ourselves persuaded to accept the truth-value of a sentence as its nominatum. By the truth-value of a sentence I mean the circumstance of its being true or false. There are no other truth-values. For brevity’s sake I shall call the one the True and the other the False. Every declarative sentence, in which what matters are the nominata of the words, is therefore to be considered as a proper name; and its nominatum, if there is any, is either the True or the False. These two objects are recognized, even if only tacitly, by everyone who at all makes judgments, holds anything as true, thus even by the skeptic. To designate truth-values as objects may thus far appear as a capricious idea or as a mere play on words, from which no important conclusion should be drawn. What I call an object can be discussed only in connection with the nature of concepts and relations. That I will reserve for another essay. But this might be clear even here: in every judgment—no matter how obvious—a step is made from the level of propositions to the level of the nominata (the objective facts).

It may be tempting to regard the relation of a proposition to the True not as that of sense to nominatum but as that of the subject to the predicate. One could virtually say: “the proposition that 5 is a prime number is true”. But on closer examination one notices that this does not say any more than is said in the simple sentence “5 is a prime number”. This makes clear that the relation of a proposition to the True must not be compared with the relation of subject and predicate. Subject and predicate (interpreted logically) are, after all, components of a proposition; they are on the same level as regards cognition. By joining subject and predicate we always arrive only at a proposition; in this way we never move from a sense to a nominatum or from a proposition to its truth-value. We remain on the same level and never proceed from it to the next one. Just as the sun cannot be part of a proposition, so the truth-value, because it is not the sense, but an object, cannot be either.

If our conjecture (that the nominatum of a sentence is its truth-value) is correct, then the truth-value must remain unchanged if a sentence-component is replaced by an expression with the same nominatum but

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Footnotes:

1. It would be desirable to have an expression for signs which have sense only. If we call them ‘icons’ then the words of an actor on the stage would be icons; even the actor himself would be an icon.

2. A judgment is not merely the apprehension of a thought or proposition but the acknowledgment of its truth.
with a different sense. Indeed, Leibnitz declares: "Eadem sunt, quae sibi mutuo substiti possunt, salva veritate". What else, except the truth-value, could be found, which quite generally belongs to every sentence and regarding which the nominata of the components are relevant and which would remain invariant for substitutions of the type indicated?

Now if the truth-value of a sentence is its nominatum, then all true sentences have the same nominatum, and likewise all false ones. This implies that all detail has been blunted in the nominatum of a sentence. What interests us can therefore never be merely the nominatum; but the proposition alone does not give knowledge; only the proposition together with its nominatum, i.e., its truth-value, does. Judging may be viewed as a movement from a proposition to its nominatum, i.e., its truth-value. Of course this is not intended as a definition. Judging is indeed something peculiar and unique. One might say that judging consists in the discerning of parts within the truth-value. This discernment occurs through recourse to the proposition. Every sense that belongs to a truth-value would correspond in its own manner to the analysis. I have, however, used the word 'part' in a particular manner here: I have transferred the relation of whole and part from the sentence to its nominatum. This I did by viewing the nominatum of a word as part of the nominatum of a sentence, when the word itself is part of the sentence. True enough, this way of putting things is objectionable since as regards the nominatum the whole and one part of it does not determine the other part; and also because the word 'part' in reference to bodies has a difference customary usage. A special expression should be coined for what has been suggested above.

We shall now further examine the conjecture that the truth-value of a sentence is its nominatum. We have found that the truth-value of a sentence remains unaltered if an expression within the sentence is replaced by a synonymous one. But we have as yet not considered the case in which the expression-to-be-replaced is itself a sentence. If our view is correct, then the truth-value of a sentence, which contains another sentence as a part, must remain unaltered when we substitute for the part another of the same truth-value. Exceptions are to be expected if the whole or the part are either in direct or indirect discourse; for as we have seen, in that case the nominata of the words are not the usual ones. A sentence in direct discourse nominates again a sentence but in indirect discourse it nominates a proposition.

