

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48

Intelligent Traffic Signals: Extending the Range of Self-Organization in the BML Model

Daniel J. Brown*

Christopher M. Danforth

Department of Mathematics and Statistics, University of Vermont, Burlington, Vermont
Vermont Advanced Computing Center, Burlington, Vermont

Word Count: approx 4500 including figures and tables

Corresponding author contact info:

Daniel Brown

UVM Department of Mathematics and Statistics

16 Colchester Ave, Burlington, VT 05401

802-522-4159, daniel.j.brown@uvm.edu

*Corresponding author

1
2
3 **Abstract**

4 The two-dimensional traffic model of Biham, Middleton and Levine (Phys. Rev. A, 1992) is a simple cellular
5 automaton that exhibits a wide range of complex behavior. It consists of both northbound and eastbound cars
6 traveling on a rectangular array of cells, each cell equipped with a traffic signal. The traffic signals switch
7 synchronously from allowing northbound flow to eastbound flow. By gating individual traffic signals, i.e. allowing
8 individual traffic signals to break from synchrony in predetermined, deterministic scenarios based on the local
9 state of traffic, the range for which the system self-organizes into a state of unimpeded flow is extended. On
10 a 100x100 cell array, this additional intelligence enables accomodation of 200 cars more than the original BML
11 model, without any reduction in average velocity.
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52

Introduction

Congestion is an increasing problem in America costing people time, money and aggravation. Increasingly, transportation engineers are looking for ways to introduce intelligent traffic management to improve flow. In fact, the U.S. Department of Transportation's Research and Innovative Technology Administration has just issued a request for information seeking available technology applications that can help fight congestion and improve the safety and performance of the nation's transportation system, without building new capacity [1]. For instance, the use of variable message signs on highways is now a standard means for warning drivers of congestion, accidents, road closures, etc... The goal is to enable commuters to avoid large jams by suggesting alternate routes. However, researchers at the University of Virginia have found that the actual percentage of cars diverted by these signs can be low [2]. Relying on people to choose to change their behavior given additional information concerning the current road conditions is unpredictable. A more rational strategy may be to force them to change routes through the use of traffic controlling technologies, e.g. traffic signals.

The cellular automaton model of traffic flow in two dimensions proposed by Biham, Middleton and Levine (BML)[3] is one of the most cited models in traffic flow studies [4-8], with over 200 citations in the scientific literature. Conceptually, the model consists of two types of cars; north moving cars and east moving cars, traveling on a square lattice with periodic boundary conditions, i.e. a torus. Practically, the model consists of a square array of cells that have three possible states: occupied by an east moving car (blue cell), occupied by a north moving car (red cell) or empty (white cell). Given a particular density ρ , defined to be the number of cars C divided by the number of cells N^2 , an initial state is chosen by uniformly distributing an equal number of each type of car (typically $C/2$) at random in the array. On odd time steps, each north moving car moves one step north if the cell above them is empty. On even time steps, each east moving car moves one step east if the cell to the right is empty. We will hereafter refer to this traffic signal operating system as *synchrony*. A car that moves one step is considered to have a velocity of 1; a car that does not move is considered to have a velocity of 0. The average velocity of the system \bar{v} is calculated as the sum of the velocities of all the cars divided by the number of cars. It is important to note that once the initial state is set, the dynamics of the system are completely deterministic. This model is typically thought of as describing a road network with a traffic signal at each cell, with each signal alternating in synchrony between north and east.

Three flow regimes were observed in the original BML model. When the model is initially populated with small ρ , the system self-organizes into free-flowing bands of slope -1 , i.e. diagonal from northwest to southeast (see Figure 1(a)). When populated with slightly higher ρ , small local jams slowly merge into one global jam of slope 1 (see Figure 1(b)). For large ρ , the small local jams merge almost immediately into a system wide, randomly populated jam of slope roughly 1 (see Figure 1(c)). It is important to note that all jam configurations result in northbound cars located immediately to the west of eastbound cars.

Until recently, the general understanding of the BML model has been that it exhibits a sudden transition from the free-flowing phase shown in Figure 1(a) to the global jamming phase shown in Figure 1(b) at a density of $\rho \approx 0.35$, with this value decreasing as the system size N increases. More recently, it has been shown by D'Souza [9, 10] that this transition is not a sudden phase transition, but that there appear to be bifurcation points, and that these three regimes are not the only possibilities. D'Souza observed densities where some initial conditions self-organize into free-flow as in 1(a), some into a global jam as in 1(b), and some conditions enter into a so-called *intermediate* state where bands of free flowing traffic intersect at jamming fronts that move upstream and dissipate before they coalesce into a global jam (see Figure 1(d)). These intermediate states persist beyond transient time scales, and the average velocity of the system in these intermediate states on square arrays seems to approach a value close to $2/3$.

