Exam I

Math 021 Section Z1

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• The exam is out of 60 points. There are a total of 6 questions, 10 points each.

• Do not discuss the contents of this exam with anyone. (including Google!)

• You must show all work to receive full credit.

Name______________________________
Problem 1: Continuity.

a: Using interval notation, give the interval where the function is continuous:

\[ f(x) = \frac{\ln(x^2 - 4)}{x^2 - 16} \]

b: Find the value of k that makes the function continuous:
\[ f(x) = \begin{cases} 
  kx^2 + 1 & \text{if } x \leq 2 \\
  x + k & \text{if } x > 2
\end{cases} \]
Problem 2: Evaluate the following limits.

a: \( \lim_{x \to -2} \frac{x^3 - 3x^2 - 10x}{x^2 + 2x} \)

b) \( \lim_{x \to 81} \frac{\sqrt{x} - 9}{x - 81} \)

c: \( \lim_{x \to \infty} \frac{8 - 4x^3}{5x^2 + 7x + 10} \)

d: \( \lim_{x \to \infty} \frac{\sqrt{5x + 3x^4}}{6 + 7x + 4x^2} \)
Problem 3: Suppose the position function of a skier during a run can be modeled by the function: \( s(t) = 3x^2 + 4x \). Where \( t \) is time measured in seconds, the position \( s(t) \) is measured in meters from the base lodge.

(a) Find the velocity function, \( v(t) = s'(t) \), using the formal definition of the derivative

\[
s'(x) = \lim_{h \to 0} \frac{s(x + h) - s(x)}{h}
\]

(b) Find the average velocity between \( t_1 = 1 \) and \( t_2 = 3 \) seconds. (Hint: This is an average rate of change, or the slope of the secant line between these two points: \( s_{\text{avg}} = \frac{s(t_2) - s(t_1)}{t_2 - t_1} \)
Problem 4: For each of the following functions find the derivative with respect to \( x \).

a) \( \frac{d}{dx}[x^5 + 3\pi^2 x^3 + \ln(6)] \)

b) \( \frac{d}{dx}[\frac{x^2 + \sqrt{x}}{x}] = \)

c) \( \frac{d}{dx}[3x^{\frac{1}{3}} + 2\sqrt{x} - \frac{1}{x}] = \)

d) Let \( f(x) = x^3 + 4x^2 + 5 \). Find \( f'(x) \). Then find the equation of the tangent line when \( x = 1 \) i.e. at the point \( (1, f(1)) \)
Problem 5: For each of the following functions find the derivative with respect to \( x \).

a: \( f(x) = (\sqrt{x^3 + 1})(x + 5)^5 \)

b: \( f(x) = \frac{\ln(x^2)}{x^2 + 1} \)

c: \( f(x) = e^{x^3}(x^2 + 1) \)
Problem 6: For each of the following functions find the derivative with respect to $x$.

a) 

$$f(x) = (2x^5 + 3x^3 + 3x + 5)^{10}$$

b) 

$$g(x) = e^{(3x^2 + \sqrt{x})^3}$$

c) 

$$f(x) = \ln(4x^2 + 3)^{4/5}$$