Our attention is thus directed to subordinate sentences (i.e., dependent clauses). These present themselves of course as parts of a sentence-structure which from a logical point of view appears also as a sentence, and indeed as if it were a main clause. But here we face the question whether in the case of dependent clauses it also holds that their nominata are truth-values. We know already that this is not the case with sentences in indirect discourse. The grammarians view clauses as representatives of sentence-parts and divide them accordingly into subjective, relative, and adverbial clauses. This might suggest that the nominatum of a clause is not a truth-value but rather that it is of similar nature as that of a noun or of an adjective or of an adverb; in short, of a sentence-part whose sense is not a proposition but only part thereof. Only a thorough investigation can provide clarity in this matter. We shall herein not follow strictly along grammatical lines, but rather group together what is logically of comparable type. Let us first seek out such instances in which, as we just surmised, the sense of a clause is not a self-sufficient proposition.

Among the abstract clauses beginning with 'that' there is also the indirect discourse, of which we have seen that in it the words have their indirect (oblique) nominata which coincide with what are ordinarily their senses. In this case then the clause has as its nominatum a proposition, not a truth-value; its sense is not a proposition but it is the sense of the words 'the proposition that ...', which is only a part of the proposition corresponding to the total sentence-structure. This occurs in connection with 'to say', 'to hear', 'to opine', 'to be convinced', 'to infer' and similar words. The situation is different, and rather complicated in connection with such words as 'to recognize', 'to know', 'to believe', a matter to be considered later.

One can see that in these cases the nominatum of the clause indeed consists in the proposition, because whether that proposition is true or false is immaterial for the truth of the whole sentence. Compare, e.g., the following two sentences: "Copernicus believed that the planetary orbits are circles" and "Copernicus believed that the appearance of the sun's motion is produced by the real motion of the earth". Here the one clause can be substituted for the other without affecting the truth. The sense of the principal sentence together with the clause is the single proposition; and the truth of the whole implies neither the truth nor the falsity of the clause. In cases of this type it is not permissible to replace in the clause one expression by another of the same nominatum. Such replacement may be made only by expressions of the same indirect nominatum, i.e., of the same customary sense. If one were to infer: the nominatum of a sentence is not its truth-value ("because then a sentence could always be replaced by another with the same truth-value"), he would prove too much; one could just as well maintain that the nominatum of the word 'morning star' is not Venus, for one cannot always substitute 'Venus' for 'morning star'. The only correct conclusion is that the nominatum of a sentence is not always its truth-value, and that 'morning star' does not always nominate the planet Venus; for this is indeed not the case when the word is used with its indirect nominatum. Such an exceptional case is before us in the clauses just considered, whose nominatum is a proposition.

In "A lied, that he had seen B" the clause denotes a proposition of which it is said, firstly, that A assured it as true, and, secondly, that A was convinced of its falsity.
When we say "it seems that . . .," then we mean to say "it seems to me that . . ." or "I opine that . . .". This is the same case over again. Similarly with expressions such as: 'to be glad', 'to regret', 'to approve', 'to disapprove', 'to hope', 'to fear'. When Wellington, toward the end of the battle of Belle-Alliance was glad that the Prussians were coming, the ground of his rejoicing was a conviction. Had he actually been deceived, he would not have been less glad, as long as his belief persisted; and before he arrived at the conviction that the Prussians were coming he could nor have been glad about it, even if in fact they were already approaching.

Just as a conviction or a belief may be the ground of a sentiment, so it can also be the ground of another conviction such as in inference. In the sentence "Columbus inferred from the roundness of the earth that he could, traveling westward, reach India" we have, as nominata of its parts two propositions: that the earth is round, and that Columbus traveling westward could reach India. What matters here is only that Columbus was convinced of the one as well as of the other and that the one conviction furnishes the ground for the other. It is irrelevant for the truth of our sentence whether the earth is really round and whether Columbus could have reached India in the manner he fancied. But it is not irrelevant whether for 'the earth' we substitute 'the planet accompanied by one satellite whose diameter is larger than one-fourth of its own diameter'.

Here also we deal with the indirect nominata of the words.

Adverbial clauses of purpose with 'so that', likewise belong here; obviously the purpose is a proposition; therefore: indirect nominata of the words, expressed in subjunctive form.

The clause with 'that' after 'to command', 'to request', 'to forbid' would appear in imperative form in direct discourse. Imperatives have no nominata; they have only sense. It is true, commands or requests are not propositions, but they are of the same type as propositions. Therefore the words in the dependent clauses after 'to command', 'to request', etc. have indirect nominata. The nominatum of such a sentence is thus not a truth-value but a command, a request, and the like.

