Non-produced means of production in Sraffa's system: basics, non-basics and quasi-basics

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1. Introduction

A principal corollary of the labour theory of value is that, since labour is the source of all value, land and other non-produced means of production can make no contribution to the value of commodities. *Rent* is consequently a deduction from the total surplus produced by labour. Although the labour theory of value has been the subject of extensive criticism for more than a century, the associated issues of non-produced means of production[1] have been largely ignored. Indeed Steedman (1977), who argues that the entire apparatus be abandoned and replaced with Sraffa's (1960) theory of production prices, makes no reference to the problem of rent. This paper investigates the extent to which the classical intuition regarding non-produced means of production is consistent with Sraffa's system.

In *Production of Commodities by Means of Commodities* Sraffa provides a rigorous reformulation of the classical theories of intensive and extensive rent noting that land and natural resources 'occupy among means of production a position equivalent to that of "non-basics" among products'. In single-product systems non-basics are typically interpreted as 'luxury goods' and it can be shown, *inter alia*, that technical progress in non-basic industries does not affect the system of relative prices and the economy-wide rate of profit. If it is true that land is always non-basic, then we might conclude that the classical view is broadly confirmed by Sraffa's modernisation of the theory of value.

Unfortunately, both the extensive and intensive systems are formally multiple-product/ process economies for which the definition of non-basics is considerably more subtle and complex than in single-product industries.[2] Important properties of single-product non-basics do not always survive the transition to multiple-product economies. Prices and profits may respond in unexpected ways to technical change, taxes and class conflict over the distribution of the surplus. The source of the perverse behaviour is traced to a class of commodities characterised here as *quasi-basics*.

The paper is organised as follows: in Section 2, we show that in systems with extensive

[1]The term 'non-produced means of production' is used in this paper synonymously with 'land' and 'natural resource'. Aluminium, for example, is a *produced* means of production, while land with a deposit of bauxite is a particular quality of natural resource.

[2]See Manara (1968), Schefold (1971) and Steedman in Pasinetti (1980).

rent, land is, like luxury goods, always non-basic. A number of important properties associated with non-basic land are enumerated. But in the following section it is demonstrated that, under intensive rent closures, land may be quasi-basic rather than non-basic. Because few properties of single-product non-basics are shared by quasi-basics, the classical view of rent as a residual payment cannot be sustained. However, the penultimate section of the paper shows that when rent is what we shall term *indirectly* intensive, land is non-basic and exhibits all of its traditional properties.

2. Extensive rent

The general problem of non-produced means of production may be illustrated by reference to the simplest Sraffian economy in which two commodities are produced by labour and produced means of production. If there exists a surplus over input requirements and wages, a positive profit rate can be paid on the value of total capital invested which is inversely related to the wage rate. Formally, the system consists of two equations and three unknowns: the profit rate, the wage rate, and the relative price of the two goods; with one of the distributional variables given exogenously, the relative price and the remaining distributional variable are determined.

The price system can be extended to determine relative prices of an arbitrary number of commodities with no logical difficulty if, for every additional commodity, we have a linearly independent equation describing its production process. For non-produced means of production, however, there is by definition no corresponding production process; there is an extra variable with no additional equation to determine its magnitude. To close the system, an additional process must be introduced without increasing the number of variables in the price determining equations. Moreover, the coexistence of the alternative process must be justified economically; that is, it must be shown that the forces of competition would not eliminate all but the most profitable process. The methods of closure traditionally discussed in the literature are *intensive* and *extensive*.

For the extensive closure, let there be n commodities, k of which use some non-produced means of production. Associated with each of the k land-using commodities are t_k qualities of land and $t_k - 1$ alternative processes.[1] By assumption, all qualities of land are required to satisfy the given level of effective demand for each of the k commodities to which land is devoted. The price of any one of these commodities must therefore cover costs of production on all plots, which implies that all but k qualities of land will earn a rent. Which lands are to receive no rent is in general dependent upon the level of wages and profits and not on any exogenously determined measure of fertility.[2] Hence, for every land-using commodity, there is at least one process which is formally equivalent to a 'synthetic process';[3] that is, a process which produces the commodity with no non-produced means of production.

