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Profit and Rent in a Classical Theory of Exhaustible and Renewable Resources

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1. Introduction

A cornerstone of the classical theory of value and distribution is the notion that there exists a surplus which is distributed in the form of profits, interest and rent. Sraffa (1960) rigorously distinguishes profit and rent by reference to a notion of scarcity. He writes:

> While the scarcity of land thus provides the background from which rent arises, the only evidence of this scarcity to be found in the process of production is the duality of methods: if there were no scarcity, only one method, the cheapest, would be used on the land and there could be no rent. (Sraffa, 1960, p. 76.)

Profit derives from the exploitation of labor, but rent depends upon scarcity of natural resources in relation to the level of effective (direct and indirect) demand.

Sraffa's theory of rent has been discussed, refined and extended by a number of authors: Quadrio-Curzio (1980), Montani (1975), Kurz (1978), Gibson and Esfahani (1983) and Gibson and McLeod (1983). But in each case, Ricardo's precept of rent as payment for the "original and indestructible qualities of land" is maintained. Nowhere in this literature is the production process

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allowed to *change* the quality of land on which it operates. But if Sraffa's system is to account for land with exhaustible mineral deposits, petroleum reserves or even agricultural land for which different methods of cultivation affect land productivity, the assumption of indestructibility is obviously inadequate. This paper generalizes the treatment of nonproduced means of production offered in *Production of Commodities* (*PCMC*) to allow for exhaustible and renewable resources by applying the basic principle of Sraffa's approach to durable capital goods: land inputs into any given production process are considered qualitatively different from land output by that process. It is seen that when the theory is appropriately extended, the Sraffian dichotomy between rent and profit still applies. Natural resources may or may not be scarce depending on the social relations of production under which they are extracted and employed; that is, the real wage, the technique and the level of effective demand.

The paper is organized as follows: the first section establishes necessary and sufficient conditions for scarcity in Sraffa's system. It is seen that Sraffa only provides a necessary condition in the passage quoted above. The full characterization is somewhat more restrictive and allows for the existence of "quasirent" in an unconventional but important sense of the term. Key propositions of this section show that while indestructible land always earns rent, durable capital goods do not. The following section adapts the fixed capital system to the particular requirements of exhaustible and renewable resources and shows that when the discounted sum of the returns to resource-bearing land is determined by the cost of discovery and development of new land, resources are not scarce and the return is profit rather than rent. A theorem on perennial resources confirms that it is not *durability* but rather *reproducibility* which is at the heart of the distinction between profit and rent. The penultimate section provides two important examples of systems with transformable land, roughly analogous to the classical categories of extensive and intensive rent, in which land does earn rent. A concluding section offers some observations on the importance of the distinction between profit, rent, capital and land.

2. Rent, Profit and Quasirent

2.1 Rent

In the simple Sraffian model with no fixed capital or nonproduced means of production, capitalists exploit workers along the well-known wage-profit line. The Perron-Frobenius theorem guarantees that, for a given level of output, capitalists cannot have more, in the form of higher profits, unless workers have less in terms of any and every commodity they produce (Pasinetti (1977)). But while profit in Sraffa's system derives from the exploitation of labor, *rent* is a product of *scarcity.*

Definition 1: Land, or more generally any input, is *scarce* if and only if in order to satisfy the given level of effective demand, an alternative process must be introduced which, for a given real wage (profit rate), *lowers* the average rate of profit (wage rate)[1].

If no resource were scarce, only the process cheapest at the prevailing rate of profit (wage rate) and prices would be employed. It follows that *reproducible* capital goods are not scarce since, according to this definition, they do not limit supply and thereby force the adoption of a more productive (and costly) alternative process. "Duality" of methods is only necessary for resource scarcity and not sufficient. As we shall see, durable means of production give rise to alternative processes, yet it is not correct to say that all durable means of production are scarce[2].

The additional process removes the degree of freedom in the simple Sraffian system introduced by the price of nonproduced means of production. The system of production equations then determines all relative prices as a function of the profit (wage) rate. Rent, in this account, is fully endogenous. Workers and capitalists struggle for their shares of the surplus, but owners of scarce, nonproduced means of production have no control over the rent their tenants can pay and still earn the average rate of profit. The "landlord" class is at the mercy of the system of relative prices corresponding to the outcome of the struggle between workers and capitalists. Indeed, Montani (1975) has shown that for the same level and composition of effective demand and technical coefficients, land may be scarce at one level of wages and profits, yet redundant at another.

