Exam II
Math 021 Section Z1
Instructor: Eric Clark
June 15, 2014

The exam is out of 60 points. There are a total of 6 questions, 10 points each.

Do not discuss the contents of this exam with anyone. (including Google!)

You must show all work to receive full credit.

Name______________________________
**Problem 1**: Implicit Differentiation and Related Rates

*(part a:)* Suppose $y$ is a function of $x$ (i.e. $y = y(x)$). Use implicit differentiation to solve the following equation for $\frac{dy}{dx}$. Express your answer with positive exponents for all variables.

$$e^{\sin(y^3)} = \ln(x^2 + y^3)$$

*(part b:)* A spherical snowball is placed in the sun. The sun melts the snowball so that its radius decreases at a rate of $\frac{1}{4}$ inches per hour. Find the rate of change of the volume of the snowball with respect to time at the instant the radius is 4 inches.
Problem 2: (Optimization) Given the function: \( f(x) = \frac{e^{x^2}(x^2 - 1)^{10}}{x^2 + 1} \)

a: Find the derivative using logarithmic differentiation. Then find all critical points.

b: Find the intervals where the function is increasing and decreasing. Use this to classify these critical points.
Problem 3: Find all relative extrema, intervals of concavity and inflection points of the following equation.

\[ f(x) = x^{7/3} + x^{4/3} \]
**Problem 4:** Find all absolute extrema of \( f(x) = e^{-\frac{4}{x} + 4x + 3} \) on the interval \([0, \infty)\)
Problem 5: You are trying to build a ski rack for your car. An open rectangular box is to be made by cutting a square corner from a 8ft by 3ft piece of metal and then folding up the sides. Let $x$ represent the side-length of the square being cut from the corners of the metal. Find the value of $x$ that maximizes the volume of the box.
**Problem 6:** Evaluate the following limits using L’hospital’s Rule when necessary.

part a: \[ \lim_{x \to 0} \cot(2x) \sin(6x) \]

part b: \[ \lim_{x \to 0} \frac{1 - e^{2x}}{\sec(x)} \]

part c: \[ \lim_{x \to 0} (1 - 2x)^{1/x} \]