[1] Typically, each quality of land is completely specialised since with prices, wages and profits determined, two crops would only pay the same rent by chance. Note that, even if land is *not* specialised by crop, only $\sum_{j=1}^{k} (t_j - 1)$ additional equations are required to specify all rents.

[2] The classical economists and Marx generally assumed identical combinations of capital and labour on lands of varying quality. This implies that the fertility ordering and, therefore, the ordering of rents were determined by the attributes of each plot. If, however, different processes are used on different qualities of land, the fertility and rental ordering may depend on the distribution of income (Kurz, 1978).

[3] It is to be stressed that the equivalence is only formal since the existence of synthetic processes implies the possibility that some subset of the available land could produce all the required output.

The k no-rent processes along with the remaining non-land-using processes serve to determine the system of $n-1$ relative prices, and either the profit or the wage rate, independently of the rents on the other qualities of land. Once all relative prices and the distributional variables are known, the remaining

$$s = \sum_{j=1}^{k} (t_j - 1)$$

rents are determined as residuals. Rent in the extensive closure may thus be considered a *price determined cost*.

More explicitly, let p_i, $i = 1, 2, \ldots, s+k$ be the price of the ith quality of land. The *rent*, ρ_i, of the ith quality of land can be defined as:[1]

$$\rho_i = r p_i \qquad\qquad i = 1, 2, \ldots, s+k$$

where r is the economy-wide rate of profit. The price determining equations in the extensive closure can thus be written:

$$[P_1 \ P_2] \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \end{bmatrix} = (1+r) [P_1 \ P_2] \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} + w[L_1 \ L_2] \qquad (1)$$

where P_1 is an n-dimensional vector of the prices of the produced goods and P_2 is a vector of the prices of the s rent-bearing lands. The input and output matrices are partitioned into submatrices: A_{11} and B_{11} represent the inputs and outputs, respectively, of the n produced good by n synthetic processes. A_{12} and B_{12} are the matrices of inputs and outputs of the n produced goods used by the s non-synthetic processes and A_{22} and B_{22} are the inputs and outputs of land used by the s non-synthetic processes. L_1 and L_2 are the vectors of direct labour coefficients for the synthetic and non-synthetic processes respectively. The rate of profit is r and w is the wage rate. Note that, since land is assumed to be unaffected by the production process, the matrices A_{22} and B_{22} are equivalent.

In systems with extensive rent, all qualities of land are non-basic. To see this, first observe that since the system of equations (1) formally involves joint production, Sraffa's single-product definition of basics as commodities which enter directly or indirectly into the means of production of *all* commodities (Sraffa, 1960, p. 8) must be generalised.[2] This Sraffa does in Part II of *Production of Commodities*; he writes:

In a system of n productive processes and n commodities (no matter whether produced singly or jointly) we say that a commodity or more generally a group of m linked commodities (where m must be smaller than n and may be equal to 1) are non-basic if of the n columns (formed by the 2m quantities in which they appear in each process) not more than m columns are independent, the others being linear combinations of these (Sraffa, 1960, p. 51).

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[1]We do not assume that this equation necessarily holds when the rate of profit is zero, since that would imply that rent would also be zero. Even with a zero rate of profit, rent is fully determinant as long as there are the same number of processes as commodities plus qualities of land.

[2]Sraffa writes:
The criterion previously adopted for distinguishing between basic and non-basic products .... now fails, since, each commodity being produced by several industries, it would be uncertain whether a product which entered the means of production of only one of the industries producing a given commodity should or should not be regarded as entering directly the means of production of that commodity (Sraffa, 1960, p. 49).

