Employment Impact Assessments: Integrating the informal sector into social accounting matrices and computable general equilibrium models

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List of abbreviations

CGE	Computable general equilibrium
EIIP	Employment-Intensive Investment Programme (ILO)
GDP	Gross domestic product
GAMS	General algebraic modeling system
I-O	Input-output
ILO	International Labour Organization
LCU	Local currency unit
LDC	Least developed country
NIPA	National income and product accounts
SAM	Social accounting matrix
US$	United States dollar

Abstract

This paper reviews methodologies for the ILO's Employment Assessment project with respect to the integration of the informal sector in social accounting matrices and CGEs. A series of theoretical models are developed to show how this can be done. The models are organized according to whether the presence of the informal sector is due to capital limitations, *functional* informality, versus *juridical* informality, which may arise as a competitive strategy on the part of entrepreneurs. The goal is to offer to policymakers some perspectives on how the informal sector could be incorporated into formal models of the economy.

Keywords: Informal sector, Functional, Juridical, Employment, Social Accounting Matrices, Computable General Equilibrium Models, Phase transition.

1. Introduction

The informal sector is often criticized as second class, both with respect to productivity and real wages as well working conditions and compliance with labour and environmental standards. Setting aside the normative critique of the informal for the moment, this paper focuses on how the informal sector can be included in a social accounting matrix (SAM) and associated macroeconomic models calibrated thereto. It is seen that the informal sector adds to GDP, stimulates *formal* sector output and employment and generally contributes to economic well-being. This conclusion is inescapable when one quantifies the presence of the informal sector in a formal model and then examines various counterfactuals with respect to its size and modes of interaction with the formal economy. It follows that using juridical means to suppress the informal sector will likely be counterproductive

Gibson and Flaherty (2016b) develop a theoretical framework for analysis of the informal sector solidly grounded in fundamental economic concepts.[1] The distinction between *functional* and *juridical* informality is developed, based on Gibson and Kelley (1994). Informal sector *workers* rarely choose informality and would prefer formal sector employment were it available. Informal shop *owners*, however, may elect to avoid labour and environmental standards as a competitive strategy, but this is not *functionally* informal according to Gibson and Flaherty (2016b). Legally, however, it is unquestionably informal and may not be registered or even counted in the economic censuses or national income and product accounts (NIPA). This is *juridical* informality according to Gibson and Flaherty (2016b).

The economic rationale for functional informality is simply that informal workers have no other option than to operate production processes that are "defective" in the sense that they would not be operated by formal sector firms Gibson and Kelley (1994). If the latter were

[1]The theoretical and empirical literature on the informal sector is immense. See Gibson and Flaherty (2016b) for a review of some of the recent literature

required to pay the market rate for formal labour its profit might well be below the market rate or even negative. In this sense, the process of production is defective and would be abandoned by any formal sector firm. Formal and juridical informality can be empirically distinguished by their comparative statics: a rise in aggregate demand, for example, will never cause an expansion in employment for both the formal and informal sectors if informality is functional. On the other hand, a rise in demand can cause formal and informal employment to rise simultaneously if the informality is juridical.

The paper is organized as follows: section 2 defines the distinction between functional and juridical informality, a distinction important again at the end of the analysis. Section 3 develops a social accounting matrix with an integrated informal sector.[2] Section 4 develops a simple model of functional informality and Section 5 examines the phase transition to a purely formal economy. Section 6 shows how the informal sector can be implemented in a CGE model and provides some numerical examples as a guide for how this might be done. Discussion of the implications of the difference between functional and juridical informality for the results of the models is presented in Section 7. Section 8 draws some conclusions from the study.

2. Functional vs. juridical informality

The distinction between functional and juridical informality is grounded in the underlying economic processes at work in informal activity. If the production processes used by informal workers would fall into disuse if the employer had to pay proper wages and benefits, then the informality is *functional* (Gibson and Kelley, 1994). The functionally informal lack the physical or human capital to operate processes that yield the prevailing rate of profit in the formal sector. The strength of this distinction between juridical and functional informality, then, is that the latter is tied explicitly to whether the informal activity can earn a sufficient return.

Juridical informality, on the other hand, is not defined by economic concepts but rather derives from policy decisions that set legal conditions required for firms to be formal, including obligations for taxes, worker benefits and compliance with health, safety and environmental regulations.[3] Further complicating the picture, governmental authorities do not and indeed cannot usually determine whether a firm is permanently unprofitable or only temporarily so, as it begins its metamorphosis to formality. The legal establishment has only blunt instruments and does not even try to separate out infant industries from their survivalist counterparts. In treating them all the same, however, government policy can block the transition of juridically informal firms to formal status. Policymakers may harbor a bias toward formality owing to its contribution to public sector revenues. However, the economy would be better off if all informal workers were transformed into formal, mainly because productivity would be higher. In reality, higher productivity is often correlated with higher

[2]Readers unfamiliar with SAMs might want to consult the ILO working paper Gibson and Flaherty (2016a) in which SAMs are discussed in detail and associated papers in the Employment-Intensive Investment Programme of the ILO

[3]These conditions may operate at several levels: firms may be juridically formal at one governmental level (city vs. state for example) but informal at another.

unemployment.

Note that juridical informality always involves a *choice* to operate production processes outside legal boundaries, boundaries that establish labour and environmental standards, as well as perhaps, taxation. As such, juridically informal firms run a risk of apprehension and must factor any potential punishment into the decision to operate formally or informally. This implies that the rate of profit must be *higher* for juridically informal firms that elect to operate illicitly. Individual firms thus face a risk-rate-of-return trade off that any other formal (that is, non-functionally informal) firm faces. The whole notion of having to operate a defective production process drops away with juridical informality. The processes operated by the juridically informal are not necessarily defective, they are simply *risky*.

3. A SAM with an informal sector

SAMs are built on the same principles as input-output (I-O) matrices. The *material balance* equation

$$X = AX + F \tag{1}$$

where X is a n-dimensional column vector $X = (x_i)$, with $i = 1, 2, ..., n$ and $A = (a_{ij})$, is an $n \times n$ matrix of technical coefficients describing the amount of good i required as an intermediate or raw material for the production of one unit of good j, with $j = 1, 2, ..., n$ and $F = (f_i)$ is the n-dimensional column vector of final demand, conventionally including consumption, investment, government expenditure and net exports Gibson and Flaherty (2016b).

The first step in including the informal sector is to expand the definition in equation 1, writing

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} C_{1,j} \\ C_{2,j} \end{bmatrix} + \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} + \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \tag{2}$$

where subscript 1 indicates the formal sector and subscript 2 the informal sector. The I-O matrix is now broken into 4 sub-matrices that show the intermediate input use of formal and informal sector goods into the production of formal and informal sector output. As a first approximation, one could set $A_{21} = 0$ in equation 2 to indicate that the use of informal output, as intermediate goods, by formal firms is zero.[4] This is unlikely to be true, but the magnitude of the other off-diagonal term, informal *use* of formal output, is typically much larger.

Here C_{ij} is the consumption matrix for the two kinds of goods and a country-relevant number of household categories, say $j = 1, 2, ..., h$. Investment *by origin* is given by I_i, and it is evident that some of the output of informal sector producers could be used to augment the

[4] This is not always done in the examples to follow.

capital stock of the country.[5] As a first approximation, one could set $I_2 = 0$ to conform to the case in which there is no informal production of capital goods. In any event the quantity will be small compared to I_1.

One major discrepancy with the NIPA is that informal sector purchases of goods used either as intermediates or capital goods are likely to be counted as *consumption* in the expression for aggregate demand on the right-hand side of equation 2. If informal sector firms are not counted in economic censuses, because they are unregistered, purchases by informals of goods and services are mistakenly considered to be for home use and not properly categorized in the data.

Government purchases, G_2, of informal sector output may also be limited, but again it is unlikely to be zero. It might not be recorded depending upon how thoroughly the informal sector is suppressed. It may well be the case that the public sector does indeed purchase informal output, but at the same time, refuses to acknowledge that it does so for political or even legal reasons. Computing the contribution of the functionally informal sector to aggregate economic activity is made more difficult by juridical informality.

Similarly net exports, N_2, may be under counted, or *under invoiced*, if informals avoid export tariffs by shipping their goods and services informally. The same is true for imports that may come into the country without notice by the relevant authorities. Here is where perhaps the most significant data problems reside, particularly if informals export contraband. Even if their trade is deemed perfectly legal, it nonetheless will be under reported since informals, both juridical and structural, will be unlikely to pay export duties and border trade will include in smuggling of a wide range of goods and services. In a perfect world, government would only spend resources up to the point at which the marginal cost of apprehension is equal to the recovered duties at the same margin. In some counties, such as Nigeria, the gap is yawning, with informal filling large tankers full of contraband oil and then integrating the shipments with the formal supply chain by way of easily forged documents. In many countries the quantities involved are undoubtedly trivial and other case, such as the border trade around Iguacu, it is not.

3.1. The intermediate sub-matrices

A closer look at the sub-matrices of A in equation 2 shows that the matrix A_{11} is square. If the number of formal processes is n_f, the dimension of A_{11} is $n_f \times n_f$. Associated with each formal process of the A_{11} matrix, however, may be zero, one or more informal processes all producing goods that are close substitutes for formal output.

It is quite unlikely that one would encounter an "orphan" informal process, one that produces a good or service for which there is no formal substitute. In the continuation of this section, it will be assumed that there are no such goods. In the paper, it is assumed that all informal processes have at least one formal process that serves to determine the price of the informal

[5]The set of multisectoral models, to which any model of formal/informal sector interaction necessarily belongs, must distinguish between investment by *origin* and by *destination* Gibson and Flaherty (2016a). The former is a component of aggregate demand, while the latter describes the sector in which the investment goods, so demanded, are used to augment the stock of capital. This distinction is unnecessary in one-sector models.

good. It is not impossible or even unlikely in principle, but the level of aggregation in which data is collected in developing countries limits the ability to identify informal sector goods for which there are no formal sector substitutes.

If there are n_i informal processes, the A_{22} sub-matrix is then $n_i \times n_i$. These are informal sector sales of intermediate goods to other informal sector processes of production. One might reasonably expect this matrix to be sparse, but in countries in which the informal sector accounts for a significant fraction of the GDP, it could be a dense as its formal sector counterpart, especially when formal firms heavily rely on imported intermediates, while informals do not.

Consider the example of informal purchase of a hammer from a formal firm. This may be considered investment, and perhaps should be, but for the moment assume that the hammer lasts less than one year and so is properly recorded in the A_{12} sub-matrix (of dimension $n_f \times n_i$) as an intermediate formal good purchase, undertaken an informal firm. Here, there are two possibilities: the first is that the hammer is used by an informal firm to which value is added in some activity, say panel beating. In the second case an informal retailer purchases the hammer and then resells it to informal repair shop owner. Here the razor-thin value added can only come in transportation or some other distribution category, and if not the panel beater would simply purchase the hammer directly from the formal seller. Intermediate purchases from the middleman then amount to an intermediate good, the hammer, and an intermediate service, the distribution. The latter is accounted for in the A_{22} matrix, sale from one informal to another. The example illustrates another basic principle of informal SAM building: the A_{21}, an $n_i \times n_f$ matrix, is likely to be vanishingly small since informals are unlikely to be able to scale their operations to the quantities demanded by the formal sector, at least in most cases. The preponderance of intermediate purchases by the formal sector will appear in the A_{11} sub-matrix and to a first approximation the $A_{21} = 0$.

Keeping in mind that the accounting is likely to be confused from the outset, with these informal intermediates counted as consumption if counted at all, one appreciates the difficulties involved in the practical construction of informal SAMs. Still, the published SAM implicitly makes an estimate of all these informal entries that is even more erroneous that the crudest estimate: namely, that they are all zero. One sees from this discussion that the accounting is prosaic while the data collection process is not.

How then does the presence of an informal sector affect the balance of savings and investment in the SAM? Clearly, if investment increases due to the wider coverage of informal activities, savings must rise as well. Here is perhaps the most important conceptual difference between formal-only SAM and that includes informality. Informals are by-and-large excluded from the capital markets of most of the economies in which they participate. It follows that heir savings, must at least cover their investment

$$\sum_{i=1}^{n_i} S_i = I_2 \tag{3}$$

where I_2 is the level of informal investment in equation 2. Since by definition, informal sector production processes do not return a rate of profit, it is personal savings rather than firm

savings that accounts for the left-hand side of equation 3. The equation also implies that *formal* savings must be equal to formal investment, but that is also true in the published NIPA. In order to add an informal sector to the SAM, it is usually necessary to assume that extra investment informals undertake is itself financed by the aggregate savings of informal sector participants. This is *not* an arbitrary assumption, but rather is an assumption required by the structure of the NIPA.[6]

3.2. An example of a SAM with formal and informal sectors

In the simplest possible case, the A_{11} sub-matrix shrinks to a scalar and there is but one formal process. The A_{22} sub-matrix can also be simplified to a scalar so that the model above has two sectors, one formal and the other informal, both producing the same good. Table 1 shows hypothetical data for a SAM with only one formal and one informal sector. This SAM has only one good, but two processes for the production of that one good. The table shows the informal intermediate contribution to formal sector production processes is minuscule; intermediates contributed to the formal sector are 1 compared to own intermediates of 10. On the other hand, the informal sector draws more intermediates from the formal sector than it does its own.

