1. The child I interviewed, K, is a seven-year-old girl. She is a very social student, who benefits from talking with other children, and utilizing "masters", well-versed students, student teachers, and adults to assist her with questions. She has an amazing imagination, and quite an eye for creating. She has headed up a puppet-making center during the class's investigation time, which she and the other girls have created elaborate scripts for. It seems that she benefits from the use of tangible objects to build ideas from.

2. This math artifact contains a series of arithmetic problems in the form of counting dots, solving addition and subtraction problems, connecting addition and subtraction problems to money, and synthesizing addition and subtraction equations.

Specifically within the Vermont Framework of Standards and Learning Opportunities this artifact seems to be working with:

7.6. Students understand arithmetic in computation and they select and use, in appropriate situations mental arithmetic, pencil and paper, calculator, and computer.
    This is evident when students:
    a. add, subtract, multiply, and divide whole numbers, with and without calculators.

7 10 Students use concrete, formal, and informal strategies to solve mathematical problems, apply the process of mathematical modeling, and extend and generalize mathematical concepts.
    Students apply mathematics as they solve scientific and technological problems, or work with the technological systems. This is evident when students:
    a. Solve problems by reasoning mathematically with concepts and skills expected in these grades
    b. Determine what the question, assignment, or problem is really asking them to do
    d. Make connections between concepts in order to solve problems.

In addition, the artifact asks students to justify their answers with written responses. By writing out how she solved the problem, the student is asked to think more deeply about the logic and strategies that she is using to get to the answer that she determined to be "right". In the interview, K was asked to further explain a lot of the vague answers she gave to justify her math responses. To K, I am very certain that the response, “I counted" makes sense in an addition problem, but I wanted her to stretch her thinking beyond that. I wanted to gain a deeper understanding of her thinking articulating her written response more clearly and more elaborately than she had on the sheet.

3. According to Wood, Piaget places a 7 year old at the level of concrete operational period. During this period, children are beginning to think logically. They have mastered the concepts of reversibility, conservation, seriation, classification, and the understanding of rules. This specific artifact of work offers examples of understanding of rules and logical thought. Kennedy and Tipps elaborate on this by saying that a concrete operational thinker can form mental images of objects and manipulate those objects in her mind. That is the basis of reversibility. Kennedy and Tipps remind teachers that concrete operational thinkers need supplements to math problems, concrete objects and pictures, to the abstract problems spoken or on paper in order to solve them. Based on those criteria, I would place K as a beginning concrete operational mathematician. She understands the rules of subtraction in taking 5 from 12, and utilizes logic in solving it. Furthermore, logical thinkers can begin to solve problems in their heads, by creating mental images of the problem. I asked if she could explain how she got to her answer. (page 18, 3)

K: I counted.
L: Did you count forward or backward?
She also demonstrates her logic when I asked her to solve a problem that was not in her packet, in order to further her discussion in a specific subtraction problem. (figure 2)

L: Well, what do you usually do to solve subtraction problems?
K: I took away the lowest number.
L: Do you do it in your head?
K: Sometimes in my head, sometimes I do it on a piece of paper.
L: Can you show me if I said take 9 away from 25. Can you show me on the paper how you would get the answer?
K: First, I would make 25 marks. Then I would take 9 away. Then I would count how many are left over. 16.
L: Sometimes you write it out, and take some away.

Talking to K also gave me the indication that she has grasped the concept of conservation. When I set up the protocol of the conservation question, K very quickly showed that she could articulate that nothing changes between the two rows.

L: I'm going to make two lines of linker cubes. How many cubes are in each row?
K: 5
L: Now are there more of them, or the same?
K: They're the same.
L: How did you know there were the same?
K: I counted them.

4. The previous example used in response three is a great one to demonstrate her understanding of the concept of subtraction. Within this interaction, K proved that she understands that subtraction is "taking away". She has brought the prior knowledge that she needs to break the whole into parts and then take away the part. In addition, she has brought the idea of the use of imagery, of pictorial representation to solve the problem so she doesn't have to rely on rote memory. She also uses this in addition in solving 10+7 = 17. (Page 20, 4)

L: The question states: Write the Answers: 10 + 7 =
It says you counted, but can you tell me a little more. Can you tell me out loud what your brain is thinking if I ask you 10 + 7?
K: Um, I'm not sure.
L: How would you solve that problem?
K: I would say, 10, 11, 12, 13, 14, 15, 16, and 17. (She counts on her fingers.)
L: So you would use 10 as a starting point, then count on your fingers?