Suppose there are n commodities in the economy of which $m < n$ are suspected of being non-basic. Reorder the matrices A and B such that the last m rows correspond to the non-basic commodities and write the matrix:

$$D = \begin{bmatrix} A_{21} & A_{22} \\ B_{21} & B_{22} \end{bmatrix}$$

where A_{22} and B_{22} are square and of order m. For the last m commodities to be non-basic,

$$\text{rank } D \leq m$$

that is, the *rank* of D must be less than or equal to m.[1]

System (1) satisfies the rank condition since the matrix D takes the form:

$$D = \begin{bmatrix} 0 & A_{22} \\ 0 & B_{22} \end{bmatrix}$$

which is of rank s. The s land qualities relevant to the price-determining equations are, therefore, non-basic according to this general definition.[2]

Systems with extensive closures can always be reduced to equivalent single-product economies for the determination of the system of relative prices and profit (wage) rate. Consequently, the well-known properties of single-product systems, the inverse wage-profit relation, existence of the standard commodity, etc., apply.[3] In addition, we may draw the following conclusions:

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[1] As Steedman has observed, the rank of D must be *equal* to m if there are to be sufficient independent equations to solve for the prices of non-basics (Steedman in Pasinetti, 1980).

[2] It may be wondered whether no-rent land is itself non-basic. In order to see that it is, note that system (1′) in which no-rent land is explicitly introduced:

$$[P_1 \ 0 \ P_2] \begin{bmatrix} B_{11} & B_{12} \\ b & 0 \\ 0 & B_{22} \end{bmatrix} = (1+r) \ [P_1 \ 0 \ P_2] \begin{bmatrix} A_{11} & A_{12} \\ a & 0 \\ 0 & A_{22} \end{bmatrix} + w \ [L_1 \ 1 \ L_2] \tag{1′}$$

is seen here to be mathematically equivalent (in the sense of having the same solution) to system (1). Since no-rent land cannot possibly affect the determination of relative prices and the profit (wage) rate, the issue of whether it is basic or non-basic is, for practical purposes, moot. No-rent land nevertheless does satisfy the rank condition and is therefore non-basic. Assume for simplicity that there is but one quality of no-rent land. The rank of the matrix D'

$$D' = \begin{bmatrix} b & 0 \\ a & 0 \end{bmatrix}$$

is unity. The rank of the matrix:

$$D'' = \begin{bmatrix} b & 0 \\ 0 & B_{22} \\ a & 0 \\ 0 & A_{22} \end{bmatrix}$$

is $s+1$ and therefore all qualities of land are non-basic.

[3] See Pasinetti, 1977.

(a) In light of the decomposability[1] of the solution, the price of land plays no role in the determination of relative prices and the profit (wage) rate. Since the price of land adjusts to absorb the difference between output prices and costs of production for non-synthetic processes, rent to non-basic land is a residual payment for a 'price determined cost'. Non-basic land rent is thus determined in a qualitatively different way from prices (of basics) in that the latter are determined logically prior to rent.

(b) There is *always* an inverse relationship between the rate of profit and the level of real wages which can be studied independently of rent. This is not to say that the existence of land has *no* effect upon the wage-profit line. Fig. 1 shows the wage-profit relation for the extensive system (1) in which there are only two qualities of land ($s=k=1$) and one land-using commodity. Technique A is drawn such that the first quality of land bears no rent while the rent on the second quality of land is zero in technique B. As Montani (1975) has shown, scarce land implies that it is the interior (shaded) locus rather than the envelope which is the relevant inverse relation between wages and profits.
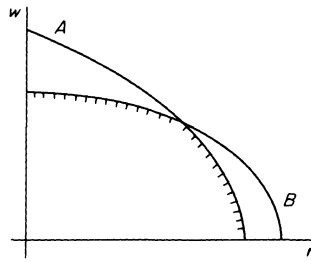

Fig. 1.

Fig. 1 shows how the existence of rent reduces the rate of profit for any given wage rate and in this sense constitutes a deduction from the surplus produced by labour. As is obvious from Fig. 1, land clearly affects the system of relative prices inasmuch as if the best quality of land were not scarce, prices corresponding to the envelope would rule. It is in precisely this way that land in extensive systems 'matters'.