[6]It follows that any financing of formal investment by informal households must *already* by included in the published national accounts. It is therefore more proper to say that the marginal informal investment is financed by the marginal increment in informal saving.

Table 1: Informal SAM[1]

			Consumption		Investment				
	Formal	Informal	Formal HH	Informal HH	Formal	Informal	Govt	Exports	Total
Formal	10	18	30	4	18	5	8	7	**100**
Informal	1	15	25	6	0	2	0	1	**50**
Value Added	62	15					8		**85**
labour-Formal	20	1					8		**29**
Labour-Informal	2	9							**11**
Capital	40	5							**45**
Savings	10	0	8	6			-4	6	**25**
Taxes	5	0	7						**12**
Imports	12	2							**14**
Total	**100**	**50**	**70**	**16**	**18**	**7**	**12**	**14**	

Addendum

GDP	100	Govt	16	Wages		Capital-labour ratio	
Formal	85	Exports	8	Formal	5.8	Formal	42.5
Informal	15	Imports	14	Informal	2.2	Informal	3.75
Val Added	100	Employment	10	Capital stock	231.25	Cost of capital	
Investment	25	Formal	5	Formal	212.5	Formal	18.8
Savings	25	Informal	5	Informal	18.75	Informal	26.7

Source: Authors' computations based on illustrative data.

The addendum to the SAM in table 1 links the SAM entries to the NIPA. This is, however, unlikely to match the published data as discussed above. In any event, these metrics must be computed to ensure that the formal/informal SAM does not openly contradict independent studies of the informal sector.[7]

4. A simple model of functional informality

An accurate auditing of informal sector participation in the macro economy is a necessary element of any coherent analysis, but it is far from sufficient. Even a complete formal/informal SAM says nothing about whether the informal sector is structural or juridical; for that a behavioral model is necessary and it will be seen that comparative statics are sufficient to distinguish the two.

One of the most celebrated models in the history of development economics is due to Lewis (Lewis, 1954). The model is based on a dual economy with modern and traditional sectors, with neither government nor foreign trade. A first model of the informal sector borrows from Lewis by reinterpreting the modern sector as formal and the traditional sector as informal. It is immediate that the result is a model of structural informality. The Lewis model does not ask whether agents *choose* to join the formal or the informal sectors, but instead departs from the assumption that workers would prefer formal to informal work and only retreat to the latter due to a *shortage* of the former.

A shortage, of course, is precisely the kind of problem economic theory is uniquely qualified to address. As long as rational actors are free to act in their own self interest, that is there is no intervention such as a minimum wage or price distortions introduced by government, noncompetitive behavior or other market failures, shortages are ephemeral. They last only until prices adjust to convince those frustrated by the shortage that they no longer want whatever it was that seemed just a short while ago to be in short supply. Even under these relatively restrictive assumptions, formal sector jobs are not allocated in this fashion, however, and it is certainly reasonable to ask why not.

The justification for the Lewis approach to functional informality lies at the heart of rational choice theory. An agent is assumed to choose a combination of utility maximizing activities in proportions that depend on how costly each is. Income generating activities are, for example, never chosen in a way that occupies all time available since the marginal utility of leisure rises rapidly as the sleep deprived come to realize that at least some time must be left for leisure. Similarly the demand for leisure has an upper bound for biological reasons. Eventually agents realize that some quantity of time must be sacrificed to obtain the means

[7] Columns of the matrix require prices since, reading down the columns of the I-O matrix, the goods are heterogeneous and must be aggregated by the price vector. To this nominal value is added the return to labour, wL, and the return to capital, rK, which are both measured in nominal terms. This suggests that the entire presentation of the I-O framework must be in *nominal* terms. If a given I-O system is also compiled for a *base year*, the values in the structured data base are both real *and* nominal. As noted above, it is convenient to normalize all prices to one for the base year, along with the base year wage rate. The units of X are then in millions of LCUs and if prices were to rise by, for example, 15 per cent, it could be said that PX is the *cost of what could have been purchased in the base year* with one-million LCUs of the base year. This is a useful convention in I-O accounting and widely adopted. See Gibson and Flaherty (2016a) for additional details.

of subsistence.[8]

The Lewis approach to functional informality presupposes these inherent limits on human choice and then simply observes that if the wage rate were to fall to a level that would encourage formal firms to hire all available labour, the wage would not cover the biologically determined minimum. Agents do not choose informality as a result, but are rather driven by necessity. The choice set shrinks to one, and only one, activity and the choice theoretic problem yields only a trivial result. Whether this is actually true for any given economy is, of course, an empirical matter but if so, functional informality will arise.

To proceed, let there be two sectors, formal and informal. The level of output in each sector is denoted by Q_i and is given by

$$Q_i = \mathcal{A}_i K_i^{\beta_i} L_i^{(1-\beta_i)} \tag{4}$$

where $i = 1, 2$ for the formal and informal sectors respectively. Here, $\mathcal{A}_i$ is an arbitrary calibration constant, K_i is the capital stock and L_i is the labour employed in each sector. The elasticity of output with respect to capital is β_i.

In this Lewis inspired model of functional informality, labour is a binding constraint and is written

$$\bar{L} = \sum_{i=1}^{2} L_i + L_s \tag{5}$$

where $\bar{L}$ is the labour supply, L_i is the *minimum labour* required to produce the output, Q_i, and L_s is the amount of *surplus labour* in the informal sector. Equation 5 is key in that it imposes "full employment" in the sense that workers are employed either in the formal sector *or* in the informal sector, either as necessary or surplus labour there.

In the *Lewis model*, but not all informal sector models, the *real wage in the formal sector* is determined by the *average product* in the informal sector, counting surplus labour

$$\frac{w}{p} = \frac{Q_2}{L_2 + L_s}. \tag{6}$$

The Lewis equation is then obtained by setting this real wage equal to the marginal product of formal labour, the first-order condition for profit maximization

$$\frac{dQ_i}{dL_i} = (1 - \beta_i)\frac{Q_i}{L_i} = \frac{Q_2}{\bar{L} - L_1}. \tag{7}$$

The right-hand side of equation 7 is the *Lewis equation* and can be simplified as follows. First, clearing fractions

$$(1 - \beta_1)Q_1(\bar{L} - L_1) = L_1 Q_2$$

and then substituting equation 4 for Q_1 and dividing by L_1

$$(1 - \beta_1)\mathcal{A}_1 K_1^{\beta_1}(\bar{L} - L_1)/L_1^{\beta_1} = Q_2.$$

[8] See Blattman and Dercon (2016) for lively–if somewhat disturbing–challenge to these assumptions.

This equation can be expressed in implicit form as

$$f(L_1) = (\bar{L} - L_1)/L_1^{\beta_1} - Q_2/(1-\beta_1)\mathcal{A}_1 K_1^{\beta_1} = 0. \tag{8}$$

Equation 8 is the *Lewis equation* in the variable L_1, taking the level of output in the informal sector, Q_2, as given. It cannot be solved by normal algebraic means. One can, however, obtain a numerical approximation to the root of this equation by simply plotting it.

Figure 1: The Lewis equation

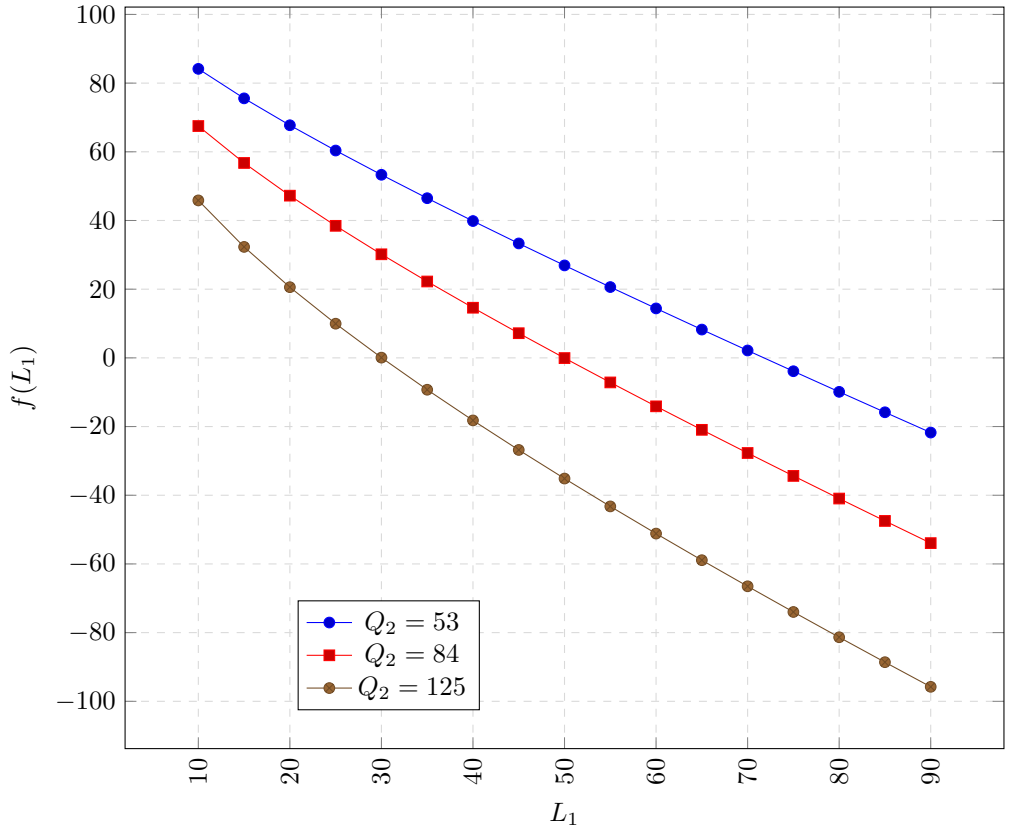


Figure 1 shows the Lewis equation for three increasing levels of output of the informal sector, $Q_2 = 53, 84$ and 125. Each curve crosses the horizontal x axis at a value of L_1 (the root) that satisfies the Lewis equation 8. If output in the informal sector is, for example, 84, the level of formal employment is 50. A rise in output to 125, however, will cause the real informal wage to increase, reducing the level of employment in the formal sector to 30. This is an irony of economies with large informal sectors for which foreign aid programs, for example, increase the productivity of the informal sector. The opportunity cost of labour increases and drives formal employment, L_1, down, because of equation 6. The rise in the wage requires a rise in the marginal productivity of labour in the formal sector, which in turn requires a *reduction* in the rate of employment there.

4.1. Functional informality in the short run

To summarize, the adjustment mechanism in the model proceeds as follows. Taking the level of Q_2 as given by some base line, historically driven productivity metric, the Lewis equation 8 can be used to solve for the level of employment in the formal sector, L_1. Using equation 5, the total amount of labour left in the informal sector is then determined, and by way of equation 6 the real wage is known. The minimum employment in the informal sector, L_2, is determined by the marginal productivity condition, equation 7,and equation 5 subsequently allows a solution for the level of surplus labour, L_s. The first-order condition in the *informal* sector

$$(1 - \beta_2)\frac{Q_2}{L_2} = \frac{Q_2}{\bar{L} - L_1} \tag{9}$$

can be used to compute the minimum labour, L_2, required to produce the given output there and subsequently the level of surplus labour. This gives the full specification for the Lewis model in the short run.

If the formal and informal sector capital stock is fixed in the short run, the only way in which the formal sector can increase employment is if the wage rate falls according to equation 7. One way this could occur is through population growth or in-migration from neighboring states to raise $\bar{L}$ and in turn increase surplus labour, L_s, in the informal sector. The increase in surplus labour has no bearing on the level of output Q_2, under the classical Lewis assumptions; therefore, the *average* product in the informal sector falls. This causes additional labour absorption in the formal sector as the opportunity cost of formal labour falls and formal firms increase output as a result.

As seen in figure 1, *a rise in the output* of the informal sector, brought about by perhaps a foreign aid program or some autonomous shock of technical change, would encourage a *reverse* flow of workers from the formal to the informal sector. This perverse adjustment would continue until sufficient labour exited from the formal sector to bring about a rise in the marginal product of labour there. This is precipitated, of course, by the rise in the real wage. At the same time, the initial increase in informal sector wages would be eroded, according to equation 6, as previously formal workers joined the informal sector.

Without an additional inflow of labour, there is no way output in the formal sector can increase absent some form of technical change or capital accumulation. The presence of the informal sector effectively blocks the ability of formal sector firms to hire more labour. The existence of the informal sector thus hems in the normal market mechanism that would otherwise aid in bringing about higher levels of employment. Before one can conclude that this argument supports a policy stance against the informal sector, keep in mind that the equilibrium wage that would arise from this market mechanism would necessarily be *lower* than what workers could earn on their own informally and according to the assumption invoked above, could drive the wage below some minimum biological level.