We meet a similar situation in the case of dependent questions in phrases like 'to doubt if', 'not to know what'. It is easy to see that the words, here too, have to be interpreted in terms of their indirect nominata. The dependent interrogatory clauses containing 'who', 'what', 'where', 'when', 'how', 'whereby', etc. often apparently approximate closely adverbial clauses in which the words have their ordinary nominata. These cases are linguistically distinguished through the mode of the verb. In the subjunctive we have a dependent question and the indirect nominata of the words, so that a proper name cannot generally be replaced by another of the same object.

In the instances thus far considered the words in the clause had indirect nominata; this made it intelligible that the nominatum of the clause itself is indirect, i.e., not a truth-value, but a proposition, a command, a request, a question. The clause could be taken as a noun; one might even say, as a proper name of that proposition, command, etc., in whose rôle it functions in the context of the sentence-structure.

We are now going to consider clauses of another type, in which the words do have their customary nominata although there does not appear a proposition as the sense or a truth-value as the nominatum. How this is possible will best be elucidated by examples.

"He who discovered the elliptical shape of the planetary orbits, died in misery".

If, in this example, the sense of the clause were a proposition, it would have to be expressible also in a principal sentence. But this cannot be done because the grammatical subject 'he who' has no independent sense. It merely mediates the relations to the second part of the sentence: 'died in misery'. Therefore the sense of the clause is not a complete proposition and its nominatum is not a truth-value, but Kepler. It might be objected that the sense of the whole does include a proposition as its part; namely, that there was someone who first recognized the elliptical shape of the planetary orbits; for if we accept the whole as true we cannot deny this part. Indubitably so; but only because otherwise the clause "he who discovered the elliptical shape, etc." would have no nominatum. Whenever something is asserted then the presupposition taken for granted is that the employed proper names, simple or compound, have nominata. Thus, if we assert "Kepler died in misery" it is presupposed that the name 'Kepler' designates something. However, the proposition that the name 'Kepler' designates something is, the foregoing notwithstanding, not contained in the sense of the sentence "Kepler died in misery". If that were the case the denial would not read "kepler did not die in misery" but "kepler did not die in misery, or the name 'kepler' is without nominatum". That the name 'Kepler' designates something is rather the presupposition of the assertion "kepler died in misery" as well as of its denial. Now, it is a defect of languages that expressions are possible within them, which, in their grammatical form, seemingly determined to designate an object, nevertheless do not fulfill this condition in special cases; because this depends on the truth of the sentence. Thus it depends upon the truth of the sentence "there was someone who discovered the ellipticity of the orbits" whether the clause 'he who discovered the ellipticity of the orbits' really designates an object, or else merely evokes the appearance thereof, while indeed being without nominatum. Thus it may seem as if our clause, as part of its sense, contained the proposition that there existed someone who discovered the ellipticity of the orbits. If this were so, then the denial would have to read "he who first recognized the ellipticity of the orbits did not die in misery, or there was no one who discovered the ellipticity of the orbits".
orbits." This, it is obvious, hinges upon an imperfection of language of
which, by the way, even the symbolic language of analysis is not entirely
free; there, also, sign compounds may occur which appear as if they
designated something, but which at least hitherto are without nominatum,
e.g., divergent infinite series. This can be avoided, e.g., through the
special convention that the nominatum of divergent infinite series be the
number 0. It is to be demanded that in a logically perfect language (logi-
cal symbolism) every expression constructed as a proper name in a gram-
matically correct manner out of already introduced symbols, in fact
designate an object; and that no symbol be introduced as a proper name
without assurance that it has a nominatum. It is customary in logic texts
to warn against the ambiguity of expressions as a source of fallacies. I deem
it at least as appropriate to issue a warning against apparent proper names
that have no nominata. The history of mathematics has many a tale to tell
of errors which originated from this source. The demagogic misuse is
close (perhaps closer) at hand as in the case of ambiguous expressions.
'The will of the people' may serve as an example in this regard; for it is
easily established that there is no generally accepted nominatum of that
expression. Thus it is obviously not without importance to obstruct once
for all the source of these errors, at least as regards their occurrence in
science. Then such objections as the one discussed above will become
impossible, for then it will be seen that whether a proper name has a
nominatum can never depend upon the truth of a proposition.