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[1]Single product systems are decomposable (i.e. recursive) if there is a permutation of the rows and columns of A such that the matrix may be written:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$$

with A_{11} and A_{22} square. Multiple-product/process systems are decomposable if there exists a simultaneous permutation of the rows and columns of the input and output matrices such that:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \end{bmatrix}$$

Note that for systems with quasi-basics, the matrix AB^{-1} is decomposable according to the single-product definition. See Steedman in Pasinetti (1980) and Schefold (1971). Of course *any* joint products system can be rendered decomposable by a suitable linear transformation (though it is significant that the B^{-1} matrix is sufficient to render systems with quasi-basics decomposable).

(c) Okishio superior technical change in any non-synthetic process will not affect the determination of relative prices and the profit (wage) rate.[1] Okishio superior technical change only causes an increase in rent.

(d) A proportional tax on rent-bearing land will not change relative prices or the profit (wage) rate and will therefore be borne entirely by landlords.

As we shall see in the following section, it is only this last property (d) of non-basic land which carries over to quasi-basic land.

3. Intensive rent

Under the extensive closure with non-basic land, k qualities of land pay no rent. If the demand for output rises, additional land is taken into cultivation and a new structure of rents, profits and prices is determined. It may, however, prove less costly to increase production by cultivating the existing plots more intensively. In this case, two processes are observed operating side-by-side on one quality of land. Rent on these qualities of land will be positive as long as the method which uses land more intensively is also more costly per unit of output. If land were not scarce, only one method—the cheapest at the prevailing level of wages and profits—would be observed operating on any one quality of land (Sraffa, 1960, p. 75).

If all $s+k$ qualities of land are in short supply, the price determining eq. (1) must be modified to read:

$$[P_1 \; P_2] \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = (1+r) \, [P_1 \; P_2] \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} + w[L_1 \; L_2] \qquad (2)$$

where land now explicitly enters the formerly synthetic properties processes. The dimensions of the vectors, P_1 and P_2 are now n and $s+k$ respectively; and the submatrices, B_{21}, B_{22}, A_{21} and A_{22}, are now of row dimension $s+k$. There is at least one quality of land upon which two processes are operating; that is, there is at least one row of the above-mentioned submatrices with two positive entries.

All qualities of land in this system are non-basic according to the rank condition of page 144. Under the assumption that land is unaffected by the production process, the last $s+k$ rows of the matrices B and A are equivalent and the rank of the matrix:

$$D = \begin{bmatrix} A_{21} & A_{22} \\ B_{21} & B_{22} \end{bmatrix}$$

is, therefore, $s+k$ which implies that all land satisfies the rank condition.

Note, however, that this system is no longer recursive. Prices and rents are determined simultaneously and, hence, it is *not* correct to conclude that, because land is non-basic, rent is a residual payment made *after* the more fundamental conflict between wages and profits has been settled. In the intensive closure, rent is consequently not a 'price determined cost' as it was with the extensive closure; it is rather a price determining cost on the same footing with other prices.

Fig. 2 depicts the solution to eq. (2) with $n=s=k=1$. Technique A in the figure is constructed such that only one of the land-using processes is employed and rent is set equal
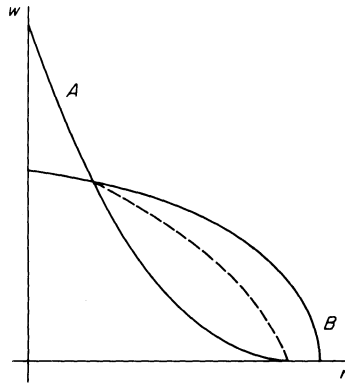
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[1]See Okishio (1961) and Roemer (1981).

Fig. 2.

to zero. In technique B, the first land-using process is replaced by the second. In the intensive rent system, however, both land-using processes are required, and consequently, it is the dotted wage-profit line that is relevant for the determination of the system of relative prices. As Sraffa (1960) has shown, the technique represented by the dotted line is formed by the linear combination of the two land-using processes such that land is eliminated. The scalars of the linear combination are easily seen to be the yields of each of the land-using processes.[1] Again rent appears as a deduction from the product of labour. In the intensive closure, however, it is obvious that Okishio superior technical change in *either* process will not only change rent, but also the system of relative price and the profit (wage) rate as well.