4.2. Comparative statics: the transition from functional to juridical informality

In the long run, both the capital stock, K_1, K_2, and the total labour supply, $\bar{L}$, adjust. In this paper, the long run is defined as the time during which these and other *state* variables adjust *between* periods. The equilibrium described by equation 8 takes place *within one period*, that is, with the capital stock and the supply of labour taken as given variables. The short-run sequence of equilibria, linked by evolving state variables, defines the long run in the model. Each short-run solution provides levels for the real wage, Q_1, Q_2, L_1, L_2, and L_s, all functions of the state variables, K_1, K_2, and the total labour supply, $\bar{L}$.

In the short-run solution, the level of surplus labour, L_s, may be zero, negative or positive. If $L_s > 0$, there is functional informality; if $L_s < 0$, functional informality no longer exists but there still may exist juridical informality. A *phase transition* occurs at $L_s = 0$, where all surplus labour has been drained from the informal to the formal sector. When $L_s < 0$, either the first or second sector (or both) must experience a labour shortage. With $L_s < 0$, equation 5 implies that the wage rate that matches the marginal productivity of labour in the formal sector will immediately increase.

At the value $L_s = 0$, the model experiences a phase change, with the Q_2 as the *order parameter* that drives the system. Defective production processes disappear and all production processes become effectively formal in that they can pay the market wage rate and still return the average rate of profit. These sectors are no longer functionally informal. All workers are paid their marginal product and are indifferent as to whether they are employed in either sector Q_1 or Q_2. Juridical informality, however, can still easily exist at or beyond this turning point. Figure 2 illustrates the transition.

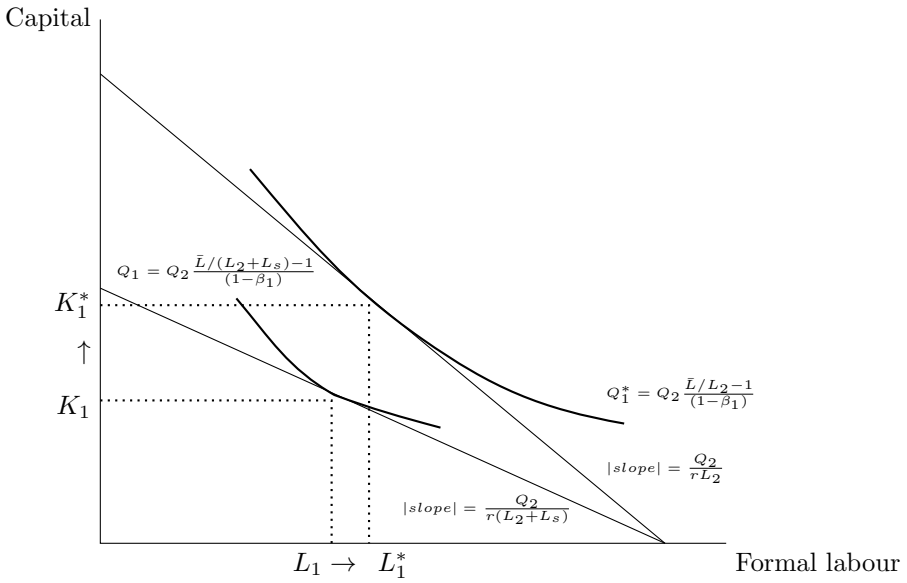
Consider a path in which output and employment in the formal sector are rising with capital accumulation. In the process, surplus labour steadily exits the informal sector until L_s goes to zero. The informal sector has now reached its maximum level of output per worker. Both sectors hire labour at a real wage that reflects the true opportunity cost of work (leisure) rather than how many workers have been crowded into the informal sector due to lack of formal job opportunities and the binding biological constraint. Functional informality has been erased.

At this juncture, labour may continue to be employed informally, producing either Q_1 or Q_2, but it is juridical not functional informality. There are no firms that are functionally informal, producing with defective processes. Juridical informality may well persist, however, as firms choose to avoid taxation, labour and environmental standards and so be recognized by the polity as informal. At this point, in theory at least, they are no longer functionally informal.[9]

Figure 2 illustrates the progression towards the turning point. Consider an isoquant that shows combinations of capital and labour that might be used to produce the formal output, Q_1. At the initial level of capital stock, K_1, there is enough capital to employ all labour

[9]International agencies may claim, nonetheless, that the nagging problem of the "informal sector" has not yet been resolved. Governments may add that the informal sector is robbing them of revenues, but from a public choice perspective, the case would have to be made that the revenues thus obtained would be spent in ways that enhance well-being.

Figure 2: The functional/juridical turning point



formally, at least in principle, but the wage cannot fall enough to bring this about. With the wage determined by the opportunity cost of labour in the informal sector in equation 6, the formal sector will only want to hire L_1 and that leaves the rest of the labour force having to crowd into the informal sector. At the given level of Q_2, only L_2 is necessary, but the actual labour informally employed is $L_2 + L_s$.

The presence of surplus labour lowers the wage rate and allows more labour to be hired in the formal sector relative to what the formal sector would absorb were there no surplus labour, but some surplus labour is still present in the system. The only solution is accumulate capital, raising the capital stock from K_1 toward the turning point capital stock, K_1^*. The rise in the stock of capital causes an increase in the marginal productivity of labour, which in turn permits the formal sector to absorb more surplus labour from the informal sector. Of course, as labour migrates from the informal to the formal sector, the wage rate rises since informal output is shared among a fewer number of workers. The equilibrium in the Lewis model comes about when the formal sector hires workers coming from the informal sector until the falling marginal product (at the new level of the capital stock) just equals the rising opportunity cost. The rising real wage thus *opposes* the progression toward the phase transition; if somehow it could be held constant, informality would disappear more quickly.[10]

Were there no surplus labour in the informal sector, it would cease to be informal and the formal sector would hire L_1 units of labour at the wage determined by the average productivity in the informal sector as in equation 6, with $L_s = 0$. Before reaching the functional/juridical turning point, the sector employs $L_2 + L_s$ units of labour and pays the lower wage of equation 6 with $L_s > 0$, which is also the wage in the formal sector. As

[10]See section 5 for a discussion of a model of this type.

the wage rises, the isocost curve in figure 2 rotates clockwise to find the tangency with the isoquant at $L = L_1$. Output is taken parametrically in this diagram, but there is no harm in thinking of increasing output in the informal sector as capital accumulates there. Technical change could also be a second factor as noted above. It causes Q_1 to increase for the same combination of factors of production, serving as stimulus for the increase in Q_1 and L_1 as seen in equation 4. Technical change thus accelerates the move toward the turning point.

Figure 2 illustrates the mechanisms at the heart of the model of the informal sector presented here. It has a number of comparative statics results discussed in the following subsections that make the model particularly amenable to policy discussions of informality.

4.3. Income distribution

As the economy approaches the functional/juridical turning point, output in the formal sector rises, but output in the informal sector remains constant. Thus, GDP has risen, formal employment has risen, as has the quality of jobs. Average productivity in the informal sector has increased and certainly economy-wide productivity, GDP per worker, has improved. The transition to an all-functionally-formal economy is a welcomed development and serves to outline a path in which output, productivity and employment grow, while income distribution improves.

How is this last point seen in the context of the model presented here? In a model with only two income classes, the Gini coefficient can be shown to be mathematically equivalent to the *difference* between the share of a labour (both formal and informal) in the population, *less* the share of income of the same in total, economy-wide, income. If the share of income is the same as the share of population of workers, the Gini is zero and income is equally distributed.

As an example of how in-migration *worsens* the distribution of income, consider an increase in the inflow of workers, δL. It can be seen that the Gini coefficient must necessarily rise as the number of workers rises. Here is the argument: with no capital accumulation or technical change in the formal sector, all newcomers will become surplus labour. Thus, the share of the total population workers represent increases. If this share of workers rises but the share of worker income in total income remains constant, the Gini coefficient will rise necessarily.

The share of worker income in total income depends on what happens to the share in both sectors, formal and informal. In the formal sector, the Cobb-Douglas technology guarantees that even if employment changes, the share of labour will remain fixed. This is because the β_1 is just the share of capital in total output in a Cobb-Douglas and so if β_1 does not change, neither do the factor shares. On the other hand, if the size of the available labour force increases, Q_1 *cannot remain fixed*. Formal employment will indeed change significantly since the real wage in the informal sector will fall with the new influx of migrants.

To see what happens to the share of labour in total output, formal plus informal, define $sigma_L$ as

$$\sigma_L = (wL_1 + Q_2)/(Q_1 + Q_2).$$

Substituting the marginal productivity condition equation 7 for the formal sector

$$\sigma_L = [(1-\beta_1)Q_1 + Q_2]/(Q_1 + Q_2).$$

This is a simple comparative static exercise of the share of labor in total output with respect to change in the labor force. Proceed as follows: let $\sigma_L' = d\sigma_L/d\bar{L}$ so that

$$sgn(\sigma_L') = sgn[(Q_1 + Q_2)(1-\beta_1) - (1-\beta_1)Q_1 - Q_2]$$

where sgn denotes the algebraic sign of the following term. Simplifying

$$sgn(\sigma_L') = sgn(-\beta_1 Q_2) < 0. \tag{10}$$

Thus, the share of labor in total income falls with a rise in $\bar{L}$ and thus the Gini rises.

A verbal explanation of the same point begins with the same observation that the inflow of workers raises the share of labour in the population. Productivity falls since output per worker in the formal sector declines as does output per worker in the informal sector. With the influx of workers into the informal sector, average output there falls, dragging the real wage in the *formal* sector down with it. In the Cobb-Douglas, the marginal productivity is given by equation 7, but the average productivity is the marginal productivity *divided by* $(1-\beta_1)$, so that they fall proportionately. This weighted average of the two falling average productivity measures must itself be falling and so the Gini coefficient, must rise unequivocally.

What then happens to employment in the formal sector? Intuitively, it rises with fall in the real wage. It may be instructive to see that the comparative statics confirm this intuition.[11] From the Lewis equation, the equilibrium level of L_1 is given by the root of

$$(\bar{L} - L_1)/L_1^{\beta_1} = Q_2/(1-\beta_1)\mathcal{A}_1 K_1^{\beta_1}$$

Differentiating with respect to $\bar{L}$

$$L_1^{\beta_1}(1 - L_1') - (\bar{L} - L_1)\beta_1 L_1^{(\beta_1 - 1)} L_1' = 0$$

where $L_1' = dL_1/d\bar{L}$. Note since nothing on the right-hand-side depends L_1 or $\bar{L}$ it is set equal to zero. Solving for L_1',

$$L_1^{\beta_1} = (\bar{L} - L_1)\beta_1 L_1^{(\beta_1 - 1)} L_1' + L_1^{\beta_1} L_1' > 0$$

. This implies that if the system experiences a rise in the supply of labour, the formal sector will increase its demand for labour. There are two effects here at play: first the real wage of labour falls, inducing a "substitution" effect that, at the same level of output, increases the labour-intensity of production. There is an "income" effect as well, in that as more labour is available,

$$L_1^{\beta_1} = [(\bar{L} - L_1)\beta_1 L_1^{(\beta_1 - 1)} + L_1^{\beta_1}]L_1'$$

[11] A convinced reader can safely skip this next section without loss of continuity.

$$\frac{L_1^{\beta_1}}{(\bar{L}-L_1)\beta_1 L_1^{(\beta_1-1)}+L_1^{\beta_1}} = L_1^{'}$$

which demonstrates that the income effect is positive for the demand for formal labour.

Policymakers frightened by the influx of labour must find a flaw in the foregoing to maintain their position that immigrant labour will be bad for formal employment. Labour unions might object however, since it is clear that real wages will decline. Thus, the same policymakers could respond that it is precisely this latter effect, decried by the unions, that is responsible for increasing poverty in the country.

This concludes the discussion of the static model of functional informality. The discussion has necessarily been incomplete since there are a wide range of parameter values that could also be investigated.

4.4. Dynamics of informality

The formal-informal sector model can be adapted to a dynamic framework to capture long-run effects. Formal sector dynamics are provided by the standard equation of capital accumulation for the state variable K_{1t}, the capital stock in the formal sector.

$$K_{1t} = K_{1t-1}(1-\delta) + I_{t-1} \tag{11}$$

where the t is the time subscript, δ is the rate of depreciation and I is investment in the previous period. With the capital stock increasing, it is now possible to have the marginal product of labour increasing in the formal sector with the same real wage. This enables formal sector producers to increase their use of informal labour.

Equation 11 links one period to the next but within each period, equation 8 can be solved to distribute labour between the formal and informal sectors. The quantity of surplus labour can then be computed for each period.[12] The model requires a data base in the form of a SAM and from there it is possible to compute the path from an initial condition to the functional/juridical turning point. Table 3 presents the formal/informal SAM to which a dynamic model with functional informality is calibrated. First note that with the nominal wage rate equal to 1, the number of formal workers, $L_1 = 70$. With a labour force $\bar{L} = 90$, the remaining labour is in the informal sector, so that equation 5 is satisfied with $L_2 + L_s = 20$. Since output in the informal sector is $Q_2 = 20$, the wage in equation 6 is $Q_2/(L_2 + L_s) = 1$. The share of output in the formal sector that accrues to capital is $\beta = 0.3$ and from the production function in equation 4 the capital stock must be

$$K_1 = [Q_1/L^{1-\beta_1}]^{1/\beta_1} = 230$$

[12]Rather than use a non-linear equation solver, such as *Mathematica* or the General Algebraic Modeling System (GAMS), it is far easier to solve the model sequentially in Excel, for which no sophisticated methods are required. The properties of the model can then studied as the foundation for larger formal/informal models. Both GAMS models and *Mathematica* worksheets are available from the authors on request.