Our considerations may be extended from these subjective clauses to the
logically related relative and adverbial clauses.

Relative clauses, too, are employed in the formation of compound
proper names—even if, in contradistinction to subjective clauses, they
are not sufficient by themselves for this purpose. These relative clauses
may be regarded as equivalent to appositions. Instead of 'the square root
of 4 which is smaller than 0' we can also say 'the negative square root of
4.' We have here a case in which out of a conceptual expression a com-
 pound proper name is formed, with the help of the definite article in the
singular. This is at any rate permissible when one and only one object is
comprised by the concept.\(^9\) Conceptual expression can be formed in such
a fashion that their characteristics are indicated through relative clauses
as in our example through the clause 'which is smaller than 0.' Obviously,
such relative clauses, just as the subjective clauses above, do not refer to
a proposition as their sense nor to a truth-value as their nominatium. Their
sense is only a part of a proposition, which in many cases, can be expressed
by a simple apposition. As in the subjective clauses an independent sub-

\(^9\) According to our previous remarks such an expression should always be assured
of a nominatum, e.g., through the special convention that the nominatum be the
number 0 if there is no object or more than one object denoted by the expression.

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ject is missing and it is therefore impossible to represent the sense of the
clause in an independent principal sentence.

Places, dates and time-intervals are objects from a logical point of
view; the linguistic symbol of a definite place, moment or span of time
must therefore be viewed as a proper name. Adverbial clauses of space or
time can then be used in the formation of such proper names in a fashion
analogous to the one we have just remarked in the case of subjective and
relative clauses. Similarly, expressions for concepts which comprise places,
etc., can be formed. Here too, it is to be remarked, the sense of the sub-
ordinate clauses cannot be rendered in a principal clause, because an essen-
tial constituent, namely the determination of place and time, is missing and
only alluded to by a relative pronoun or a conjunction.\(^{10}\)

In conditional clauses also, there is, just as we have realized in the case
of subjective, relative and adverbial clauses, a constituent with indetermi-
nate indication corresponding to which there is a similar one in the con-
cluding clause. In referring to one another the two clauses combine into
a whole which expresses, as a rule, only one proposition. In the sentence
"if a number is smaller than 1 and greater than 0, then its square also is
smaller than 1 and greater than 0," this constituent in the conditional clause
is 'a number' and in the concluding clause it is 'its'. Just through this in-
determinacy the sense acquires the universal character which one expects
of a law. But it is in this way also that it comes about that the condi-
tional clause alone does not possess a complete proposition as its sense,
and that together with the concluding clause it expresses a single proposi-
tion whose parts are no longer propositions. It is not generally the case
that a hypothetical judgment correlates two judgments. Putting it in that
(or a similar) manner would amount to using the word 'judgment' in the
same sense that I have attributed to the word 'proposition.' In that case I
would have to say: in a hypothetical proposition two propositions are re-

\(^{10}\) Regarding these sentences, however, several interpretations are easily conceivable.
The sentence 'after Schleswig-Holstein was torn away from Denmark, Prussia and
Austria fell out with one another' could also be rendered by 'after the
separation of Schl.-H. from Denmark, Prussia and Austria fell out with one another.'
In this formulation it is sufficiently clear that we should not regard it as part of this
sense that Schleswig-Holstein once was separated from Denmark; but rather that
this is the necessary presupposition for the very existence of a nominatum of the
expression 'after the separation of Schl.-H. from D.' Yet, our sentence could also be
interpreted to the effect that Schl.-H. was once separated from D. This case will be
considered later. In order to grasp the difference more clearly, let us identify ourselves
with the mind of a Chinese who, with his thrilling knowledge of European history,
regards it as false that Schl.-H. ever was separated from D. This Chinese will regard
as neither true nor false the sentence as interpreted in the first manner. He would deny
it to any nominatum because the dependent clause would be lacking a nominatum.
The dependent clause would only apparently indicate a temporal determination; but
if the Chinese interprets our sentence in the second manner, then he will find it
expressing a proposition which he would consider false, in addition to a component
which, for him, would be without nominatum.
lated to each other. But this could be the case only if an indeterminately denoting constituent were absent; 11 but then universality would also be missing.