In *PCMC*, Sraffa revives the classical conceptions of *intensive* and *extensive* rent. For the simplest intensive system, two processes produce the same final commodity on a single quality of land.

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[1] Definition 1 can be expressed graphically by noting that it is an *interior* wage-profit line which is relevant when the most profitable technique cannot produce all the required output. See Montani (1975) and Gibson and McLeod (1983).

[2] Of course two processes may also cooperate at switchpoints, but as a matter of coincidence only.

The low-cost process is incapable of satisfying total effective demand and, thus, there is room for a second, more costly method to co-operate with the first. In this case, all land is scarce and earns intensive rent. The second method, however, may be so costly that profit-maximizing capitalists may reasonably choose to employ land of inferior quality. Capitalist competition for scarce, first quality land enables landlords to earn an extensive rent. Whether rent is intensive or extensive amounts to a problem of the choice of technique and rent may switch (and reswitch) according to the level of wages and profits.

To illustrate these and other points in a formal model, let there be n produced commodities which use $m \leq n$ nonproduced means of production. Write the price system as:

$$PB = (1+r)\,PA + wL \tag{1}$$

where A and B are the nonnegative input and output matrices respectively. P is a row vector of prices and L is a nonnegative vector of labor coefficients. r and w are profit and wage rates. In order to have a meaningful solution, A and B must be square and of order $n+m$. L and P are, therefore, of order $n+m$ as well.

Land is fully specialized in that each quality defines its own method of production. Partition the matrices A and B into four submatrices such that the first n rows correspond to produced and the last m rows correspond to nonproduced goods:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

where A_{11}, B_{11} are square, indecomposable and of order n and A_{22}, B_{22} are square of order m. Let the last m processes be insufficient to satisfy the level of final demand and let m of the remaining n processes be alternative to the last m processes. Note that if $m=n$ nonproduced means of production are required for all n goods. By the assumption of $m \leq n$ we rule out the possibility of one process using more than one nonproduced resource. With little additional loss of generality, we shall hencefor m assume that $m=n$. The units of measurement of A, B and L are defined such that each process employs one unit of land. We have the following fundamental definitions:

Definition 2: Rent to nonproduced means of production is rp_i where p_i is the price of the ith nonproduced resource.

Definition 3: Land is *indestructible* if:

$$A_{21} = B_{21} \quad \text{and} \quad A_{22} = B_{22}.$$

Definition 4: Rent to indestructible land is *extensive* if:

$$A_{21} = 0.$$

Rent to indestructible land is *intensive* if:

$$A_{21} = A_{22}.$$

Definition 5: Commodities corresponding to last m rows of A and B are *quasibasic* if there exists an m by m matrix Q such that:

$$A_{21} = A_{22}Q \quad \text{and} \quad B_{21} = B_{22}Q.$$

If $Q = 0$, commodities corresponding to the last m rows of A and B are *nonbasic* (Gibson and McLeod (1983)).

The impotence of landlords is illustrated by an essential property of the price system with nonproduced means of production.

Theorem 1: Indestructible land in systems with extensive rent is nonbasic. The structure of relative prices and the profit (wage) rate is therefore invariant with respect to a tax on land.

Proof: Rent-bearing land in extensive systems is nonbasic by Definitions 3—5. The system of equations determining relative prices and the profit (wage) rate is:

$$P_1 B_{11} = (1 + r)\, P_1 A_{11} + w L_1$$

where P_1 is the subvector of $P = [P_1\, P_2]$ and L_1 is the corresponding subvector of L. A tax on land obviously does not disturb the solution since the first n processes employ no rent-bearing land.

Theorem 2: Indestructible land in systems with intensive rent is quasibasic. The structure of relative prices and the profit (wage) rate is, therefore, invariant with respect to a tax on land.