She demonstrates an understanding that addition is another part/whole relationship, in the reverse, and she can combine the 10, the starting point, and incorporate the other 7 by using a concrete style of counting on her fingers.

5. When K was asked to create her own equations, it seems that she seems to be placed into a situation of disequilibrium. She is able to synthesize addition equations, but not to create subtraction equations. It appears that K may be establishing hypotheses about reversibility. She seems to understand the combining functions of addition and the separation functions of
subtraction, but when asked to create a group of equations, she really struggled to explain the subtraction equation that her father gave her for her homework packet. (page 18, 1)

L: K, for the first one, how did you think of five equations that equal 15. How did you come up with these? (Points to paper.) What sort of strategies did you use?
K: I kind of put numbers together that make fifteen.
L: Did you do that for addition and for subtraction. I noticed that you have and equation that is 20-5 = 15.
K: Well, my dad told am that one. He helps me with homework.
L: What sorts of questions did he ask you?
K: He tried to help me understand it better.

It seems that K is at level of understanding with addition that allows her to "put numbers together" but she has a harder time taking numbers away. She can utilize pictorial or concrete strategies to solve subtraction problems, but when asked turn that thinking around, it presents a level of difficulty that stretches her beyond her developmental level. There appears to be no disequilibrium in the concept of conservation with K. She stated that there was the same number of cubes in both lines, and that she determined that by counting them.

6. In this artifact, K seemed to building content about the arithmetic operations of addition and subtraction. I feel that K really needs either concrete or pictoral/representational based lessons to strengthen her understanding of operations, like addition and especially subtraction. She could do and explain how to do a subtraction problem fine when I allowed her to show me how she understands it best, by making the lines to represent the whole, and then marking off the part she is taking away. Similarly, since she is such a social being, she would probably be able to take some important ideas from those who have "mastered" subtraction, and by working in pairs or small groups with manipulatives, she could "see" subtraction as the same part/whole concept that addition is, but in a more visual form. If she did that, she may be able to become more versed in synthesizing her own problems, instead of just being able to solve subtraction problems.

I also feel that K would benefit from a more authentic math experience. Granted, I was not present for the lesson that led to the homework packet, it seems that using the MathLand worksheets seem to have a tendency to take the math and take it out of context. Creating scenarios where a child would need to use addition or subtraction in more of a real life situation would make the formation of equations much more clear to K instead of arbitrarily asking a child to synthesize a set of equations without any basis of origin. 7. According to Underhill, there are five different areas that a child develops in: content readiness, pedagogical readiness, maturational readiness, affective readiness, and contextual readiness. K has demonstrated content readiness in addition within this artifact, she has the basic knowledge of addition, and she has some addition math facts.

L: How did you know that 5 and 5 is 10 and 6 and 6 is twelve, and 7 and 7 is 14?
K: Because I know.
L: Those are ones you just know? Do you just remember them in your head?
K: I know 5’s and 6’s and 7’s. Because 5 and makes 10 and 6 and 6 makes 12.

She can also demonstrate it concretely, by counting on her fingers. She has content readiness in subtraction, too, She has some strategy to represent and solve subtraction, too.

Similarly, she demonstrates pedagogical readiness by utilizing the pictures in the worksheets to count the number of objects, to determine how much money is being represented. She can create
her own pictures, too, to represent "taking away". (Fig. 2)

Maturational readiness is what Piaget describes, and is the concrete operational stage that I believe K is moving into. She possesses a lot of the knowledge that a concrete operational thinker has, like conservation, and the idea that she has mental images to solve problems, but she has yet to master the concept of inverse operations, such as addition and subtraction.

I observed K's affective readiness to be very receptive. Even when she could not explain to me her reasoning, she always seemed to have a good attitude about the problems in front of her.

This seems to be one of the most important areas of readiness, because she might not be capable of moving ahead if she is experiencing anxiety about her math abilities.

I think that K's edge of her zone of proximal development is the application of math concepts. She seemed to be able to demonstrate to me her knowledge of concepts, but when she was asked to apply addition and subtraction to a "real-life" problem, it was a problem about buying something, she struggled. She had the correct answer on the paper, but had a lot of difficulty in describing how she got to it.

The problem is Jose has these coins, (a dime, a nickel, and a penny) How much does he have, can he buy a ball for $.16. Show your thinking.

L: How much is this one?
K: 10, I'm not sure.
L: how many cents would Lake make?
K: 5, then 11, 12, 13, 14, 15, 16.

It seems that she has difficulty applying addition to a problem where she first has to extrapolate the numbers from the coin symbols. I feel that her ceiling on her zone of proximal development is contextual readiness, and in her concrete operations specifically the concept of reversibility of subtraction and addition.