If a time point is to be indeterminately indicated in a conditional and a concluding clause, then this is not infrequently effected by tempus praesens of the verb, which in this case does not connote the present time. It is this grammatical form which takes the place of the indeterminately indicating constituent in the main and the dependent clause. “When the sun is at the Tropic of Cancer, the northern hemisphere has its longest day” is an example. Here, too, it is impossible to express the sense of the dependent clause in a main clause. For this sense is not a complete proposition; if we said: “the sun is at the Tropic of Cancer” we would be referring to the present time and thereby alter the sense. Similarly, the sense of the main clause is not a proposition either, only the whole consisting of main and dependent clause contains a proposition. Further, it may occur that several constituents common to conditional and concluding clause are indeterminately indicated.

It is obvious that subjective clauses containing ‘who’, ‘what’, and adverbial clauses with ‘where’, ‘when’, ‘whenever’, ‘wherever’ are frequently to be interpreted, inasmuch as their sense is concerned, as conditional sentences, e.g., “He who touches pitch soils himself”.

Conditional clauses can also be replaced by relative clauses. The sense of the previously mentioned sentence can also be rendered by “the square of a number which is smaller than 1 and larger than α, is smaller than 1 and larger than α.”

Quite different is the case in which the common constituent of main and dependent clause is represented by a proper name. In the sentence: “Napoleon who recognized the danger to his right flank, personally led his troops against the enemy’s position” there are expressed two propositions:

1. Napoleon recognized the danger to his right flank.
2. Napoleon personally led his troops against the enemy’s position.

When and where this happened can indeed be known only from the context, but is to be viewed as thereby determined. If we pronounce our whole sentence as an assertion we thereby assert simultaneously its two component sentences. If one of the components is false the whole is false. Here we have a case in which the dependent clause by itself has a sense in a complete proposition (if supplemented by temporal and spatial indications). The nominatum of such a clause is therefore a truth-value. We may therefore expect that we can replace it by a sentence of the same truth value without altering the truth of the whole. This is indeed the case.

11 Occasionally there is no explicit linguistic indication and the interpretation has to depend upon the total context.

but it must be kept in mind that for a purely grammatical reason, its subject must be ‘Napoleon’; because only then can the sentence be rendered in the form of a relative clause attaching to ‘Napoleon’. If the demand to render it in this form and if the conjunction with ‘and’ is admitted, then this limitation falls away.

Likewise, in dependent clauses with ‘although’ complete propositions are expressed. This conjunction really has no sense and does not affect the sense of the sentence; rather, it illuminates it in a peculiar fashion. 18 Without affecting the truth of the whole the implicate may be replaced by one of the same truth value; but the illumination might then easily appear inappropriate, just as if one were to sing a song of sad content in a cheerful manner.

In these last instances the truth of the whole implied the truth of the component sentences. The situation is different if a conditional sentence expresses a complete proposition; namely, when in doing so it contains instead of a merely indicating constituent a proper name or something deemed equivalent to a proper name. In the sentence: “if the sun has already risen by now, the sky is heavily overcast”, the sense is the present—therefore determinate. The place also is to be considered determinate. Here we can say that a relation is posited such that the case does not arise in which the antecedent sentence nominates the True and the consequent sentence nominates the False. Accordingly, the given (whole) sentence is true if the sun has not as yet risen (no matter whether or no the sky be heavily overcast), and also if the sun has risen and the sky is heavily overcast. Since all that matters are only the truth-values, each of the component sentences can be replaced by another one of the same truth value, without altering the truth-value of the whole sentence. In this case also, the illumination would usually seem inappropriate; the proposition could easily appear absurd; but this has nothing to do with the truth-value of the sentence. It must always be remembered that associated thoughts are evoked on the side; but these are not really expressed and must therefore not be taken account of; their truth-values cannot be relevant. 13

We may hope we have considered the simple types of sentences. Let us now review what we have found out!