Proof: Land in systems with intensive rent is quasibasic by Definitions 3—5. (Set $Q = I$, the identity matrix.) Let T be a diagonal mth order matrix of taxes $(1 + t_i)$ where t_i is the tax rate on the ith quality of land. The price equations (1) are now modified to read:

$$[P_1\, P_2] \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = (1 + r)\, [P_1 P_2] \begin{bmatrix} A_{11} & A_{12} \\ A_{21}\, T & A_{22}\, T \end{bmatrix} + w\, [L_1\, L_2].$$

Now postmultiply by the matrix:

$$M = \begin{bmatrix} I_{n \times n} & 0 \\ -Q & I_{m \times m} \end{bmatrix}.$$

Since land in system with intensive rent is quasibasic, the first n price determining equations can be written as:

$$P_1 [B_{11} - B_{12} Q] = (1+r) P_1 [A_{11} - A_{12} Q] + w [L_1 - L_2 Q]$$

which is again seen to be independent of taxes. Theorems 1 and 2 establish the subsumption of owners of nonproduced resources in that even if their real incomes are diminished by taxes, no changes in the real wage, profit rate or relative prices of produced goods will occur.

2.2 Profit

In this section I argue that the distinction between rent and profit does not turn on the *durability* of means of production. Though it may seem natural to consider used capital equipment as a sort of nonproduced means of production, as is common in neoclassical theory, this section shows that the distinction between rent and profit is undisturbed by the introduction of fixed capital goods. The discussion follows the work of Schefold (1971, 1980).

In place of nonproduced means of production, let the last m processes employ *used* capital goods among their means of production. There are then n processes[3]

$$B^1 = \begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} \qquad A^1 = \begin{bmatrix} A_{11} \\ 0 \end{bmatrix}$$

which are *primary* in that only new goods are employed as means of production (even though used goods are produced). These processes are partitioned such that first n rows refer to new goods and the last m rows are used goods. Processes

$$B^2 = \begin{bmatrix} B_{12} \\ B_{22} \end{bmatrix} \qquad A^2 = \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix}$$

[3] The fixed capital system has been discussed extensively. See Sraffa (1960), Roncaglia (1978), Schefold (1971), Schefold (1980), Varri (1980), Baldone (1980), van Schaik (1976), and Abraham-Frois and Berrebi (1979).

are referred to as *secondary* processes. Associated with the ith primary process, there are m_i secondary processes and we have:

$$m = \sum_{i=1}^{n} m_i.$$

The processes are assumed to be ordered such that the first m_1 secondary processes belong to the first primary process, the second m_2 to the second primary process and so on. Each process may use any number of machines, but each machine at each vintage defines a separate secondary process. For simplicity, let there be no superimposed joint production so that each sequence of secondary processes produces only one new good. Also assume that the same used capital good does not participate in processes which produce different goods.

Theorem 3 insures that used capital goods are neither nonbasic nor quasibasic and cannot therefore be said to earn rent.

Theorem 3: Capital goods (whether new or used) in fixed capital systems are neither nonbasic nor quasibasic.

Proof: Assume the converse is true and construct a counterexample to show that it is not. (Note that by Definition 5, if capital goods are not quasibasic then they are not nonbasic.) Consider an economy in which two processes produce a commodity (good 1) which when used as means of production lasts for two periods. Output and input matrices can be written:

$$B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \qquad A = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix} \; L = [l_1 \, l_2]$$

with $b_{21} = a_{22}$, that is, the output of the used good in process one is the input of the used good in process two. Even for this simplest example of the fixed capital system, it is not possible to find a Q such that

$$0 = a_{22} Q \quad \text{and} \quad b_{21} = 0 Q$$

and, consequently, used capital goods are basic commodities.

Theorem 3 shows that owners of new and used capital equipment exploit labor on the same footing. A tax on used capital equipment will generally alter the real wage in terms of all but the numeraire commodity and change the rate of profit and associated structure of relative prices.

While it is now obvious that used capital goods are basic, Schefold (1971) has shown that there exists a matrix M' [4]

$$M' = \begin{bmatrix} I_{n\times n} & 0 \\ Q & I_{m\times m} \end{bmatrix}$$

with submatrix $Q=\{q_{ii}\}$, $i=1,2,\ldots,n$ where n is the number of primary processes. Each q_{ii} is itself a vector of length m_i; that is, $q_{ii}=\{\pi^{-1},\pi^{-2},\ldots,\pi^{-m_i}\}$ with $\pi=(1+r)$, such that when post multiplied by M', the price-determining equations reduce to:

$$P_1\,[B_{11}+B_{12}\,Q]+P_2\,B_{21}=\pi\,\{P_1\,[A_{11}+A_{12}\,Q]+ \\ +P_2\,A_{22}\,Q\}+w\,[L_1+L_2\,Q]$$

but since the output of used capital goods of the ith process is the input of $i+1$th process

