Computational Study of Ferromagnetic Phase Transitions in the Ising and XY Models via the Monte Carlo Method

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Phase transitions surround us, from the freezing of water into ice to the critical opalescence of liquids. In order to understand the emergence of universality near continuous second order phase transitions, we have performed Markov Chain Monte Carlo simulations of two simple models of ferromagnetism on a two dimensional square lattice. The first consists of Ising spins (magnetic moments) which have a discrete symmetry and are constrained to point in only two directions, while the second employs XY spins allowed to point anywhere on the two-dimensional unit circle. For the Ising model, we use finite size scaling to confirm the existence of a phase transition and compare our results with the exact analytical solution discovered by Onsager. When the symmetry is continuous, the Mermin-Wagner-Hohenberg theorem precludes long range ferromagnetic order at finite temperature and we instead observe the binding of vortices and anti-vortices at the Kosterlitz-Thouless transition. Due to its differing universality class from the Ising model, this transition has no known analytical solution, and thus we predict the temperature of the phase transition by an analysis of the helicity-modulus of the XY spins.