

Consider a collection of n dots (vertices) where each pair of dots is connected by a line segment (edge). This forms the complete graph on n vertices, denoted K_n . For certain values of n , we can partition the edges of K_n into a collection of triangles. The triangles form a Steiner triple system on n points. For other values of n , we can partition the edges into a collection of quadrilaterals. The squares form a 4-cycle system on n points. In this presentation, we will examine which values of n enable us to "glue" the edges of a Steiner triple system on n points together with the edges of a 4-cycle system on n points to produce an embedding of the graph K_n . This embedding has faces shaped like either triangles or quadrilaterals so that no two triangles touch and no two quadrilaterals touch, except at vertices. After establishing the admissible values of n , we will present a construction yielding the desired embedding for an infinite class of values. This is a highly visual and accessible presentation of original research.