$$P_2\,B_{21}=\pi\,P_2\,A_{22}\,Q \tag{2}$$

so that we have:

$$P_1\,[B_{11}+B_{12}\,Q]=\pi\,P_1\,[A_{11}+A_{12}\,Q]=w\,[L_1+L_2\,Q] \tag{3}$$

$$P_1\,B_{12}=\pi\,[P_1\,A_{12}+P_2\,A_{22}]+w\,L_2. \tag{4}$$

Eqs. (3) and (4) constitute an "integrated system" and must be interpreted with care since they can create the (false) impression that used capital goods are nonbasic in that their prices, P_2 appear as residuals. From (3) it is obvious, however, that the secondary production processes affect the prices of new goods from which prices of used machines are deduced. Though it may appear that after a linear transformation by matrix M', used capital goods are indistinguishable from land, it should be noted that *any* all-basic system can be rearranged in a similar fashion[5]. Note that prices of used capital goods in the fixed capital system are determined essentially by the rule that the rate of profit be everywhere uniform. There is no rent: the surplus is appropriated entirely in the form of profit, whether on new or used equipment.

[4] Sraffa (1960, p. 75) describes this transformation which is essentially the same as the Manara transformation. See Manara (1980). For a detailed discussion see Schefold (1971).

[5] The foregoing is not intended to suggest that the transformation M' is of no significance. Indeed, Schefold has shown the fixed capital system is immune to most of the pathologies of general joint-product systems (Schefold, 1980, p. 140). The integrated system of Eqs. (3) behaves essentially like a single-product economy in that there is an inverse relationship between wages and profits, Sraffa's standard commodity exists, etc.

2.3 Quasirent

Is there any sense in which used capital equipment can be said to earn a quasirent? Sraffa suggests that "fossils", that is, superseded capital equipment, earn quasirent analogous to payment for nonproduced means of production (Sraffa, 1960, p. 78). These machines are "worth employing for what they can get" which is determined by the ruling rate of profit and associated prices. Note that since, by definition, fossils are without primary processes, they are nonbasics and therefore mathematically equivalent to nonproduced means of production. Fossils may be taxed without disturbing the relative prices and the profit (wage) rate, etc. There is, nevertheless, an important difference which justifies the term "quasirent". Rent to fossils is not due to scarcity according to Definition 1 above. By Okishio's theorem, the introduction of a superior alternative process causes the rate of profit to rise rather than fall. Fossils are machines which have been replaced by *more* efficient reproducible capital goods and hence there is no analogy to scarce fertile land[6]. Processes employing efficient, reproducible capital goods could, if permitted, satisfy the entire effective demand. Nothing, however, prevents operators of fossil processes from selling

[6] One might be tempted to think of the efficient reproducible capital goods as in some sense temporally scarce and thereby commanding a superprofit. Indeed, in orthodox (Marshallian) usage this superprofit is called a "quasirent". In the traditional approach, quasirent is a transitional phenomenon which disappears in the long run as the efficient technique is fully diffused. The text departs from the traditional definition for several reasons. First, it would not be possible to allow the less efficient fossil-using process to determine the rate of profit (wage rate) and relative prices simply because without a production process for fossils, the system of reproduction prices is logically and mathematically incoherent. Neither would it do to think of the return to a new and efficient capital good as rent arising from its scarcity since it is precisely the reproducibility of capital goods which guarantees an equal rate of profit. Sraffa's system effectively partitions commodities into reproducible and nonreproducible sets based on currently existing conditions of production. Since Sraffa does not follow Marshall in the distinction between the short and long run, there is no set of commodities for which production processes exist in the long run but not in the short. Finally, there is no comfort in seeking to restrict the relevance of the Sraffian system to the "long run" (as some of Sraffa's neoclassical critics have done) since in the long run the problem of fossils disappears. For these reasons the text maintains a definition of quasirent which while at variance with the orthodox notion, is consistent with Sraffa's general approach.

output at the established price and thereby siphoning off part of the aggregate demand for the product. Mathematically, fossil prices are determined in the same way as scarce land, but it is clear that the return has nothing to do with their scarcity.

The existence of quasirent-bearing fossils shows that while all scarce means of production earn rent, not all rent is generated by scarcity. Thus, in order to maintain a clear distinction between rent and profit, we will refer to rent to nonscarce means of production as *quasirent*.