Table 2: A SAM for the functionally informal model

	Formal	Informal	Consumption	Investment	Total
Formal	-	-	350	50	**400**
Informal	-	-	50	-	**50**
Value Added	400	50	-	-	**450**
Labour	150	42	-	-	**150**
Capital	250	-	-	-	**250**
Savings	-	-	50	-	**50**
Total	**400**	**50**	**450**	**50**	-

- = N/A
Millions of LCUs.
Source: Authors' computations based on illustrative data.

In table 2 there are two sectors, formal and informal. Here the GDP, computed as the sum of value added in both sectors, is 400. Investment *by origin* is 50 million LCUs and is added to the capital stock in the formal sector.[13]

Labour for the modern sector has become more expensive and so it must substitute capital for labour.[14] This increases the marginal productivity in the formal sector and employment also increases there. For an increase in Q_1 to occur under this circumstance, K_1 must rise. This is the source of the increase in the output of the formal sector.

5. Phase transition in the model of functional informality

How then can the insights from the theoretical model be combined with a data set for a SAM to give a dynamic model of the functionally informal sector? Begin by specifying a simple SAM for the example.

In addition to the data of the SAM, additional parameters must be specified as shown in table 4.

The settings for the first simulation are simple and although somewhat unrealistic, are designed to reveal the principal adjustment mechanisms of the model as clearly as possible. There is no growth in either the labour force or the output of the informal sector. Half of formal sector profits are invested in the capital stock of the formal sector. The capital

[13]The distinction between *origin* and *destination* is necessary in multi-sectoral models. The former is a component of aggregate demand whereas the latter changes the capital stock by sector, according to equation 11.

[14]Observe that the increase in the marginal productivity of labour that must come about due to a rise in the formal sector wages is due to this higher level of capital that the formal sector employs.

Table 3: Informal SAM[1]

	Formal	Informal	Consum-ption	Invest-ment	Total
Formal	-	-	83.52	16.48	**100**
Informal	-	-	50	-	**50**
HH	100	50	-	-	**150**
Value added	-	-	-	-	-
Labour-Formal	70	-	-	-	**70**
Labour-Informal	-	50	-	-	**50**
Capital	30	-	-	-	**30**
Savings	-	-	16.48	-	16.48
Total	**100**	**50**	**150**	**16.48**	-

[1] Nominal LCUs.
Source: Authors' computations.

Table 4: Basic parameter settings

	Sim 1 (%)	Sim 2 (%)	Sim 3 (%)	Sim 4 (%)
Depreciation[1] (δ) rate	5.0	5.0	5.0	5.0
Savings rate out of profits [1] (s)	0.50	.50	0.50	0.50
Rate of growth of informal output Q_2	0.0	0.02	0.0	0.0
labour force growth	0.0	0.001	0.0	0.0[3]
Technical change ($\mathcal{A}$) growth	0	0	0	0.5[3]
Share of capital β_1 formal	0.3	0.3	0.3	0.3
Share of capital β_2 informal	0.2	0.2	0.2	0.2
Initial capital stock formal[2]	230	230	230	230
Initial capital stock informal	60	60	60	60
labour supply[2,3]	120	120	120	120

1. Parameters *not* calibrated from SAM. 2. From the base SAM.
3. With wages = 1.
Source: Authors' computations

stock of the informal sector remains constant. Table 5 shows the results of simulation for 30 periods.

Surplus labour is shown in the first column and from the data of the table, it is evident that the economy exhausts its informal sector surplus labour by period 22. The decline proceeds at a rate of 15 percent per year. Thereafter, the economy is *all* formal with the two sectors still producing the same good, but with different formal processes[15] No "defective" process is in operation.

Column 3 of the table shows the formal capital stock, which is growing at a rate of 1 percent in response to the 50 percent of profits that have been reinvested in the capital stock. The rate of depreciation slows the rate of capital accumulation in the formal sector, which in turn reduces the demand for labour that pools in the informal sector. Formal sector capital stock thus takes pride of place in determining the rate at which the economy approaches the turning point, from formal/informal to all formal.[16]

With Q_2 given, the next two columns *simultaneously* determine the level of output, in column 5, and employment, in column 8, in the formal sector. The Lewis equation (equation 8) in column 7 determines the ratio of employment in the two sectors, but this depends on the quantity of formal output in column 5, Q_1, which in turn depends on L_1, the quantity of formal labour. With formal labour known, *total* informal labour is also determined (not shown) as just the difference between the labour supply and total formal labour.

5.1. A defective process in the informal sector

Splitting this quantity of labour into that required for the production of informal output, Q_2, and surplus labour, L_s, requires some effort but is highly instructive. Functionally informal labour, as defined here, is the quantity of labour *beyond the quantity of labour necessary for the formal production of* Q_2. Thus, it is necessary to define how much labour would be required to produce Q_2 formally, by way of the production function in equation 4. Taking the level of Q_2 and K_2 as *given*, the level of L_2 is then computed in column 9. Note that this level of labour is *not* consistent with the first-order condition of equation 7. The production process employed in the informal sector is thus *defective* in that profit is not maximized, as it would be in the formal sector. In order to produce the level of output in column 6, with the capital stock in column 5, the labour demand in column 9 would require a real wage of 0.8[17] This wage is 20 percent *lower than the real wage determined by the average product in the informal sector*, shown in column 11. The process employed in the informal sector is therefore *defective*. If the average product in the informal sector were paid entrepreneurs producing Q_2 formally, the rate of profit would fall to that shown in column 14 and the process would be abandoned. This is the meaning of "defective" as used in this paper.

[15]The capital stock in the previously informal sector is lower, set at 60, and the share of profits is also lower, $\beta_2 = 0.25$.
[16]Here the reference is to functional informality; juridical informality, as noted above, can persist after the turning point.
[17]This is obtained by setting equation 7 to w, the required wage, and then solving for L_2, setting $Q_2 = 50$ and $K_2 = 60$.

5.2. Capital accumulation and growth

Column 12 of table 5 shows that prior to the turning point, the formal sector real wage is equal to the average product in the informal sector. The rate of formal sector profit is then computed in column 13 and column 14 shows the virtual rate of profit in the informal sector, if Q_2 were produced formally. Formal output is shown in column 14 and investment as the savings rate times the *mass* of profits is shown in column 15. The last two columns show the GDP and the share of labour in the formal sector as the economy approaches the turning point.

As the economy reaches the turning point, the surplus labour in the informal sector steadily approaches zero. All informal activity thereafter is juridical as functional informality disappears. All production is formally produced with the real wage equal to the marginal product.

Table 5: The decline of functional informality as formal capital accumulates

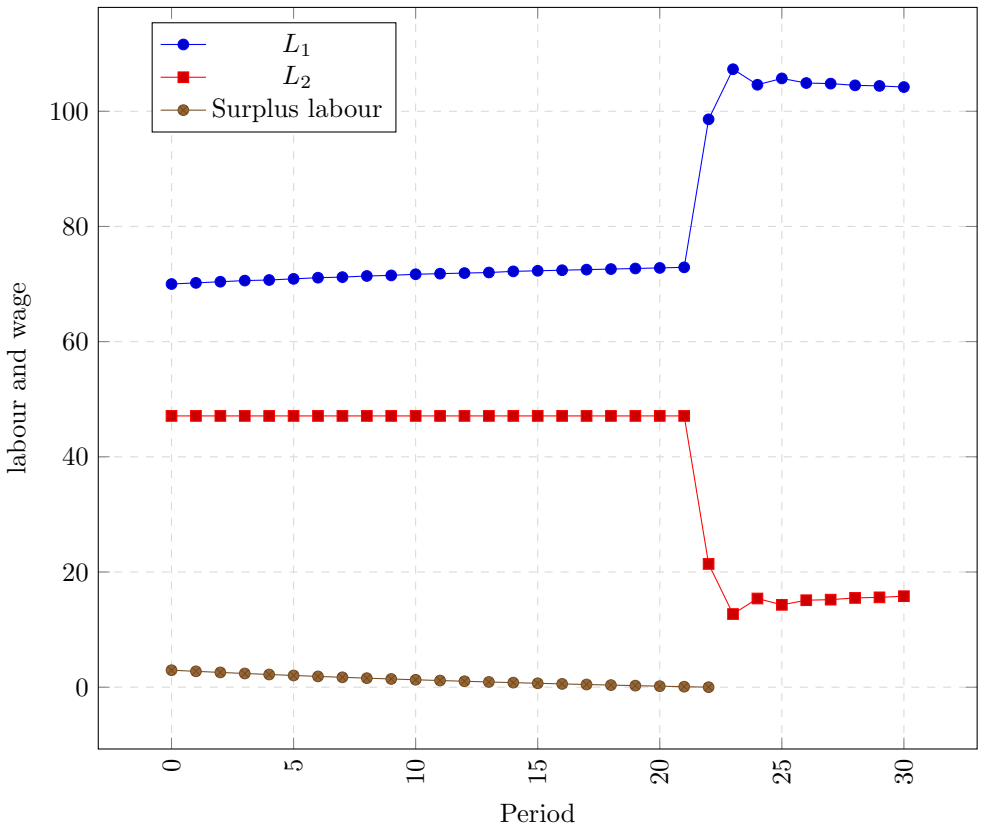
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
							Fml/			Fml	Avg					
	Surplus	Labour	Capital		Output		Infml	Labour		Share of	Infml		Profit		Invest-	
Time	Labour	Supply	Fml[1]	Infml	Fml	Infml	Labour	Fml	Infml	Labour	Product	Wage	Fml	Infml	ment	GDP
0	2.9	120	230	60	100	50	1.40	70.0	47.1	0.58	1	1	0.13	0.05	16.5	150.0
1	2.8	120	233	60	101	50	1.41	70.2	47.1	0.58	1	1	0.13	0.05	16.7	150.7
2	2.6	120	237	60	101	50	1.42	70.4	47.1	0.59	1.01	1.01	0.13	0.04	16.7	151.3
3	2.4	120	241	60	102	50	1.43	70.6	47.1	0.59	1.01	1.01	0.13	0.04	16.7	152.0
4	2.2	120	244	60	103	50	1.44	70.7	47.1	0.59	1.02	1.02	0.13	0.04	16.7	152.6
5	2.0	120	248	60	103	50	1.44	70.9	47.1	0.59	1.02	1.02	0.13	0.03	16.7	153.2
6	1.9	120	251	60	104	50	1.45	71.1	47.1	0.59	1.02	1.02	0.13	0.03	16.7	153.8
7	1.7	120	254	60	104	50	1.46	71.2	47.1	0.59	1.03	1.03	0.12	0.03	16.7	154.3
8	1.6	120	257	60	105	50	1.47	71.4	47.1	0.59	1.03	1.03	0.12	0.03	16.7	154.9
9	1.4	120	260	60	105	50	1.48	71.5	47.1	0.60	1.03	1.03	0.12	0.02	16.7	155.4
10	1.3	120	263	60	106	50	1.48	71.7	47.1	0.60	1.03	1.03	0.12	0.02	16.7	155.9
11	1.2	120	266	60	106	50	1.49	71.8	47.1	0.60	1.04	1.04	0.12	0.02	16.7	156.4
12	1.0	120	269	60	107	50	1.50	71.9	47.1	0.60	1.04	1.04	0.12	0.02	16.7	156.8
13	0.9	120	272	60	107	50	1.50	72.0	47.1	0.60	1.04	1.04	0.12	0.02	16.7	157.3
14	0.8	120	274	60	108	50	1.51	72.2	47.1	0.60	1.05	1.05	0.12	0.01	16.7	157.7
15	0.7	120	277	60	108	50	1.51	72.3	47.1	0.60	1.05	1.05	0.12	0.01	16.7	158.2
16	0.6	120	280	60	109	50	1.52	72.4	47.1	0.60	1.05	1.05	0.12	0.01	16.7	158.6
17	0.5	120	282	60	109	50	1.53	72.5	47.1	0.60	1.05	1.05	0.12	0.01	16.7	159.0
18	0.4	120	284	60	109	50	1.53	72.6	47.1	0.60	1.05	1.05	0.12	0.01	16.7	159.3
19	0.3	120	287	60	110	50	1.54	72.7	47.1	0.61	1.06	1.06	0.12	0	16.7	159.7
20	0.2	120	289	60	110	50	1.54	72.8	47.1	0.61	1.06	1.06	0.12	0	16.7	160.1
21	0.1	120	291	60	110	50	1.55	72.9	47.1	0.61	1.06	1.06	0.11	0	16.7	160.4
22	0.0	120	304	63	138	28	3.46	98.6	21.4	0.78	1.04	0.98	0.14	0.12	25.3	166.3
23	0.0	120	331	68	142	28	-	98.9	21.1	1	0	1.01	0.14	0.11	27.0	170.4
24	0.0	120	359	74	146	29	-	99.2	20.8	1	0	1.03	0.13	0.10	27.6	174.5
25	0.0	120	387	80	150	29	-	99.4	20.6	1	0	1.05	0.12	0.10	28.1	178.4
26	0.0	120	414	85	153	29	-	99.7	20.3	1	0	1.07	0.12	0.09	28.4	181.9
27	0.0	120	441	91	156	29	-	99.9	20.1	1	0	1.09	0.11	0.09	28.8	185.3
28	0.0	120	467	96	159	30	-	100.0	20.0	1	0	1.11	0.11	0.08	29.1	188.4
29	0.0	120	492	101	161	30	-	100.2	19.8	1	0	1.13	0.10	0.08	29.4	191.2
30	0.0	120	516	106	164	30	-	100.4	19.6	1	0	1.14	0.10	0.07	29.7	193.9

Source: Authors' computations. Note: Fml = formal sector. Infml = informal sector.