8. K's mathematical reasoning strengths seems to lie in patterning (See the interview on her rationale for even/odds), in her 1:1 correspondence (See her justification of addition problems in the interview transcript), and may have a little problem in the reversibility of addition and subtraction problems. She demonstrates that in her inability to rationalize her subtraction equation, but has little difficulty making addition ones.

In connecting K's math learning and literacy learning, I see the strength being in the socialness of learning, and the group activities benefiting her better. She is articulate in her verbalizations, and can explain her reasoning fairly well, but when it is dependent upon her literacy skills, it seems that her spelling and writing could be limiting her responses on a worksheet like the one in focus. I do notice about K that she struggles a lot with her reading and writing, but her affective readiness is so positive in all areas. She keeps her positive attitude when she has to tackle a large writing assignment, and proved to have a positive attitude when asked to stretch the justification of her answers in the packet.

She uses a lot of the same strategies for writing, and math, specifically the use of more experienced writers, or mathematicians. She has demonstrated that during journaling time, asking me to spell words, or using her dictionary. Similarly, she used her father as a resource to find an equation with the desired answer. I also see her concrete cognition evident in watching her brainstorm journaling topics. I can see
the words being formed in her head, and the manipulation she performs to order the sentences in a logical fashion, and a descriptive one as well.

Her artwork, in terms of the bicycle I asked her to draw was fairly detailed. She began with the wheels, and then she drew the handlebars. After she connected the two, she drew the seat, then the spokes, and finally after exclaiming she forgot something, she drew pedals. She paid close attention to details, the wheels are at the same size, the spokes are equal quantities on both wheels, the pedals are the same size, and on opposite sides of the bike, and the seat is proportional to the rest. However, there are at least two perspectives being represented in her drawing, which is indicative of some "illogical" theories, or some difficulty explaining, as was the case with K. She definitely was getting the answers correct, but she could not explain how consistently.

K proved a lot of different things about mathematics in an elementary school context to me.

She proved different levels of cognition can effect how a child responds to a math problem on a worksheet, and that talking about it can open up deeper levels of understanding, She also reinforced my feelings that authenticity really deepens a math experience, and creating an artificial experience, like a worksheet may make it more difficult for a child to justify her answers.
Works Cited


Vermont's Framework of Standards and Learning Opportunities, Vermont State Board of Education. Vermont, 1996. pp 7.2, 7.4

Transcript of the Student Interview

L. K. you did this for homework the other day, and I wanted to ask you a few questions about your thinking when you came up with your math answers. For the first one, how did you come up with five equations that equal 15. How did you come up with these? (Point to paper) What sort of strategies did you use?

K. I kind of put numbers together that make fifteen.

L: Did you do that for addition and for subtraction? I noticed that you have an equation that is 20-5 15.

K: Well, my dad told me that one. He helps me with homework.

L: What sorts of questions did he ask you?

K: He tried to help me understand it better

L: and this one?

K: I counted.

L: Did you count forward or backward?

K: Forward. I stamped the highest number, and then I counted. I took away the number it says.

L: You made twelve stamps, and then too away the number?

K: No, I did it in my head.

L: How did you know this was ten cents? There are only six coins, (She counts them.)
K: Those aren't coins.

L: No?

K: No, they're pennies
L: But there are only six, right? So, how did you know it was ten cents? She reads K's answer again. There are pennies and a nickel. That makes ten cents?

K: Yeah.

L: How conic?

K: My dad helped me.

L: He helped you with that one, too? Well, did you know that a nickel is worth five cents?

K: Yeah

L: So, (points to nickel) 5, (points to penny) 6... (points to next)

K: 7, 8, 9, 10. yeah.

L: How did you know that 5 and 5 is 10 and 6 and 6 is twelve, and 7 and 7 is 14?

K: Because I know.

L: Those are ones you just know? Do you just remember them in your head?

K: I know 5s and 6s and 7s. Because 5 and makes 10 and 6 and 6 makes 12.

L: okay, you said you subtracted to find the answer for... (number 4 page 19). It looks like you wrote 25 -IS and you marked 10 as your answer. Is that the same idea as before, did you stamp the number, and then take some away?

K: Um, I can't remember.

L: Well, what do you usually do to solve subtraction problems?

K: I. take away the lowest number.

L: do you do it in your head?

K: sometimes in my head. sometimes I do it on a piece of paper.

L: can you show me if I said take 9 away from 25. Can you show me on the paper how you would get the answer?

K: first, I would make 25 marks. Then I would take 9 away. Then I would count how many are left over. 16.

