Instructions: The format for this exam is open-note and open-book. No collaboration or consultation is acceptable nor will it be tolerated; any violation of this constitutes academic dishonesty and will be handled according to University policy. This policy is not limited to fellow classmates in ME304: it extends to all people external to the class including faculty and other graduate students. Please note the due date and time will be strictly enforced: I reserve the right to reject any exam submitted late.

1. Laplace’s Equation in an Annular Sector. A 2-D annular sector of electrically-conducting material is defined by \( a \leq r \leq b \) and \( 0 \leq \theta \leq \pi/2 \). The edges of the annulus and the inner radius are all electrically-grounded; the outer radius is set at some electric potential \( u(b, \theta) = f(\theta) \). Mathematically, the electric potential is determined by

\[
\nabla^2 u = 0
\]

with boundary conditions

\[
u(a, \theta) = 0 \quad u(r, 0) = u(r, \pi/2) = 0 \quad u(b, \theta) = f(\theta).
\]

(a) Find the general solution to this problem for an arbitrary \( f(\theta) \) in terms of undetermined series coefficients.
(b) Find a particular solution for the case \( f(\theta) = 1 \).

2. 1-D Heat Diffusion with Lateral Cooling. Suppose that heat is lost from the lateral surface of a thin rod of length \( L \) into a surrounding medium at temperature zero. The heat diffusion equation then takes on the form

\[
\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} - hu
\]

where \( \alpha \) and \( h \) are constants related to the thermal diffusivity and surface convection, respectively.

(a) Assume the initial temperature distribution is \( u(x, 0) = f(x) \). At \( t = 0 \) the ends at \( x = 0 \) and \( x = L \) are instantaneously brought to zero and held at that temperature. Find a solution for the transient temperature distribution \( u(x, t) \) using the method of separation of variables. (Hint: Be sure to carefully consider the \( \lambda = 0 \) eigenvalue case)
(b) Find the particular solution corresponding to the case \( f(x) = x/L \).