3. Profit with Changing Land Qualities

This section relaxes the crucial assumption of a given and fixed structure of land qualities. In what follows, we allow the production process to change the quality of land on which it operates in order to account for exhaustible resources such as petroleum reserves or mineral deposits. We shall see that this approach is also well suited to renewable resources such as fish populations and agricultural land.

Though the distinction between exhaustible and renewable resources is common in the literature on resources, it is well known that it cannot be made entirely rigorous (Dasgupta and Heal, 1979, p. 113). Renewable resources are in fact exhaustible as Peruvian anchovies, the Blue Whale, and the Great American Dustbowl have made plain. Similarly, reserves of exhaustible resources can be augmented through exploration, discovery and development (Devarajan and Fisher (1982)). In principle, then, we need not distinguish exhaustible and renewable resources other than by the specific characteristics of the production processes in which they are employed. Indeed, we shall argue that resource scarcity has much less to do with the intrinsic exhaustibility of the resource in question than the social relations under which it is extracted.

It is quite natural to consider land which undergoes a change in quality due to the production process in which it is employed as a sort of fixed capital. The rate at which the resource is depleted may then be computed endogenously and will be subject to variations in the level of wages and profits in precisely the same way that fixed capital depreciation rates depend on the outcome of the struggle between capital and labor. The generalization of the fixed capital system to land is not trivial, however, since we must be careful to identify a process analogous to the fixed capital system's primary process as well as a process which *truncates* the sequence of secondary processes. In other words, there are two degrees of freedom which must be eliminated in order to arrive at a determinant system: an initial and final price of the changing resource.

3.1 Exhaustible Resources

Consider first, land with a deposit of a depletable or exhaustible resource. Clearly, land quality changes as the resource is extracted; land from which resources have been extracted for t years should be considered a different commodity than the same land after $t+1$ years. For exhaustible resources, the primary process which "produces" land may be conceived as a process for exploration and development of land with unexploited resource deposits. This approach obviously requires unexplored land as an input but since virgin land is not itself a produced means of production, we do not yet have a surrogate primary process. One solution to this dilemma is to simply assume that there are excess supplies of unexplored land such that its owners are unable to command a rent. In this case, exploration is primary.

In addition to a primary process, there must be some final process in which land does not appear as an output. Fixed capital systems are truncated when the cost of additional labor and produced means of production rises to the point that the next secondary process cannot earn the average rate of profit. Were the eldest vintage machine employed in an additional secondary process, its imputed price would be negative. The truncating process retires the capital good from service by either producing a vintage with a zero price or converting the capital good to scrap (Sraffa, 1960, p. 64).

The generalization of the fixed capital system to exhaustible resources is now relatively straightforward. The process for the discovery and development of new land together with the new-land using process constitute the primary processes to which processes employing land at various stages of exploitation are secondary. The secondary processes jointly produce final commodities along with land of changed quality and may do so with varying efficiency in that more or less labor and/or other inputs may be required per unit of output. Some secondary processes may even use commodities not required by the primary or other secondary processes. If the resource is exhaustible, the efficiency of the exploited land presumably declines until the imputed value of the land becomes negative and is abandoned.

Since this approach to exhaustible resources is wholly analogous to the fixed capital system, it follows that here landlord income is not rent but profit. Return in the ith process, ϱ_i, is now the simple

Ricardian rent, rp_i of Definition 2, plus a resource depletion charge, $p_i - p_{i+1}$. We have:

$$\varrho_i = rp_i + p_i - p_{i+1} \tag{5}$$

during the period in which the ith quality of land is transformed into the $i+1$th quality. Theorem 4 shows that the discounted sum of the returns to land is just equal to the cost of exploration and development.

Theorem 4: Consider the following simplified economy in which a sequence of processes produces final commodity 0 by means of labor, the final good itself and land of qualities 1 through m. There is a process for the exploration, discovery and development of land of quality 1. Write the price determining equations as:

$$\begin{aligned} p_1 &= \pi\, p_0\, a_{00} + w l_0 \\ p_0\, b_{01} + p_2 &= \pi\, (p_0\, a_{01} + p_1) + w l_1 \\ p_0\, b_{02} + p_3 &= \pi\, (p_0\, a_{02} + p_2) + w l_2 \\ &\vdots \\ p_0\, b_{0m} &= \pi\, (p_0\, a_{0m} + p_m) + w l_m \end{aligned} \tag{6}$$