The rank condition of page 144 is a generalisation of the criterion by which non-basics are identified in single-product industries, but it is a much weaker condition than decomposability. Even though all decomposable systems satisfy the rank condition, it is not true that all non-basics qualifying under the rank condition have the same characteristics. Since non-basics in joint-product systems can qualify under either the criterion of decomposability *or* the rank condition, there are effectively two subclasses of non-basics. We shall

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[1]It is perhaps most instructive to see this in the case of a simple example of the intensive closure with only one quality of land and one produced good. Write eq. (2) as:

$$[p_1 \; p_2] \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = (1+r) \, [p_1 \; p_2] \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + w[l_1 l_2]$$

In order to construct the wage-profit line, form the linear combination of the processes such that the input and output of land cancels. With the price of first (produced) good as the numeraire, the equation of the wage-profit line is thus:

$$w = \frac{(b_{11} - q b_{12}) - (a_{11} - q a_{12})}{(l_1 - q l_2)}$$

where $q = a_{22}/a_{21} = b_{22}/b_{21}$. Land input coefficients are thus the weights of the linear combination of the two land-using processes. Note that a multiplicative tax on land, which may be thought of as a proportional increase in all land input and output coefficients, does not disturb the wage-profit relation.
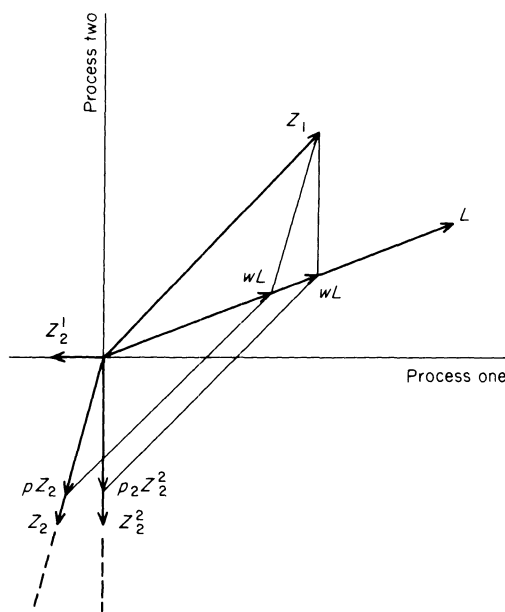
Fig. 3.

refer to products which qualify under decomposability as simply non-basics, while non-basics which do not qualify by virtue of decomposability yet satisfy the rank condition shall be referred to as *quasi-basics*.

The distinguishing feature of quasi-basics is the fact that a tax applied to a quasi-basic in all its uses does not distrub the system of relative prices or profit (wage) rate. To appreciate the differences and similarities between the non-basics and quasi-basics, consider Fig. 3 in which we have graphed the solution to systems (1) and (2) (again with $n=s=k=1$) for both the intensive and extensive closures. Taking the profit rate as given and the price of the produced good as the numeraire, the *rows* of the matrices B, A and $Z=B-vA$ are plotted in process space along with the vector L.[1] Here $(1+r)$ is written as v for simplicity. The real wage and the price of land can be read from the diagram as the scalar multiples w and p_2 which satisfy the linear combination:

$$Z_1+p_2Z_2=wL$$

where subscripts on matrices refer to *rows*. In the intensive case, the Z vector for the uniform quality of land is labelled Z_2. The Z vector for land of the first quality in the extensive case is Z_2^1 and for the second quality Z_2^2. As the figure is drawn, land of the second quality earns a rent since L falls inside the convex cone formed by Z_1 and Z_2^2.