Average growth in output is slow before the turning point, only one-half percentage point per period. Thereafter, growth accelerates to almost two percent. Part of the reason is the large jump in investment that takes place at the phase transition, when the second sector becomes formal. Part of the reason is the decline in the wage rate, dropping by more than 6 percent as the surplus labour is formally absorbed.

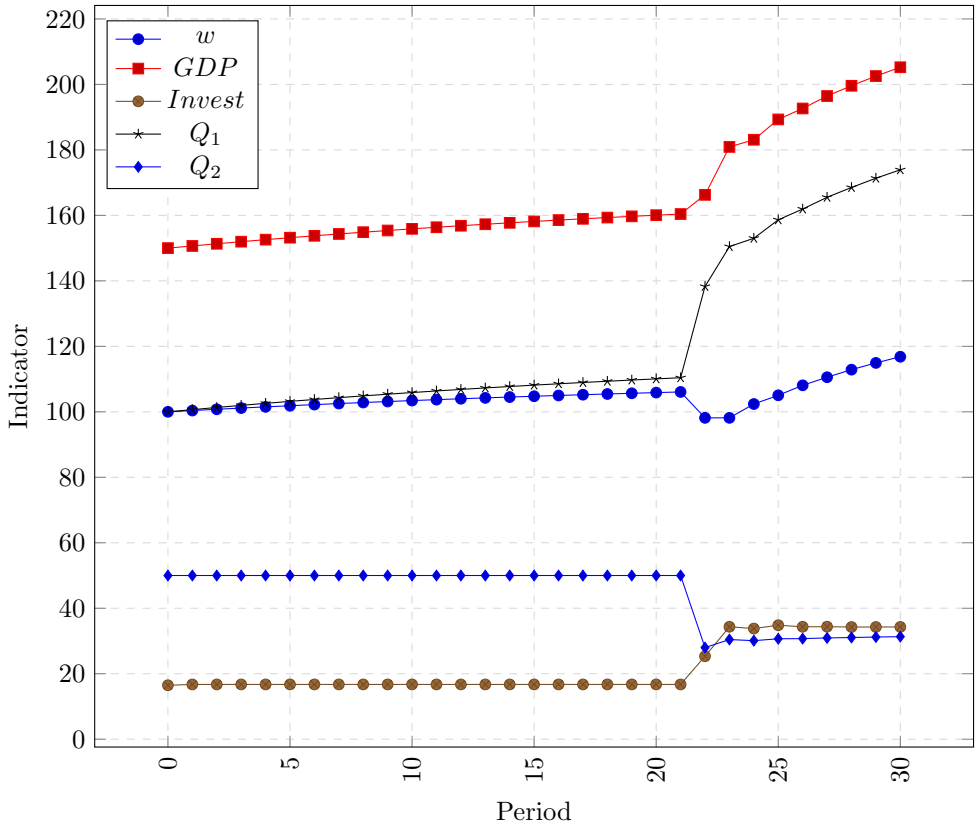
Why does this happen? The change in the state of the system begins with a *collapse* in the level of Q_2 as the employment, L_2, falls to a level consistent with the marginal productivity of labour. The cause of the transition is this change from functionally informal to formal status. The drop in the wage rate encourages the formal sector to employ much more labour, rising from 72.9 in period 21, to 98.6 in period 22 to 107.3 in the first period after the transition. The wage reduction sets the stage for an acceleration in the rate of formal employment.

Figure 3: Employment before and after the functional/juridical turning point



So it is clear from figures 3 and 4 that if the wage rate falls, formal sector processes will be able to absorb more labour and in the limit the entire labour force. This could have happened at any point in the time path of the economy, but the assumption is that if the wage rate does indeed fall to a level supporting full employment, the wage would hit some minimum biological level. Now that capital has accumulated, a market clearing wage could be feasible. The simulations shows, however, that at the transition, the formal wage is *lower* that it had been with functional informality. To answer the "how does this happen" question of the previous paragraph one must dig a bit deeper.

Figure 4: Macro indicators before and after the functional/juridical turning point



The first observation made by (Blattman and Dercon, 2016) is that "it doesn't always happen." In a widely disseminated paper, the authors report on a 5-year experiment that addresses precisely this question. It appears that in Ethiopia, at least, the lure of formal sector employment is not dispositive. Of the workers offered formal sector jobs in the context of a randomized controlled trial, some two-thirds quit after one year, seemingly preferring the more "entrepreneurial" alternative offered by informal sector activity. The paper does not, lamentably, distinguish functional and juridical informality, but nonetheless casts a somewhat dark shadow on the simulations in table 5. Conventional wisdom has robustly held that workers prefer formal to informal work, because of benefits, learning by doing, stability and potential wage growth, all features that are markedly absent in functional informality. (Blattman and Dercon, 2016), however, show that workers quit for valid reasons: many of them get sick or are hurt on the job. Informal work offers a flexibility of working hours that cannot be matched by factory work. In short, the latter is no picnic and this raises the crucial question of why workers in period 22 would give up their informal jobs for a *lower* formal wage.

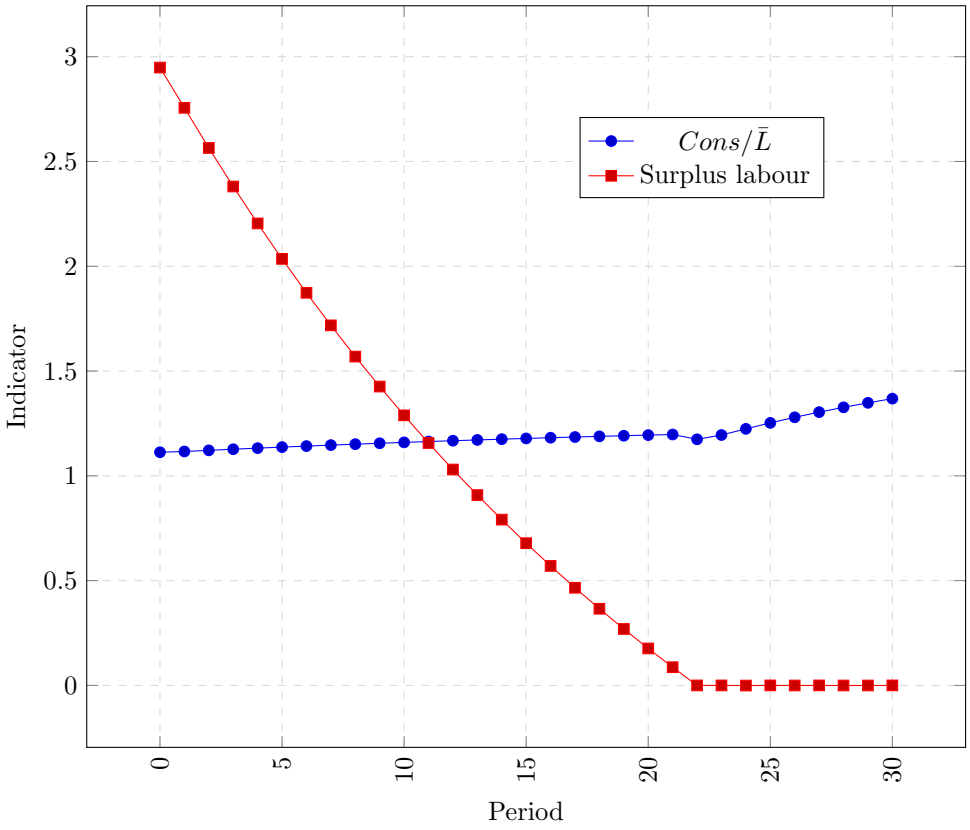
The short answer is the potential for wage growth that the simulation clearly reveals. (Boudreaux and Cowen, 2008) observe that women prefer informal credit markets with obligatory loan amortization, even though the interest rates can be unreasonably high. Without going into their argument in detail, it boils down to this: women will pay a premium to prevent the extended family from dissipating any positive savings on consumption, such as alcohol or unreasonably speculative commercial ventures. They will accept a negative return on savings to keep their wealth safe, not from theft, but from unproductive consumption Schindler (2007).

One could easily describe the transition at the turning point in similar terms: workers forgo higher wages in the informal sector, effectively depositing the difference with formal sector employers, again for "safekeeping." Workers need to have faith that this surplus will indeed be invested in new, employment generating capital stock. It is merely an assumption in the simulation that it will and there it clearly pays off. Wage growth before the turning point is less than one percent and more than doubles thereafter.

The narrative of this transition could unfold in the following way. Workers may hear of jobs newly available in the formal sector and be attracted to them. So long as there is no herding, there would only be an imperceptible drop in the wage rate as workers begin to trickle in. This subtlety is necessarily lost in the simulation, however, such that in the final days of the informal sector's existence, all informal workers simultaneously pool into the formal labour market. Naturally, the wage will take a steep decline. As discussed in detail below, the wage recovers, but informal sector workers with high discount rates could indeed forestall the turning point. Reality will certainly be clouded by the considerations of these paragraphs and will be unlikely to follow the crisp path shown in the simulation of table 5.

There has been a phase change in this data at the turning point. Beginning with the uppermost series, GDP, there is an observable jump as the functional informal sector fades. As noted, GDP grows, prior to the turning point, at only about 0.5 percent per period, while after the turning point (from period 23-30), GDP grows at slightly less than 2 percent.

Figure 5: Consumption per worker and surplus labour



The real wage growth behaves in a similar fashion. Prior to the turning point, the growth rate is 0.3 percent. Despite the sharp drop at the phase change, the growth rate of labour increases dramatically to 2 percent and by the 4th period in the post-turning point period, wages exceed the maximum achieved prior to the turning point. There is full employment in the formal sector, by definition, after the phase transition.

This shows the power of formality but the question arises as to precisely why it occurs. The answer lies in the rate of capital accumulation in the two sectors. Prior to the transition, there is no net capital accumulation in the informal sector, by assumption. There is no profit to fuel the accumulation until the turning point, when sector 2 becomes formal. The fall in the wage rate causes profit in sector 2 to rise dramatically and total profits jump from 21 percent of GDP to 29 percent of GDP (not shown), providing for a qualitative change in the growth path of the economy. The capital stock growth rate responds appropriately; after the transition it rises to nearly 7 percent for both sectors, having grown in the pre-transition period at less than one percent for the formal and zero for the informal sector.

Figure 5 further confirms that informals effectively invest in formality with an expectation of a brighter future. There consumption per worker, defined as GDP less investment divided by the labour force, in table 5, rises slowly as surplus labour is absorbed. At the phase transition, consumption per capita falls, but then quickly recovers as workers' wages rise with increasing marginal productivity. GDP growth also rises dramatically after the turning point, as noted, primarily because investment growth rebounds. The latter falls to nearly zero right before the phase transition since wages are determined by the average product in the informal sector and as the economy nears the critical point, investment growth virtually comes to a halt. Just after the phase transition, investment growth rises abruptly and then slows back to a steady growth as the capital stock in both sectors expands at a common rate.

There is nothing about the pre-turning point phase of the economy that is desirable. In effect, profits that would have been generated by formality are keeping the wage rate above subsistence for functionally informal. After the transition, those profits are devoted to capital accumulation, growth and higher wages.