where as before, all coefficients are defined per unit of land. Here the first process is for exploration and development; the second is the primary land-using process; the third is the first secondary and the last is the truncating or closing process. We then have:

$$p_1 = \sum_{i=1}^{m} \varrho_i\, \pi^{-i}.$$

Proof: Substituting Eq. (5):

$$\sum_{i=1}^{m} \pi^{1-i}\, p_i - \pi^{-i}\, p_{i+1} = p_1 + \sum_{i=2}^{m} \pi^{1-i}\, p_i - \sum_{i=2}^{m} \pi^{1-i}\, p_i - $$
$$- \pi^{-m}\, p_{m+1} = p_1 - \pi^{-m}\, p_{m+1}.$$

But by definition of the truncating process, $p_{m+1} = 0$.

p_1 is then the discounted sum of the stream of surplus profits accruing to the owner of the resource. The process by which surplus is distributed across the primary and secondary processes may create the illusion that exhaustible resources are scarce in the sense of Definition 1. This danger is especially present in that Theorem 4 is a well-known equilibrium condition of neoclassical capital theory, a theory which does not typically distinguish produced and nonproduced means of production. But again, the condition that the discounted sum of returns is equal to the costs of reproduction of durable goods expresses nothing more than the tendency toward an equalization of the rate of profit (Schefold (1980)).

3.2 Renewable Resources

The last secondary or truncating process need not actually retire land from service. It must only impute to its output of land a zero price so as to insure the equality of the number of processes and number of commodities. Land formally analogous to discarded machines may not be abandoned at all but may serve as an input into the exploration-development process. In this process, labor clears, regenerates, renews or recycles "dead" land so that it again becomes serviceable. This process may produce as a byproduct some commodity which in the process of production actually improves the quality of the soil. Alfalfa is a well-known example of a crop which restores nitrogen to land depleted through excessive cultivation. In other instances the process may require very little labor, as land which must lay fallow in order to replenish its productive powers. Given that the formal model needs no modification to account for renewable resources of this kind, it is immediate that land may be renewed or recycled without affecting any of the conclusions of the last section. Renewable resources, like exhaustible resources, are not in themselves scarce and consequently do not necessarily command a rent.

Of course we need not assume that the productivity of land diminishes to the point at which it must be recycled. In some agricultural applications land quality may vary cyclically without ever forcing a truncation. Land in this case is the logical equivalent of a perennial machine, that is, a machine which lasts an arbitrarily large number of periods (Schefold, 1971, p. 78). The perennial machine case resembles the system described in Theorem 4 except that there is no closing process since the resource is never exhausted. Theorem 5 shows that perennial resources do not alter the conclusion reached so far by demonstrating that the effect on prices and the rate of profits (wages) of an additional secondary process diminishes as the number of secondary processes increases.

Theorem 5: Consider the sequence $p_1, p_2, \ldots, p_m$ where p_i is the price of the ith quality of land in economy of Theorem 4. If this sequence is bounded from above, and we denote the supremum of the set of solutions for p_1 with m secondary processes as $p_1(m)$, then we have:

$$\lim_{m \to \infty} p_1(m) - p_i(m+1) = 0.$$

Proof: First observe that if the sequence of prices were not bounded from above, all prices in the economy would be unbounded as number of processes increased without limit. This follows from the assumed indecomposability of the all-basic price determining system. Replace Eq. (6) with:

$$p_0 b_{0m} + p_{m+1} = \pi (p_0 a_{0m} + p_m) + w l_m$$

and assume that the coefficients b_{0m}, a_{0m} and l_m are also bounded for all m. Now construct an integrated system by taking the linear combination of the primary and secondary processes with weights, $\pi^{-1}, \pi^{-2}, \ldots, \pi^{-m}$. Write the result as:

$$p_1 (m) = z^m + p_{m+1} \pi^{-m}$$

where $z = p_0 b - \pi p_0 a - wl$; and $b = \sum_{i=1}^{m} b_{0i} \pi^{-i}$; $a = \sum_{i=1}^{m} a_{0i} \pi^{-i}$; and $l = \sum_{i=0}^{m} l_i \pi^{-i}$. Thus,

$$p_1 (m) - p_1 (m+1) = (p_0 a_{0m+1} + l_{m+1} - p_0 b_{0m+1} - p_{m+2}) / \pi^{m+1}.$$

As m increases without bound, the left hand side vanishes since the numerator on the right is bounded but the denominator is not.