Land in both regimes satisfies the rank condition and, consequently, a tax on scarce land, which would be represented by an extension of the vectors Z_2 and Z_2^2, is wholly borne by landlords. The real wage in terms of the produced good, as well as the profit rate, remains unchanged since the Z vectors for land do not rotate with the application of the tax. On the other hand, observe that land plays a far more crucial role in the intensive than in the extensive closure. In the latter, wages, profits and the relative price are inde-

[1]See Schefold (1971) and Gibson (1983) for details of this graphical procedure.

pendent of land productivity. In Fig. 3, the real wage in terms of the produced good is seen not to depend upon the length of the vectors $Z_2{}^1$ and $Z_2{}^2$. In the intensive system, however, land productivity does matter: land/output ratios determine the orientation of the vector Z_2 which clearly affects the real wage. For the intensive closure the tax property shows that one may dispense with the absolute amount of land employed in each process; relative quantities of land are, nevertheless, essential to the determination of prices and the profit (wage) rate.

4. Indirect intensive rent

Consider an economy in which there are two commodities, the first of which uses only produced inputs (in addition to labour) while the second employs land of uniform quality. If the demand for the second good rises past the point at which the available land can supply the required output, any of a number of possible scenarios could be relevant. If an alternative process which uses less land at a higher unit cost is employed the resulting rent is intensive. Alternatively, an additional process for the *first* commodity could be introduced which uses less of the second good per unit output. As the output of the first good rises, the input-saving process produces a larger and larger proportion of the total effective demand for that commodity. Total demand for the second commodity would then be reduced, thereby eliminating the need to cultivate existing land more intensively.

Note that in this economy the first good is basic, the second is quasi-basic and land is non-basic. To see this, first write the price-determining equations:

$$[p_1 \; p_2 \; p_3] \begin{bmatrix} b_{11} & b_{12} & 0 \\ 0 & 0 & b_{23} \\ 0 & 0 & b_{33} \end{bmatrix} = (1+r) \, [p_1 \; p_2 \; p_3] \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} + w[l_1 \; l_2 \; l_3]$$

This system is decomposable since the subsystem

$$[p_1 \; p_2] \begin{bmatrix} b_{11} & b_{12} \\ 0 & 0 \end{bmatrix} = (1+r) \, [p_1 \; p_2] \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + w[l_1 \; l_2]$$

is sufficient to determine the profit rate and the relative price of good one and two. The second good, on the other hand, is *quasi-basic* since the rank of the matrix

$$D = \begin{bmatrix} 0 & 0 & b_{23} \\ 0 & 0 & b_{33} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$$

is only two.

We conclude from this example that like the category 'non-basic', which may refer to either non-produced means of production or luxury goods, 'quasi-basic' is a general category of commodities. Quasi-basics may be produced, as the second commodity in this example, or non-produced as in the case of land in the intensive closure. Note further that quasi-basics are always associated with either direct or indirect intensive closures and do not arise in systems closed extensively. The converse is not, however, true. As this example shows, non-basic land may well appear in systems with indirect intensive rent.

5. Conclusions

Land in Sraffa's reformulation of the classical theories of intensive and extensive rent is always *non-basic*. In extensive systems, rent (like the prices of luxury goods) does not enter into the equations which determine the prices of basic commodities or the profit (wage) rate. A number of important properties follow from this: extensive rent is a residual, price-determined, deduction from the product of labour; there is always an inverse relationship between wages and profits; Okishio superior technical change in non-synthetic processes will not affect basic prices or profits (wages); and, a proportional tax will be borne entirely by landlords. Thus the classical conception of rent in extensive systems is essentially undisturbed by Sraffa's reformulation of the theory of value.

Unfortunately, systems with intensive rent involve multiple-product industries for which the definition of non-basics is considerably more complex. The analogy to luxury goods is much less clear; indeed, the first three properties of the extensive rent system mentioned above need not apply. For this reason land under the intensive closure has been character- ised as *quasi-basic* rather than non-basic. Though we have identified a special case (indirect intensive rent) in which land remains non-basic, the intensive closure generally undermines the classical conception of rent as a residual payment.

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