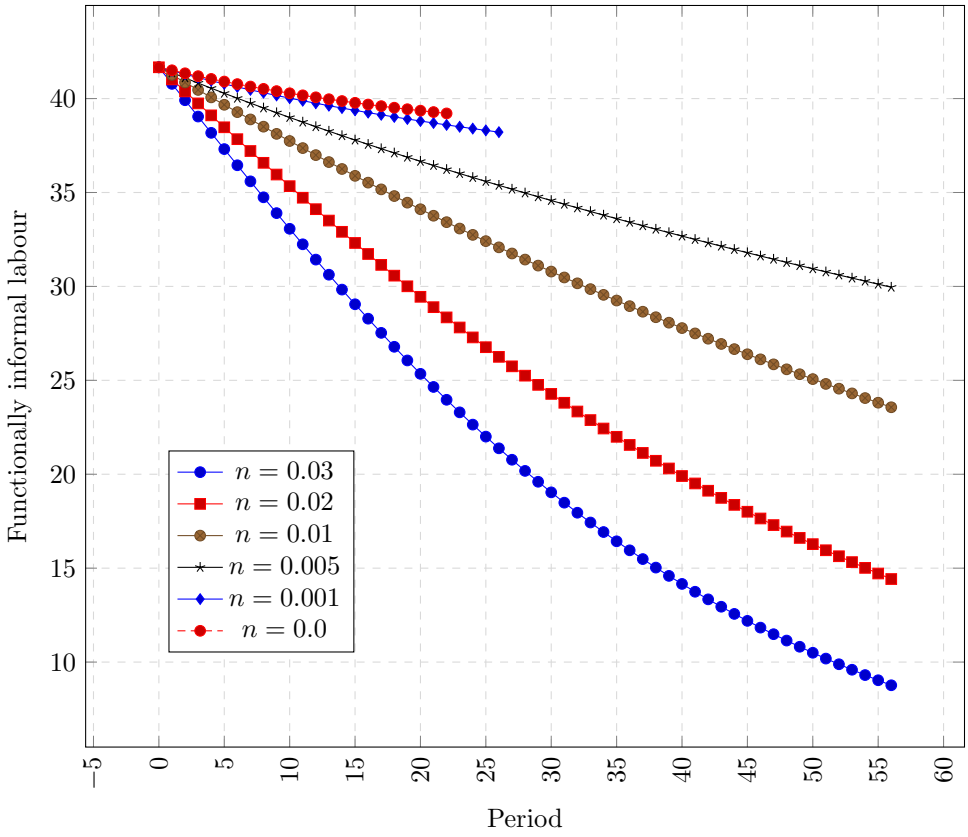
5.3. Growth in the labour force

The first simulation above assumed that the growth in the labour force is zero. This is obviously unrealistic for developing countries and was only assumed to facilitate the explanation of the model. Figure 6 shows the dramatic effect of raising the rate of growth of the labour force on the quantity of functional informality. Even a one tenth of one percent increase in the growth rate forestalls the transition by 4 years. Higher growth rates cause functional informality to rise above the base level before falling, even though as a percentage of the labour force, the functionally informal still falls.

Figure 6 plots the total amount of informality, the labour required to produce Q_2, *plus* the surplus labour *not* required for the production of Q_2, as a function of the number of periods before the phase change. Note that in the first simulation, the quantity of informal labour is

some 39.2 percent of the labour force at the critical point and zero thereafter. This reveals a highly unrealistic aspect of the model in that such a large fraction of the labour force would magically convert to formality, virtually over night. The figure shows that as the growth rate of the labour force moves into a more reasonable range, the process unfolds much more gradually. Figure 6 shows that with a 3 percent population growth rate, total functional informality declines to only 8 percent of the labour force before the critical point and slightly less than 15 percent when the labour force grows at 2 percent. The number of periods before the turning point increases rapidly with population growth, highlighting the real-world difficulty of entirely disposing with functional informality.

Figure 6: Functionally informal labour at various population growth rates
Percent of the labour force



5.4. Growth in informal output

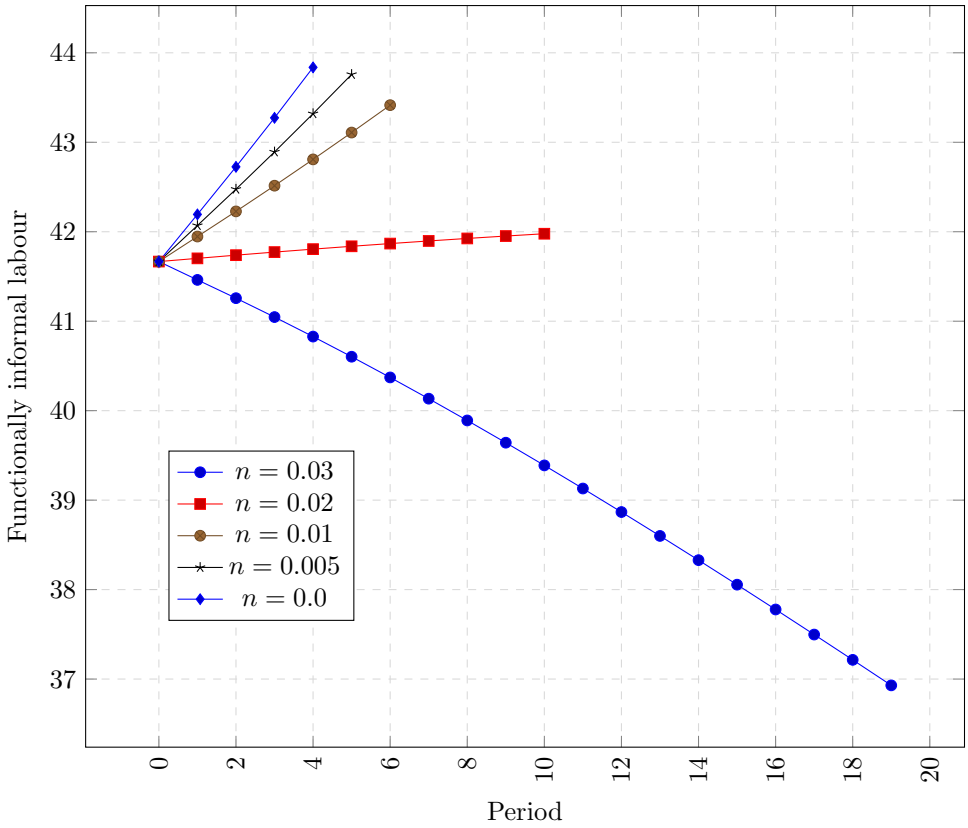
Offsetting population growth is the growth rate of informal output, Q_2. Again, the first simulation unrealistically claimed a zero growth rate for informal production, even though productivity of the informal sector, Q_2 divided by the total number of functionally informal, rises vegetatively. Zero growth of informal output, however, is a strong assumption and it is instructive to see how the model behaves when it is relaxed.

Figure 7 shows the effect of growth in informal output. The series plotted at the top shows

the effect of raising output with no compensatory increase in the growth rate of the labour force. This results in the clearing of surplus labour very rapidly. By the fifth period, the transition to formality or juridical informality is initiated. The lack of population growth removes the model from serious consideration since, again, a large fraction of the labour force goes formal in a very short amount of time. Moreover, with Q_2 rising at 2 percent, the number of informals actually *rises* before the critical point.

The explanation of this seemly bizarre behaviour of the model is actually quite straight forward, keeping in mind the reluctance of Q_1 operators to hire labour when the wage is above the marginal product of labour. The higher growth rate of Q_2 ensures that the average product in the informal sector slows the flow of labour, quite impressively, from the informal to formal, Q_1, production. In fact, it even *reverses*. This mystery solved, the realism of the model can then be restored by adding a bit of labour force growth to ensure that progress is still made in the proper direction, that of reducing informal sector activity over time. The growth rate of Q_2 at 2 percent is hefty, admittedly, and figure 7 shows that a correspondingly large rate of labour force growth is required, 3 percent, to arrest the perverse trend.

Figure 7: Functionally informal labour at various population growth rates
Percent of the labour force with growth of informal output at 2 percent



This brief foray into the mechanics of a supply driven model of informality concludes the discussion of the Lewis-based system. Sceptics will claim, of course, that it is not the level of formal capital stock that drives an actual economy toward the phase transition and

that many other factors should have been included. The analytics of the simple model are nonetheless necessary for a thorough understanding of how informality actually affects an economy. Structural details left out in the discussion, above all demand and the role of the public sector in stimulating it, the exchange rate, education and many, many more features are all in the province of CGE modeling. This more realistic framework can still accommodate the functionally informal mechanism for reaching the phase transition.

6. A CGE model with an informal sector

The next step is to generalize the model to include a demand side in a full CGE framework (Davies and Thurlow, 2010; Gibson, 2005). Much of the structure elaborated above will be preserved to facilitate the understanding of the framework. Most CGEs are multisectoral with many categories of goods, processes and household income. They include a full I-O structure to model intermediates and more complex production functions than are described here. Rather than fully develop the CGE modeling framework, this section focuses on the essential links between the simple model above and a model in which demand, derived from factor incomes, plays an essential role.

In what follows, the formal economy is still aggregated into one sector and the informal sector is broken out. There is still only one good, produced by the formal and informal sectors. It would not require much to have a multiplicity of formal sectors, but it would add little to the discussion to follow and complicate matters significantly.

Households incomes are determined by the factor-to-household income matrix that relates the functional to the size distribution of income. This means that the model records both "rich" and "poor" households but does not assert that all the poor work in the informal sector. Thus, there are no"informal households"; poor households, nonetheless, will be far more likely to work in the informal sector than their rich counterparts. If the latter are observed to participate in the informal sector, they are more likely be juridical rather than functionally informal. It is to the latter that the model is principally addressed.

Households are related to factor income in CGE models through an income distribution matrix that takes the form of

$$\phi(i,j) = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} \\ \phi_{2,1} & \phi_{2,2} \end{bmatrix}$$

where the i index is for factors, labour and capital, and the j is for households, poor and rich. Here, for example, ϕ_{21} is the amount of informal income received by the poor households and is likely to be large, while ϕ_{22} is the amount of informal income going to rich households and is likely to be correspondingly small.

Here again there are two production processes, one for the formal and one for the informal sector. All this, of course, assumes that the economy has not yet reached the turning point or phase transition, discussed above, at which there emerge two production processes, both formal, that hire factors in according with the conventional profit maximizing criteria.

The more realistic environment of the CGE model allows a slightly more elaborate account of the dynamics of formal/informal sector evolution. This was alluded to above, but now can be seen more clearly. The key difference is that the total amount of aggregate demand in the model determines the level of economic activity. Taking the level of Q_2 as endogenously determined (or, more realistically, by the growth of informal capital), Q_1 is now determined by the level of aggregate demand in the system.

Formal firms can enter or exit a market based on profitability and there is no problem when a formal firm shuts down. It simply decides that its resources are better (more profitably) used elsewhere. Informal firms do not and cannot shut down since they provide the only means of subsistence for those who operate these "defective" processes. It follows that the share of *formal* firms in the total volume of demand is a residual, left after informal firms sell all they can at the going price.

How the level of informal output is determined depends on the structural characteristics of the economy. If the output is given, the suggestion is that the *marginal product of labour in the informal sector is effectively zero.* Labour can thus leave the informal sector without having any impact on the level of Q_2. Thus the real wage rises as the formal sector draws labour away from the informal sector. A second option is to assume that the labour coefficient, output per worker in the informal sector, is constant and as workers depart the informal sector, output falls in proportion. This is, in a sense, the opposite assumption; the marginal productivity of labour is not zero, but rather equal to the average productivity of labour in the informal sector.

It is important to see the effects of these two assumptions on the the behavior of the model. In the first case, informal income per worker rises when labour is drawn away from the informal sector. The real wage in the system therefore increases and so for a given amount of capital accumulation in the formal sector, fewer workers will be hired. In the second case, informal output per worker is constant and therefore, so is the real wage. Formal employment rises in proportion to the capital stock and so its growth is unimpeded by the effect of rising wages. In both cases, informals are assumed to be able to sell as much as they produce. Why is this assumption invoked? The answer comes from the price side of the model. Informal firms are, above all, price takers in that they have no way to alter the price of the good they produce. If it is not cheaper, weighted by quality, than the corresponding formal good, informals will sell nothing. Hence, the informal sector price is determined by formal sector costs.

The interlacing of the model is now emerging; formals determine price, which in turn determines incomes of informals, given their level of output. On the other hand, informals determine the quantities the formal sector sells through the productivity of the processes that informals operate. This "criss-crossed" relationship between the formal and informal sectors is key to how the CGE model functions and must be understood.

Rather than having formal sector demand for labour depend on the current level of the capital stock and the real wage rate determined in informal sector, the approach allows aggregate demand to determine the demand for labour in the formal sector. Again, there are two options. The first is to assume that demand for labour depends on the marginal

productivity of labour as in equation 7, so that as the real informal wage rises, the demand for labour will contract. A second approach is to let the labour coefficient in the formal sector determine the quantity of labour hired there. In the first case, the level of the capital stock is important as well as aggregate demand. In the second case, it is only the level of aggregate demand that determines the growth in formal employment.

In either case, the formal sector is assumed to operate at less than full capacity, defined as the quantity it could produce if it hired labour up to the marginal product as determined by the full use of its capital stock. In a demand determined CGE model, however, it is not the quantity of the capital stock that limits production but rather the aggregate demand. For the formal sector, this is the amount of aggregate demand left unsatisfied by *informal* sector activity.

Case I: *The demand for labour is sensitive to the real wage* is. The question arises as to what is the level of Q in the marginal productivity equation 7 above. In the earlier model, it was determined by the capital stock and the real wage determined in the informal sector.[18] To have a model of functional informality operating inside the CGE model, it is only necessary to introduce $Q'_1 \leq Q_1$, where the prime indicates that the level of value added in the formal sector, as determined by aggregate demand, is *less* than what would be produced by the formal sector if the formal sector fully used its capital stock.