The sense of a subordinate clause is usually not a proposition but only part of one. Its nominatum is therefore not a truth-value. The reason for this is either: that the words in the subordinate clause have only indirect nominata, so that the nominatum, not the sense, of the clause is a proposition, or, that the clause, because of a contained indeterminately indicating constituent, is incomplete, such that only together with the principal

18 Similarly in the cases of ‘but’, ‘yet’.

13 The proposition of the sentence could also be formulated thus: “either the sun has not as yet risen or the sky is heavily overcast”. This shows how to interpret this type of compound sentence.
by the restitution of Alsace-Lorraine" there are expressed two propositions, which, however, do not correspond to the main and the dependent clause—namely:

1. Bebel believes that France's desire for vengeance could be assuaged by the restitution of Alsace-Lorraine;
2. France's desire for vengeance cannot be assuaged by the restitution of Alsace-Lorraine.

In the expression of the first proposition the words of the dependent clause have indirect nominata; while the same words, in the expression of the second proposition, have their usual nominata. Hence, we see that the dependent clause of our original sentence really is to be interpreted in a twofold way; i.e., with different nominata, one of which is a proposition and the other a truth-value. An analogous situation prevails with expressions like 'to know', 'to recognize', 'it is known'.

A condition clause and its related main clause express several propositions which, however, do not correspond one-to-one to the clauses. The sentence: "Since ice is specifically lighter than water, it floats on water" asserts:

1. Ice is specifically lighter than water.
2. If something is specifically lighter than water, it floats on water.
3. Ice floats on water.

The third proposition, being implied by the first two, would perhaps not have to be mentioned expressly. However, neither the first and the third, nor the second and the third together would completely render the sense of our sentence. Thus we see that the dependent clause 'since ice is specifically lighter than water' expresses both our first proposition and part of the second. Hence, our clause cannot be replaced by another of the same truth-value; for thereby we are apt to alter our second proposition and could easily affect its truth-value.

A similar situation holds in the case of the sentence: "If iron were lighter than water it would float on water". Here we have the two propositions that iron is not lighter than water and that whatever is lighter than water floats on water. The clause again expresses the one proposition and part of the other. If we interpret the previously discussed sentence: "After Schleswig-Holstein was separated from Denmark, Prussia and Austria fell out with one another" as containing the proposition that Schleswig-Holstein once was separated from Denmark, then we have: firstly, this proposition, secondly, the proposition that, at a time more precisely determined by the dependent clause, Prussia and Austria fell out with one another. Here, too, the dependent clause expresses not only one proposition but also part of another. Therefore, it may not generally be replaced by another clause of the same truth-value.

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14 This may be of importance in the question as to whether a given assertion be a lie, an oath or a perjury.
It is difficult to exhaust all possibilities that present themselves in language; but I hope, in essence at least, to have disclosed the reasons why, in view of the invariance of the truth of a whole sentence, a clause cannot always be replaced by another of the same truth-value. These reasons are:

1. that the clause does not denote a truth-value in that it expresses only a part of a proposition;
2. that the clause, while it does denote a truth-value, is not restricted to this function in that its sense comprises, beside one proposition, also a part of another.

The first case holds

a. with the indirect nominata of the words;
b. if a part of the sentence indicates only indirectly without being a proper name.

In the second case the clause is to be interpreted in a twofold manner; namely, once with its usual nominatum; the other time with its indirect nominatum; or else, the sense of a part of the clause may simultaneously be a constituent of another proposition which, together with the sense expressed in the dependent clause, amounts to the total sense of the main and the dependent clause.

This makes it sufficiently plausible that instances in which a clause is not replaceable by another of the same truth-value do not disprove our view that the nominatum of a sentence is its truth-value and its sense a proposition.

Let us return to our point of departure now.

When we discern generally a difference in cognitive significance between "a = a" and "a = b" then this is now explained by the fact that for the cognitive significance of a sentence the sense (the proposition expressed) is no less relevant than its nominatum (the truth-value). If a = b, then the nominatum of 'a' and of 'b' is indeed the same and therefore also the truth-value of "a = b" is the same as that of "a = a". Nevertheless, the sense of 'b' may differ from the sense of 'a'; and therefore the proposition expressed by "a = b" may differ from the proposition expressed by "a = a"; in that case the two sentences do not have the same cognitive significance. Thus, if, as above, we mean by 'judgment' the transition from a proposition to its truth-value, then we can also say that the judgments differ from one another.