## 1. CYU: Rules of Integration

Let $k$ be a constant

1. **Constant Rule**: If there are no limits of integration, this is an *indefinite integral*

$$\int k\,dx = kx + c$$

where $c$ is an arbitrary constant. To see that this is the right answer differentiate what is on the right-hand side to get what is under the integral sign. This will always work and that is why the integral is known as an *anti-derivative.* If there are limits, then we drop the $c$

$$\int_3^4 k\,dx = kx]_3^4 = 4k - 3k = k$$

Here we have evaluated the integral at the upper limit ($4k$) and subtracted the integral evaluated at the lower limit ($3k$).

2. **Power Function:**

$$\int x^k\,dx = \frac{1}{k+1}x^{k-1} + c$$

this is not valid for $k = -1$. The same idea applied to the definite integral.

3. **Integral of** $1/x$

$$\int \frac{1}{x}\,dx = \ln x + c$$

this is only valid for $x > 0$.

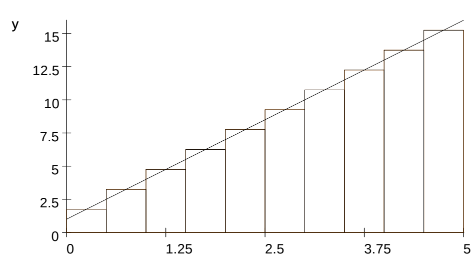
4. **Exponential function**

$$\int ak^x dx = \frac{a^{kx}}{k \ln a} + c$$

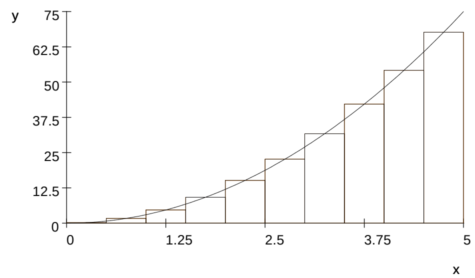
All these rules are important and should be memorized. Integrals can be added or subtracted as the next section shows.

## 2. Areas

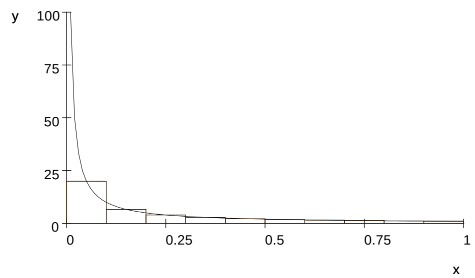
Integrals find areas under a curve that is integrated. Here is what you are calculating in $\int_0^5 k\,dx = kx + c$.



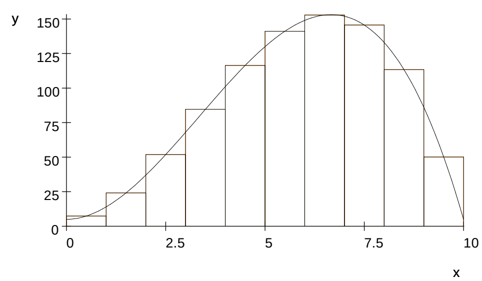
For the power function $\int x^k \, dx = \frac{1}{k+1} x^{k-1} + c$




For $1/x$




For $-x^3 + 10x^2 + 5$




## 3. Practice

Show that

1. $\int 3.5 dx$
2. $\int_0^5 -\frac{1}{2} dx$
3. $\int_{-1}^1 x^5 dx$
4. $\int_1^3 x^{-5/2} dx$

5. $\int_2^k \sqrt{x} dx$
6. $\int_1^3 (5x^3 + 2x^2 + 3x) dx$
7. $\int (x^{-1/3} + 2x^{2/5}) dx$
8. $\int_1^{10} 2^{4x} dx$
9. $\int e^x dx$
10. $\int_{0.1}^1 (6e^{3x} - 8e^{-2x}) dx$

## 4. Answers

1. $\int 3.5 dx = 3.5x + c$
2. $\int_0^5 -\frac{1}{2} dx = -2.5$
3. $\int_{-1}^1 x^5 dx = 0$
4. $\int_1^3 x^{-5/2} dx = 0.538\,37$
5. $\int_2^k \sqrt{x} dx = \frac{2}{3} k^{\frac{3}{2}} - 1.8856]$
6. $\int_1^3 (5x^3 + 2x^2 + 3x) dx = 129.33$
7. $\int (x^{-1/3} + 2x^{2/5}) dx = \frac{3}{2} x^{\frac{2}{3}} + \frac{10}{7} x^{\frac{7}{5}} + c$
8. $\int_1^{10} 2^{4x} dx = 3.965\,6 \times 10^{11}$
9. $\int e^x dx = e^x + c$
10. $\int_{0.1}^1 (6e^{3x} - 8e^{-2x}) dx = 34.738$

## 5. Consumer surplus

1. Given the demand function
$$q = 90 - 2p$$
find the consumer surplus when $q = 25$. $[Answer: 156.25]$
2. Given the supply function
$$p = (q + 3)^2$$
Find the producers' surplus when price is $p = 81$. $[Answer: 252]$

3. Given the demand functions

$$q = \sqrt{25 - p}$$

and the supply function

$$p - 2q - 1 = 0$$

Find the consumers' and producers' surplus. $[Answer : 42.67; 16]$

4. Given the demand function

$$q = \sqrt{113 - p}$$

and the supply function

$$p - (q + 1)^2 = 0$$

Find the consumers' and producers' surplus. $[Answer : 228.67; 277.67]$

5. A monopolists sets marginal revenue equal to marginal cost to determine output. She then plugs this result into the demand function to determine price. If the demand curve is

$$p - 274 + q^2 = 0$$

and marginal cost is

$$C = 4 + 3q$$

Find the consumer surplus. $[Answer : 486]$

## 6. Solutions

1. We have

$$S_c = \int_{32.5}^{45} (90 - 2p)\, dp = 156.25$$

2. The area under the supply curve has to be subtracted from the $pq$ to get the answer

$$S_p = 81(6) - \int_0^6 (q + 3)^2\, dq = 252$$

3. First find the equilibrium price

$$\begin{aligned} p - 2q - 1 &= 0 \\ \sqrt{25 - p} &= q \end{aligned}$$

The solution is: $\{[p = 9.0, q = 4.0]\}$

$$S_c = \int_9^{25} \left(\sqrt{25 - p}\right) dp = 42.667$$

and the supply function

$$\int_1^9 \left(\frac{p}{2} - \frac{1}{2}\right) dp = 16$$

4. First find the equilibrium price

$$\begin{aligned} q &= \sqrt{113-p} \\ p-(q+1)^2 &= 0 \end{aligned}$$

Solution is: $[p=64.0, q=7.0]$

$$S_c = \int_{64}^{113} \left(\sqrt{113-p}\right) dp = 228.67$$

$$\int_{1}^{64} (\sqrt{p}-1)\, dp = 277.67$$

5. Revenue, $R=pq$. From the demand curve, $274-q^2$ we have

$$R = pq = 274q - q^3$$

so that marginal revenue is $dR/dq = 274-3q^2$ and marginal cost is

$$274 - 3q^2 = 4 + 3q$$

Solution is: $[q=9.0]$ ,Solution is: $q=9$ and $p=274-9^2=193$. The maximum price a consumer will pay is 274

$$S_c = \int_{193}^{274} \left(\sqrt{274-p}\right) dp = 486$$