6.1. Case I: A simple example

Rather than jumping to a fully developed CGE model to illustrate how the informal sector can be incorporated, consider the simple example discussed in table 3. The table is reproduced below with intermediates to add some realism. For simplicity the distinction between rich and poor consumers has been suppressed. As in standard Keynesian models, there is only one consumption function

$$C = \bar{C}c(Q_1 + Q_2) \tag{13}$$

with a marginal propensity to consume of $c = 0.7$ and an intercept of $\bar{C} = 28.5$. The GDP is the same as in table 3 above, as is the distribution of value added by the formal and informal sectors. The labour force is again equal to $\bar{L} = 120$.

To see how demand modifies the basic structure of the model, consider the fact total aggregate demand, F, or GDP, Y, must be equal to total value added, Y

$$F = Y = Q'_1 + Q_2 \tag{14}$$

where Q_i represents value added for both the formal and informal processes and where the price is taken to be unity for convenience. Here Q' has been substituted for Q in the formal

[18] To see this, simply substitute the production function into the marginal productivity condition to get

$$(1 - \beta_1)(K_1/L_1)^{\beta}/L_1 = w \tag{12}$$

where the real wage is determined by the average productivity in the informal sector. It is evident that demand plays no role here in the determination of the quantity of labour hired in the formal sector.

Table 6: Informal SAM[1]

	Formal	Informal	Consum-ption	Invest-ment	Total
Formal	25.2	0.26	83.52	16.50	**125.78**
Informal	0.63	2.65	49.79	-	**52.91**
HH	100	50	-	-	**150**
Value added	-	-	-	-	-
Labour-Formal	70	37.50	-	-	**70**
Labour-Informal	-	50	-	-	**50**
Capital	30	12.50	-	-	**30**
Savings	-	-	16.50	-	**16.5**
Total	**125.78**	**52.91**	**150**	**16.50**	-

[1] Nominal LCUs.
Source: Authors' computations.

sector to capture the idea that the value added in this sector no longer depends on the capital stock, but instead on aggregate demand.[19]

Equation 7 can then be expressed, after clearing the fractions, as

$$(1-\beta_1)Q_1'(\bar{L}-L_1) = Q_2 L_1$$

substituting equation 14 into

$$(1-\beta_1)(Y-Q_2)(\bar{L}-L_1) = Q_2 L_1$$

or solving for L_1

$$\frac{(1-\beta_1)\bar{L}}{[(1-\beta_1)+Q_2/(Y-Q_2)]} = L_1$$

where now it is clear that an increase in the level of demand, Y, will *increase* the demand for formal labour. Raising Q_2 reduces the level of demand for the formal labour, increasing functional informality, just as in the model above. Finally, an increase in $\bar{L}$ will cause an increase in the demand for formal labour, as the real wage in the informal sector falls. All this follows the pattern of the simple model in section 4.

The model is solved as shown in table 7. The table shows each variable and parameter of the simplified model. Parameters are determined either from the SAM or taken as given exogenously as shown in the table. In the case of the behavioural equations, the expression

[19] Equation 14 may require some additional explanation. In table 6, the equation holds, but only in the aggregate. Specifically, one cannot write that $F_1 = Q_1$ and $F_2 = Q_2$. This is evident from the presence in off-diagonal terms in the I-O flow matrix. The material balance for the formal sector in the SAM is written

$$a_{11}X_1 + a_{12}X_2 + F_1 = a_{11}X_1 + a_{21}X_1 + Q_1$$

While the first term on both sides of this equation cancel, $F_1 = Q_1$ requires that $a_{21}X_1 = a_{12}X_2$, which does *not* generally hold.

Table 7: Solving the CGE model with an informal sector

Symbol	Concept	Source	Value	Equation
X_1	Fml GVP[1]	SAM	125.79	-
X_2	Infml GVP[1]	SAM	52.91	-
a_{11}	IO-coef	A_{11}/X_1	0.20	-
a_{12}	IO-coef	A_{12}/X_2	0.01	-
a_{21}	IO-coef	A_{21}/X_1	0.01	-
a_{22}	IO-coef	A_{22}/X_2	0.05	-
β_1	Fml capital share	SAM	0.30	-
β_2	Infml capital share	External	0.25	-
c	Marginal propensity	External	0.70	-
$\bar{C}$	Autonomous consumption	SAM	29	-
$\bar{L}$	Labour supply	SAM	120	5
L_1/L_2	Formal/informal labour	$(1-\beta_1)Q_1'/Q_2$	1.40	8
C_1	Fml consumption	$\bar{C}+c(Q_1+Q_2)-C_2$	83.87	13
C_2	Infml consumption	$Q_2-(a_{12}+a_{22})X_2$	49.64	-
I_1	Fml investment demand	SAM	16.50	-
I_2	Infml investment demand	SAM	0.00	-
L_1	Fml labour demand	$(1-\beta_1)Q_1/w$	50.00	7
L_2	Infml labour demand	$\bar{L}-L_1$	70.00	9
50.00 Q_1	Fml value added	$K_1^{\beta_1}L_1^{(1-\beta_1)}$	100.0	4
50.00 Q_1'	Fml value added[3]	$wL_1+(1-a_{11}-a_{21})X_1$	100.0	-
50.00 Q_2	Infml value added	SAM	50.0	-
K_1	Fml capital	External	230	-
60.00 K_2	Infml capital	External	60	-
w	Wage[4]	Q_2/L_2	1.00	6
1.00002 L_s	Surplus labour	$\bar{L}-L_1-L_{2f}$	2.95	-
L_{2f}	Infml labour demand	$(Q_2/K_2^{\beta_2})^{(1/(1-\beta_2))5}$	47.1	9
δ	Depreciation	External	0.05	

Source: Authors' computations. Notes: 1. Gross value of production. 2. Fml = formal sector. Infml = informal sector. 3. Determined by aggregate demand. 4. Average product in informal sector before turning point, while after turning point set to maintain zero surplus labour. 5. Notional.

determining the value of the variable is shown along with its value in the first row of table 8.

The model is first calibrated to the SAM. The consumption function takes the marginal propensity to consume, c, as given and then computes the level of autonomous consumption consistent with the SAM value of total consumption, both formal and informal. Formal consumption is a residual after the informal consumption is deducted. Informal consumption is set to the gross value of production of the informal sector, less intermediate demand (which is small) for informal output. Informal investment is set to zero for simplicity, but in a more complete model, final demand for informal output would have to be distributed across the categories of final demand by some method not discussed here.

Table 8: The decline of functional informality with aggregate demand

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
						Fml/		Output								
	Surplus	GVP[1]				Infml	Fml	Fml	Infml		Labour		Capital		Labour	
Time	Labour	Fml[1]	Infml[1]	Cons	Invest	Labour	Q_1'	Q_1	Q_2	Wage	Fml	Infml	Fml	Infml	Infml[3]	GDP
0	2.95	126	52.9	133.5	16.5	1.40	100.0	100.1	50.0	1	70.0	50.0	230.0	60.0	47.1	150.0
1	2.79	126	52.9	133.9	16.7	1.41	100.6	101.5	50.0	1	70.2	49.8	235.0	60.0	47.1	150.6
2	2.63	127	52.9	134.3	16.8	1.42	101.1	102.8	50.0	1.01	70.3	49.7	239.9	60.0	47.1	151.1
3	2.47	128	52.9	134.7	17.0	1.42	101.7	104.1	50.0	1.01	70.5	49.5	244.8	60.0	47.1	151.7
4	2.30	129	52.9	135.1	17.2	1.43	102.2	105.3	50.0	1.01	70.6	49.4	249.5	60.0	47.1	152.2
5	2.14	129	52.9	135.5	17.3	1.44	102.8	106.5	50.0	1.02	70.8	49.2	254.2	60.0	47.1	152.8
6	1.98	130	52.9	135.9	17.5	1.45	103.4	107.6	50.0	1.02	71.0	49.0	258.8	60.0	47.1	153.4
7	1.82	131	52.9	136.3	17.7	1.46	104.0	108.6	50.0	1.02	71.1	48.9	263.4	60.0	47.1	154.0
8	1.65	132	52.9	136.7	17.9	1.46	104.6	109.6	50.0	1.03	71.3	48.7	267.9	60.0	47.1	154.6
9	1.49	132	52.9	137.1	18.0	1.47	105.2	110.6	50.0	1.03	71.5	48.5	272.4	60.0	47.1	155.2
10	1.32	133	52.9	137.5	18.2	1.48	105.8	111.5	50.0	1.03	71.6	48.4	276.8	60.0	47.1	155.8
11	1.16	134	52.9	138.0	18.4	1.49	106.4	112.4	50.0	1.04	71.8	48.2	281.2	60.0	47.1	156.4
12	0.99	135	52.9	138.4	18.6	1.50	107.0	113.2	50.0	1.04	72.0	48.0	285.6	60.0	47.1	157.0
13	0.83	135	52.9	138.8	18.8	1.51	107.6	114.0	50.0	1.04	72.1	47.9	289.9	60.0	47.1	157.6
14	0.66	136	52.9	139.3	19.0	1.52	108.2	114.7	50.0	1.05	72.3	47.7	294.2	60.0	47.1	158.2
15	0.49	137	52.9	139.7	19.2	1.52	108.9	115.4	50.0	1.05	72.5	47.5	298.4	60.0	47.1	158.9
16	0.32	138	52.9	140.1	19.3	1.53	109.5	116.1	50.0	1.06	72.6	47.4	302.7	60.0	47.1	159.5
17	0.16	139	52.9	140.6	19.5	1.54	110.1	116.8	50.0	1.06	72.8	47.2	306.9	60.0	47.1	160.1
18	0.0	165	31.2	141.0	19.7	3.11	131.2	152.5	29.5	0.95	96.7	23.3	311.1	60.0	23.3	160.7
19	0.0	165	31.3	141.0	19.7	-	131.1	153.0	29.6	0.95	96.6	23.4	312.0	60.2	23.4	160.7
20	0.0	165	31.4	141.0	19.7	-	131.0	155.0	29.7	0.95	96.6	23.4	316.2	60.4	23.5	160.7
21	0.0	165	31.5	141.0	19.7	-	131.0	157.0	29.8	0.95	96.5	23.5	320.1	60.5	23.5	160.7
22	0.0	165	31.6	141.0	19.7	-	130.9	158.8	29.8	0.95	96.5	23.5	323.8	60.6	23.5	160.7
23	0.0	165	31.6	141.0	19.7	-	130.9	160.5	29.9	0.95	96.4	23.6	327.3	60.7	23.6	160.7
24	0.0	165	31.6	141.0	19.7	-	130.8	162.2	29.9	0.95	96.4	23.6	330.7	60.7	23.6	160.7

Source: Authors' computations. Notes: 1. Gross value of production. 2. Fml = formal sector. Infml = informal sector.
3. Labour demand if informal process operated formally.

Before the turning point, the process is defective in the sense that it hires more labour than would a modern, profit maximizing firm. The extra labour hired is given by

$$(Q_2/K_2^{\beta_2})^{(1/(1-\beta_2))} > (1-\beta_2)Q_2/w$$

where $w = Q_2/(\bar{L} - L_1)$ and measures, as mentioned above, the defectiveness of a "defective process". Observe that as the wage increases the process is even more defective since the gap between L_2 required to produce Q_2 (the term on the left-hand side of the inequality) and the marginal productivity of labour, on the right, increases. This accounts for the abrubt change in labour demand at the phase transition discussed above.

Table 8 shows the results of the model simulated over time. Although the columns are not precisely identical to table 5, it is instructive nonetheless to compare the two simulations. Table 8 shows progress toward the turning point that is essentially driven by a growth rate in investment, set exogenously to 1 percent per period. This rather anaemic growth still beats the simple model of table 5 to the turning point by four periods (18 versus 22). This suggests that the two simulations are broadly comparable and it is seen that there is, again, a phase transition as surplus labour disappears. As the informal sector turns formal, the quantity of labour it can absorb falls dramatically. As a result, the wage must fall to enable the formal sector to increase its demand for labour. This follows the simulation in table 5 closely.

It is important to see that while the phase change involves some discontinuities, there is nothing unrealistic about the transition. The essence of the problem is that the informal sector becomes formal, with new operators of the production processes, that follow the standard first-order conditions for profit maximization. This causes the informal sector to discharge a great deal of labour in a relatively short amount of time. The paper has argued that informals operate defective processes in the sense that more labour is used than is strictly necessary for the production of their output when modern methods of production are employed. While no post-phase change SAM is presented, there would be no assumption, for example, that the pattern of intermediate use shown in table 6 would remain in force for the informal sector. In short, the disappearance of the informal sector changes *everything*.

Formal value added is determined by the total amount of aggregate demand, less what the informal sector produces above. After the phase change, both formal and informal output are determined as in a standard CGE model, that is. by the factors of production as discussed above. The model solves for the wage rate that balances supply and demand for labour. As shown below, this causes the level of the wage to fall precipitously at the turning point (similar to the model in table 5). Thereafter, real wage growth follows capital accumulation and investment is determined by savings.