What Theorem 5 shows is that if all land-using processes are combined by taking a linear combination with geometrically declining weights, the impact of the last process on basic prices, profits and wages is negligible as the number of secondary processes increases without bound. Together with a primary process for exploration and development, the perennial closure is essentially a direct extension of the fixed capital system and, consequently, reproduces its properties. Even if durable capital goods last "forever" they nevertheless participate in the division the surplus extracted from labor in the same way as any other capital good.

4. Rent with Changing Land Qualities

It might be argued that the assumption of a primary land using process is unduly restrictive in that it resolves the problem of non-produced means of production by assuming it away. Land is not scarce since it can be "produced" by labor and other produced means of production. It remains to consider alternatives to the assumption of unlimited supplies of unexplored land. If land is in fixed total supply either the final commodity must be produced by a synthetic, or "backstop" process or there will be room for a second *sequence* of processes to operate along side the first. In these cases, landlords earn what corresponds to simple Ricardian extensive and intensive rent.

4.1 Extensive Rent

If the level of effective demand cannot be satisfied with the quantity of land in production, it may be possible to introduce a synthetic process, that is, a process in which nonproduced means of production appear neither as inputs nor as outputs. If the synthetic process causes the average rate of profit to *fall*, land of *all* qualities is scarce, nonbasic and earns rent. But if the synthetic process causes the average rate of profit in the economy to rise rather than fall, the return, if any, is quasirent. In this case, capitalists operating synthetic processes are unable to undersell tenant capitalists who can pay rent and still earn the normal rate of profit.

Note that *integrated* exhaustible and renewable systems are effectively synthetic in that they neither use nor produce rent-bearing resources. Such integrated systems can be thought of as analogous to no-rent processes in Ricardian extensive systems and may well cooperate with a second integrated process that is able to pay rent to land employed in its *primary process*. The no-rent integrated process is part of the basic system which determines prices and the profit (wage) rate while land in the second integrated process is nonbasic. No quality of resource participating in the first integrated process earns rent while all qualities involved in the second integrated process do.

4.2 Intensive Rent

If there is no synthetic process available for the production of the final commodity, it will be possible for two integrated processes to cooperate. In this case, all land qualities are scarce and all earn rent. Though each process in the simple Ricardian intensive system is replaced by a sequence of processes, we shall see that the essential properties of the classical intensive system are reproduced even when land is transformed in production.

Consider a system with two sequences of processes both operating on the same initial quality of land and returning that land, through rotation or some other means, to its original quality. There are two integrated processes since by assumption, the output of the cheapest is insufficient to satisfy total effective demand. There is no presumption that the number of processes in each sequence is the same. Indeed, the more productive sequence may well exhaust land more rapidly as a result of more intensive cultivation. The price determining equations for the first sequence of processes can be written:

$$p_0 b_{01} + p_2 = \pi (p_0 a_{01} + p_1) + w l_1$$
$$p_0 b_{02} + p_3 = \pi (p_0 a_{02} + p_2) + w l_2$$
$$\vdots$$
$$p_0 b_{0m_1} + p_1 = \pi (p_0 a_{0m_1} + p_{m_1}) + w l_{m_1}$$

which can be expressed as an integrated system:

$$p_0 b^1 + p_1 \pi^{1-m_1} = \pi (p_0 a^1 + p_1) + w l^1 \tag{7}$$

where $b^1 = \sum_{i=1}^{m_1} b_{0i} \pi^{1-i}$; $a^1 = \sum_{i=1}^{m_1} a_{0i} \pi^{1-i}$; $l^1 = \sum_{i=1}^{m_1} l_i \pi^{1-i}$.

The second integrated system is then written as:

$$p_0 b^2 + p_1 \pi^{1-m_2} = \pi (p_0 a^2 + p_1) + w l^2. \tag{8}$$

Taken together Eqs. (7) and (8) determine the price of the final good *and* the initial quality of land, p_1 in precisely the same way as the simple intensive rent system[7].

Unfortunately, it appears that first quality land in this system is basic rather than quasibasic since if Eqs. (7) and (8) are written as:

$$[p_0\ p_1] \begin{bmatrix} b^1 & b^2 \\ \pi^{1-m_1} & \pi^{1-m_2} \end{bmatrix} = \pi [p_0\ p_1] \begin{bmatrix} a^1 & a^2 \\ 1 & 1 \end{bmatrix} + w [l^1\ l^2] \tag{9}$$

there is, in general, no Q such that:

$$\pi^{1-m_1} = Q \quad \text{and} \quad \pi^{1-m_2} = Q \tag{10}$$

unless, of course, $m_1 = m_2$.