L: Sometimes you write it out, and take some away. I understand. Oh. tricky question: How did you know that 6 is an even number.

K: OH, cause I know even, and I know uh......I know even and I know odd.

L: what's an even number? What would an even number other than 6 be?

K: I would do... maybe, 1, 3...

L: those are even numbers?
K: I think so.

L: What is an odd number?

K: 2, 4, 6, 8. I'm saying the whole numbers, but I'm skipping some, like 1, 2, 3, 4, 5, 6.

L: That's how you 01 them apart? I understand. How about this: 20 #2. How much is this one?

K: 10, I'm not sure. A nickel and a penny. I know how much he has.

L: How many cents would take make?

K: 10, then 5, 10, 11, 12, 13, 14, 15, 16.

L: 20 # 4, it says you counted, but can you tell me a little more. Can you tell me out loud what your brain is thinking if I ask you 10 + 7?

K: Um, I'm not sure.

L: How would you solve that problem?

K: I would say, 10, 11, 12, 13, 14, 15, 16, 17.

L: So you would use 10 as a starting point, then count on your fingers?

K: I couldn't count from 1.

L: Would you run out of fingers?

K: Yeah.

L: What were you doing on this page? Just on this line?

K: I knew what 2+2 was, 4 + 3, I counted that. I counted 7+7 on my fingers. But I knew the others. My math teacher says, 'just say what you know.'

L: I'm going to make two lines of linker cubes. How many cubes are in each row?

K: 5

L: Now are there more of them, or the same?

K: They're the same.

L: How did you know there were the same?

K: I counted them.

L: Now here's the fun part. I would like you to draw me a bicycle (Started with wheels, then handlebars, then spokes then pedals.)
The Answer Is 15

Write at least 5 equations that have this answer.

6 + 9 = 15
9 + 6 = 15
10 + 5 = 15
20 - 5 = 15

Write the Answer

How many dots are here?

I Counted

Choose the Answer

12 - 7 = 5

A 6
B 8
C 15
D 5

Tell how you know.

I Counted

True or False?

This is 10¢.

Prove it. They are pennies and a nickel.

Note to the Teacher: Permission is given to reproduce this page.
1. At the Pool
   Carlos swam 3 laps at the pool 3 times.
   Aaron swam 4 laps at the pool 4 times.
   How many laps did the boys swim in all?

2. The Answer Is 16
   Write 2 addition equations and 2 subtraction equations that have this answer.
   \[ 2 \times 2 = 4 \]
   \[ 4 + 4 = 12 \]
   \[ 3 - 2 = 1 \]

3. Write the Answers
   \[ 5 + 5 = 10 \]
   \[ 6 + 6 = 12 \]
   \[ 7 + 7 = 14 \]

4. Choose the Answer
   Marla had 25¢. She spent 15¢. How much did she have left?
   - 10¢
   - 15¢
   - 5¢
   - 20¢

   \[ 25 - 15 = 10 \]
True or False?

6 is an even number. Prove it. **I counted in even numbers.**

How Much?

José has 3 coins. How much does he have in all? Can he buy a ball for 15¢? Show your thinking.

\[
\begin{align*}
10 + 5 + 1 &= 16 \\
\text{yes} &= 15
\end{align*}
\]

The Answer Is 20

Write 7 equations that have this answer.

\[
\begin{align*}
10 + 10 &= 20 \\
5 + 5 + 5 + 5 &= 20 \\
9 + 1 + 6 &= 16 \\
8 + 2 + 5 + 1 &= 16
\end{align*}
\]

Write the Answers

\[
\begin{align*}
10 + 7 &= 17 \\
10 + 3 &= 13 \\
10 + 9 &= 19
\end{align*}
\]

Tell how you know. **I counted**
Race to the Finish

How fast can you add and subtract numbers in your head?
Start at the bicycle. Race to the finish line.
Add or subtract the numbers along the way.
Write your subtotals in the squares.
Write your total on the finish line.

2 - 1 ✔ + 3 ✔ 4 + 6 ✔ 10 = ✔

3 + 4 ✔ 7 + 2 ✔ 9 - 9 ✔ 0 - 0 = ✔

5 - 3 ✔ a + 5 ✔ 7 - 0 ✔ 7 + 3 = ✔

5 + 3 ✔ e - 0 ✔ 8 + 7 ✔ 15 + 1 = ✔

9 + 4 ✔ 13 + 1 ✔ 14 + 4 ✔ 18 - 0 = ✔

2 + 2 ✔ 4 + 3 ✔ 7 - 0 ✔ 7 + 7 = ✔