Demonstration of Fourier Transforms: Signal Averaging

When a signal contains noise, one means for reducing its effects is through averaging. Here a collection of signals are acquired, their respective FFTs computed, and then ensemble averaged. The idea is that random noise in the FFTs will, in the limit of an infinite number of averages, go to zero.

Noisy Signal Generation

Given the following:

- \( f := 10 \) frequency in Hz
- \( \text{samp} := 100 \) sampling rate in Hz
- \( \text{npt} := 1024 \) number of data points acquired

Define our simulated waveform:

\[
\text{signal}_i := \sin(2 \pi f \cdot \text{time}_i) + 0.5 \left( \text{rnd}(1) - \frac{1}{2} \right)
\]

Compute the FFT of this signal and then plot the magnitude (modulus) of the Fourier coefficient:

\[
j := 0 \ldots \frac{\text{npt}}{2}
\]

\[
\text{freq}_j := j \cdot \text{df}
\]

\[
\text{MyFFT} := \text{fft} (\text{signal})
\]
**Filtering Properties of FFT Averaging**

\[ n_{\text{max}} := 100 \]

\[ k := 0 \ldots n_{\text{max}} - 1 \]

\[ i := 0 \ldots \text{npt} - 1 \]

Compute FFTs for \( n_{\text{max}} \) different signals

\[ \text{signal}_{i, k} := \sin(2 \pi f \cdot \text{time}_i) + \left( \text{rnd}(1) - \frac{1}{2} \right) \]

\[ M^{(k)} := \text{fft(signal}^{(k)}) \]

Compute the average Fourier Spectrum

\[ \text{MyAvg} := \frac{1}{n_{\text{max}}} \cdot \sum_{k = 0}^{n_{\text{max}} - 1} M^{(k)} \]

A Single FFT

\[ \langle |M^{(k)}| \rangle_i \]

Average of 100 FFT's

\[ \langle |\text{MyAvg}| \rangle_i \]