But notice that m_1 and m_2 are the number of processes over which land is restored to its original quality. Theorem 6 shows that when land is affected by the production process, it is not the discounted sum of rent that is equalized between (integrated) processes but rather the discounted sum of rent *per process*.

Theorem 6: For the economy consisting of a *sequence* of m_2 integrated processes as in Eq. (7) and a sequence of m_1 integrated processes as in Eq. (8), the discounted value of the rent is the same in both of the sequences of integrated processes.

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[7] Indeed, note that when $m^1 = m^2 = 1$, Eqs. (7) and (8) constitute a simple intensive system.

Proof: The return from the operation of each of the integrated processes is

$$\varrho_1 = p_1 \, (\pi - \pi^{1-m_1}); \quad \varrho_2 = p_2 \, (\pi - \pi^{1-m_2}).$$

Operating these integrated processes m_2 and m_1 times, respectively, yields by elementary manipulations:

$$\sum_{i=1}^{m_2} \varrho_1 \, \pi^{-i} = p_1 \, (\pi - \pi^{1-m_1}) \, (\pi^{m_2} - 1)/(r \, \pi^{m_2}) = p_1 \, (\pi - \pi^{1-m_1} - \pi^{1-m_2} + \pi^{1-m_1-m_2})/r,$$

$$\sum_{i=1}^{m_1} \varrho_2 \, \pi^{-i} = p_1 \, (\pi - \pi^{1-m_2}) \, (\pi^{m_1} - 1)/(r \, \pi^{m_1}) = p_1 \, (\pi - \pi^{1-m_1} - \pi^{1-m_2} + \pi^{1-m_1-m_2})/r$$

which are seen to be equivalent.

What Theorem 6 shows is that we may link integrated processes until the total number of processes is the same for both sequences. If so, then Eqs. (9) show that land is quasibasic and hence a tax on rent will not disturb the structure of relative prices and the profit (wage) rate. Theorem 6 is crucial in that it demonstrates that whether resources are transformed in the process of production is of no consequence to the distinction between rent and profit. Through the device of an integrated system, it is always possible to construct a Ricardian indestructible analogue for systems which allow for variable quality land. Thus is true even with different numbers of secondary processes, provided we are willing to wade through the algebra of an "integrated sequence of integrated processes".

5. Conclusions

This paper has argued that Sraffa provides a rigorous distinction between rent and profit rooted in an objective definition of scarcity. A necessary condition for scarcity is the existence of two processes producing the same final commodity. This condition is found to be sufficient if we add the proviso that the additional process cause the equilibrium rate of profit to fall. If the second process raises the rate of profit, the return is not rent but quasirent.

The rent/profit distinction was developed and applied to an environment in which land is assumed to be indestructible. Though natural resources do not conform to this specification, no new theory is required. We need only apply the theory of fixed capital in order to account fully for even the most complicated patterns of resource exploitation. When this is accomplished we see that the dichotomy between rent and profit is robust and impervious to the modifications required to handle complex problems in the theory of natural resources.

The distinction drawn in this paper between profit and rent is not intended to suggest that the origin of rent is "scarcity" while only profit derives from the exploitation of labor. Profit, rent and quasirent are but forms in which the surplus extracted from labor appears in capitalism[8]. The primary motive for disentangling these concepts is to lay foundation for a more diverse class structure as suggested above. Not all classes struggle for a share of the surplus on the same footing. This paper shows that returns to owners of nonproduced means of production are determined not by their success in waging class war, but by competition for the resources they control. And this is true whether landlords actively struggle for a share of the surplus or not.

It is important to see that the central concern of this paper is class structure rather than what the orthodoxy would refer to as the temporal nature of production. In neoclassical thinking, the durability of capital equipment insures that produced goods will appear in the economy's "endowment" on par with land and other nonproduced means of production. Because there is no distinction between produced and nonproduced means of production, there is no need to differentiate profit and rent. The return to all "capital" may then be conceived as rent, determined by its "scarcity" rather than rooted in social relations characterized by the subsumption of workers to capital. It should come as no surprise, then, that neoclassicals have been willing to perpetuate a confusion between the categories of capital and land.

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