One of the important findings of the model building exercises is that the phase change that occurs when surplus labour is exhausted also involves a change in the "closure" of the model. When Q_1' rises to equal Q_1 it is no longer proper to say that aggregate demand determines the valued added. The aggregate demand equation 14 becomes essentially redundant since value added is determined by the available factors of production. The equation does not, however, go away; it must still be respected and the only way that this can happen is to

make the level of investment an endogenous variable. This changes the nature of the model fundamentally, from one in which output and employment are demand driven to one in which the key macroeconomic variables are supply driven, driven by the supplies of the factors of production. Thus, after the phase transition, the formal/informal model behaves as any other computable general equilibrium system with a so-called "neoclassical" closure. Moreover, it rejoins the simple model of functional informality, discussed above in section 4.

This claim does not mean that the model must remain at full employment of the factors of productions thereafter. Sluggish adjustment of factor prices could easily lead to unemployment of labour or capital and then the aggregate demand equation 14 would reassert itself to determine the levels of output and employment, as well as the other variables of macroeconomic interest.

Readers familiar with the CGE modeling methodology might reasonably complain that a central feature of CGEs has been ignored in the development of the model so far, the relative price system that does the work of allocating resources (capital and labour) to their efficient end uses. Against this charge there is literally no defense since from the outset it has been assumed that the formal and informal sectors share a common price. There is no role, therefore, for relative prices to allocate resources or, indeed, do anything else. This does not, however, undermine the claims mode in this paper about the generality of the informal sector analysis presented: it would be entirely feasible to embed the formal/informal dichotomy into a multi-sectoral CGE model in which relative prices are active. There may well be a formal/informal sector pair for *every* sector in the model and the balance of supply and demand, whether from formal or informal suppliers, could adjust by way of relative prices. The advantage of the criss-cross of prices and quantities in the formal/informal pair is evident: even if price is determined by multi-market, or general, equilibrium, the informal sector still takes its price as effectively given. No fundamental change in the structure of the model need be introduced.

6.2. Case II: fixed coefficients

A striking feature of LDCs is the absence of a smoothly functioning labour market, such that firms can easily expand hiring until the marginal productivity of labour falls to the real wage. Moreover, the smooth adjustment of the real wage in the formal sector to the opportunity cost of labour in the informal sector may also strike some as unreasonable, given the chaos, bustle and other distorting imperfections of labour markets in developing countries. It may be also empirically incorrect to say that there is no gap between the wage earned in the formal and informal sectors.

One way to address this criticism is to relax the assumption that the first-order conditions for profit maximization hold in the formal sector. An alternative is to simply introduce constant labour coefficients for both formal and informal processes. In this case the marginal productivity of labour in the informal sector is *not* equal to zero as in the models above. There, any reduction in informality would not have any effect on informal output whatsoever. This assumption, which dates back to the original Lewis model, perhaps calls for an even

more capricious theory of the real wage; that it is, somehow, determined *institutionally*, outside of a competitive labour market. Perhaps neither assumption is more palatable than the other and so it is worthwhile to address precisely how the model would change if the more structuralist assumptions of fixed labour coefficients were applied.

The model with fixed labour coefficients still describes functional informality if the labour constraint binds. The rigidity of fixed coefficients imparts a slightly more robust response to an increase in investment demand. In this case an increase in aggregate demand raises the demand for labour in the formal sector. This immediately drains the functionally informal labour pool, but now *the output of the informal sector falls*, due to the fixity of the labour coefficient. The real wage may be related, or not, to the average product in the informal sector, but the point is that the average product is not changing as labour moves. The labour coefficient per unit of output therefore remains fixed in both sectors, impervious to the optimizing behavior built into the model above.

Given the fixed coefficient view embedded in this version of the model, it is no longer appropriate to define surplus labour as the total labour supply less what would be required to produce the level of output Q_2.[20] In the previous model, the quantity of labour required to produce Q_2 was obtained by solving the production function for L_2. Now, with fixed coefficients, $Q_2 = (\bar{L} - L_1)/(l_2 X/(1 - a_{11} + a_{21})$ where l_2 is the direct labour coefficient in the informal sector. In other words, both formal and informal employment are directly linked to aggregate demand through fixed labour coefficients.

The criss-cross relationship between the formal and informal sectors is still active in this version of the model. The adjustment proceeds as follows: first aggregate demand determines the *sum* of output in the two sectors. The level of output of the formal sector is the residual since, as before, it cannot block the informal sector from producing and selling informal output at the price determined by formal sector. The phase transition is still defined as when all labour is employed formally.

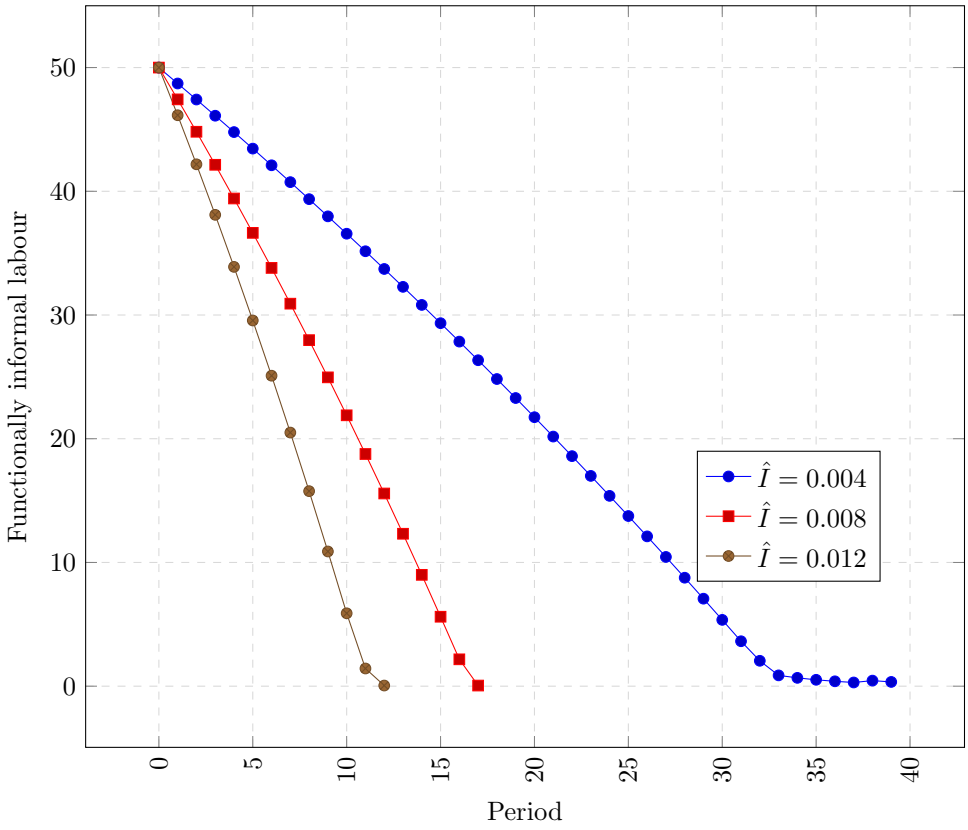
Figure 8 shows the relationship in this model with fixed coefficient between the formal and informal sectors.

Figure 8 shows three different levels of the growth in investment spending in the model of table 8 and the associated level of informal labour. Observe that a one percent rate of growth takes almost 4 decades to eliminate the informal sector. The rate of growth of GDP is anaemic, only 0.4 percent. In the second case, with investment growth of 2 percent, the time horizon is cut in half (GDP growth of 0.8 percent). Finally, a more rapid rate of growth of investment at 3 percent (GDP growth of 1.2 percent) causes the informal sector to lapse after approximately 13 years.

The discussion of the fixed coefficients model suggests a hierarchy of the effect of spending

[20] Is a fixed labour coefficient consistent with profit maximization? The answer to this question is that "it could be" so long as the wage rate does not change. Observe that the labour coefficient is L/X whereas the labour coefficient derived from the production function is $L/Q \times Q/X$, where $L/Q = (1 - \beta)/w$. With a constant capital share and wage rate, there is no change L/Q, although there might be a change in the ratio of value added (Q) to gross value of production $X = Q/(1 - a_{11} + a_{21})$. The structuralist penchant for fixed coefficients suggests that this ratio might indeed be constant as well, enabling the conclusion that fixed coefficients *are* consistent with profit maximization. The structure is admittedly rickety since any change in the wage rate could set in motion any sequence of changes.

Figure 8: Functional informal labour at various growth rates of aggregate demand



multipliers in demand constrained systems. There are three options:

1. The capital-constrained model with a demand for labour that depends on the wage rate discussed in section 4.
2. The demand-constrained model with a demand for labour that depends on the wage rate (Case I) discussed in subsection 6.1.
3. The demand-constrained model with a demand for labour that does not depend on the wage rate (Case II) discussed in subsection 6.2.

The first is the simplest: the spending multiplier in a capital-constrained model is zero. The second gives a *lower* multiplier than the third. The reason is that profit-maximizing firms will not hire additional workers unless the marginal product rises above the wage. This is precisely what the increase in demand does: it shifts the marginal product up causing more labour to be hired. All good, but here is the rub: when workers leave the informal sector the real remuneration there *rises*. This causes a secondary effect that reduces employment in the formal sector. There is a headwind in Case I of the demand-driven models. With fixed coefficients, Case II of the demand-driven models, the real wage is constant and so the headwind is absent. There is no change in the wage rate and thus no reduction in demand for formal labour. Case II speeds the economy along its way to the phase transition more rapidly while Case I is impeded by the resistance of formal employers to increasing demand for labour if wages rise.

7. From functional to juridical informality

The last section seems to wrap up the analysis of functional informality. Recall that the paper began with an assumption of *full employment*; those that are not employed formally do *not* enjoy leisure but are forced by the necessities of subsistence to join the ranks of the functionally informal. Juridical informality, on the other hand, is not about survival, it is about risk. The juridically informal run afoul of the law and in so doing reap rewards in the form of higher profits than in the formal sector. This is a strategy, pure and simple, vis-a-vis government and its legal structure.

The model of functional informality is based on the premise that workers prefer formal to informal work–independent of the wage rate–but this is not sacrosanct. In the functional model the central dynamic is the flow of labour from the informal to the formal sector, driven by a gravitational force that is impossible to resist. There can never be excess demand for formal labour so long as the reserve army of informals is available. In particular, without some growth in the labour force, there can never be an increase in *both* formal and informal labour simultaneously. With the labour-leisure trade-off in the background, the simultaneity is entirely possible so long as the remuneration in the informal sector outweighs, at the margin, the subjective value of a unit of free time. For the functionally informal this impossible; for the juridically informal it is not. A rise in aggregate demand with juridical informality could cause the following scenario to unfold. A rise in demand causes the risk-rate-of-return trade off to favor participation in the juridically informal sector. This implies that the worker

must either leave the functionally informal sector or abandon her formal sector employment. In this case, the measured informality, both functional and juridical, does not appear to have changed in the eyes of government statisticians. Yet functional informality has declined.

Thus, the empirical evidence suggesting that formal labour has its drawbacks may simply be a reflection of theoretical confusion between functional and juridical informality. Some of those leaving formal jobs may be doing so at the behest of opportunities to skirt labour and environmental laws that apply to enterprises they intend to start up. Moreover, if there is no specific preference for formal sector work, those who are juridically informal may also prefer an increase in leisure over formal employment. This has dramatic implications for the models discussed above.

The presence of juridical informality allows workers to escape the binding subsistence constraint. The juridically informal have an appetite for risk and in the best case will earn incomes that exceed either their formal or functionally informal counterparts. In the worst case, they can always retreat to informal sector. If not, just as in developed economies, they face a labour leisure trade-off. Consider then an increase in effective demand in the model with some juridical informality. This model is unconstrained by labour supply since even the smallest increase in the wage will encourage additional informal participation. Now the multiplier of an increase in final demand is at its highest. A rise in investment or government spending is likely to *increase* informality rather than decrease it. This gives an empirical foundation for the determination of the nature of informality. The policy response to this kind of informality is entirely distinct from that of the functional sort, requiring the closing of tax and regulatory loopholes and tighter implementation of labour standards. In a word, juridical informality requires juridical, that is, legal solutions. All these would be, now somewhat obviously, counterproductive if the informality were functional.

8. Conclusions

The analysis here addresses issues concerning the mechanisms of adjustment, the type of informality and possible policy implications of the application of CGEs to informality and yields the following main conclusions.

- The simulations of this paper show that the model of functional informality developed here is entirely compatible with computable general equilibrium modeling.
- In general it has been seen that the informal sector adds to GDP, stimulates *formal* sector output and employment and generally contributes to economic well-being. Efforts to remove the informal sector directly are highly likely to be counterproductive.
- Informal or traditional sector productivity is the key to addressing the question of extreme poverty since they have often have many more dependents than their formal sector counterparts. They are also more vulnerable to income shocks.
- Policies that seek to raise the opportunity cost of labour reduce the flow of workers from the informal to the formal sector.

- Economists and policymakers who take active steps to eliminate the informal sector neglect the fact that as Q_2 increases so too does Q_1, formal sector output. This is all made clear by equation 8.
- A number of models have been developed in this paper and it must be left to the practitioner to decide which version of the model is more appropriate for any given economy.

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