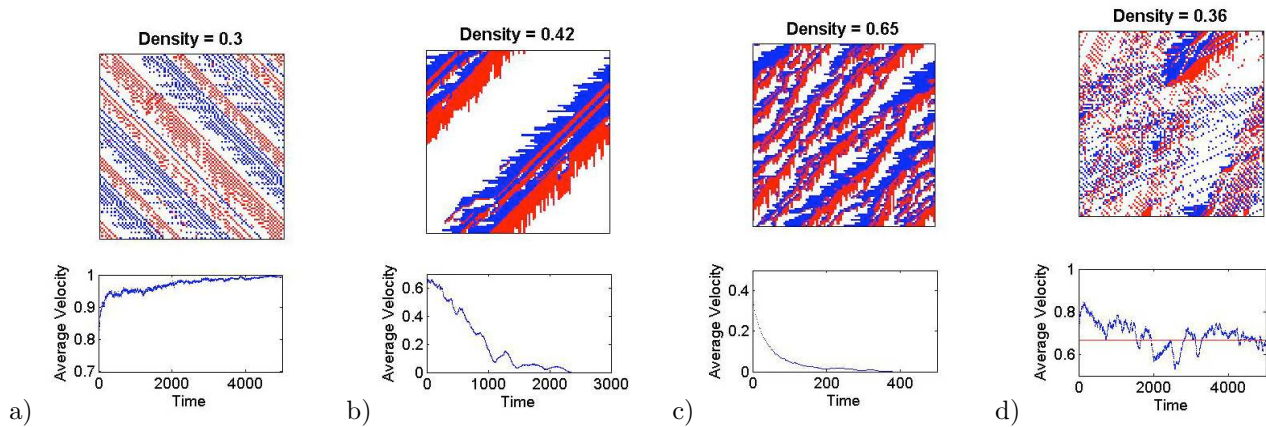


Figure 1: The three regimes observed in the original BML model, and the recently discovered intermediate state, shown for $N = 100$. a) For low densities ($\rho < 0.35$) the system self-organizes into free-flowing bands of slope -1 that no longer interact. Here the average velocity of the system approaches 1 as each car moves unimpeded at each step. b) At slightly higher densities the system produces small local jams which slowly merge into one global jam, where the average velocity of the system is 0. c) At high densities the small local jams coalesce immediately into a system wide jam. It is important to note that in all jam regimes the eastbound cars (blue) are to the west of the northbound cars (red). d) The intermediate state discovered by D'Souza [9] with \bar{v} approaching $2/3$ (red horizontal line). These intermediate states have been shown to persist beyond transient time scales, and to coexist with both the free-flowing state and the global jamming state at specific densities.

Effects of intelligent traffic signals

The so-called *green wave*, a traffic signal strategy widely implemented in cities, was investigated using the BML model by Török and Kerétsz in 1996 [11]. The green wave strategy coordinates the traffic signals so that traffic can flow continuously through several intersections. Implementation of this strategy using the BML model results in *caravanning*, where contiguous columns of northbound cars (rows of eastbound cars) move as a group. Surprisingly, the green wave strategy decreased the density at which the system jammed, and failed to enable self-organization at any higher densities. Globally, the green wave strategy did not improve the flow of traffic. Many studies have been done on the effects of random changes to the BML model, such as cars changing directions randomly, or traffic

1
2
3 signals randomly alternating directions of flow [4] and even asynchronous updating of the automaton [10]. Despite
4 moderate success reported on implementation of these stochastic strategies, engineers are not likely to set traffic
5 signals to switch randomly.
6

7 The focus of this research is investigating the possibility of improving traffic flow in the BML model with a
8 deterministic modification to traffic signal operation, based on observations of the local state. In the original model,
9 all traffic signals are set to alternate in synchrony from northerly flow to easterly flow (or vice-versa) regardless of
10 the current conditions. We pose the question: If the individual traffic signals were equipped with knowledge of the
11 current local conditions, could they apply deterministic rules to break this synchrony in very specific instances to
12 improve the overall flow in the system?
13
14

15 To address this question, we implement several variations of the what is known as the *gating* technique [12],
16 inspired by the suboptimal situation in which synchrony results in no movement for half of every traffic signal cycle.
17 Essentially, the method enables traffic signals to pass unimpeded cars through any intersection which is congested in
18 the orthogonal direction. Take for example the situation depicted in Figure 2a, where it is currently the north moving
19 cars (red) turn to advance. If the traffic signal shown in Figure 2a were to break from synchrony, as in Figure 2b,
20 and instead let the eastbound (blue) star car through, and then return to synchrony, the resulting situation would be
21 that the northbound (red) star car is in the exact same place 3 steps later as in the original, but the eastbound star
22 car has advanced 2 cells east. This *gating* technique manages to move the eastbound car safely to the right of the
23 northbound cars without impeding any northerly flow, and thus reduces the chance that the blue car will nucleate a
24 jam. This simple maneuver forms the basis of our gating modification of the BML model.
25
26
27
28
29

30 There are several instances similar to the one depicted in Figure 2b which could cause a traffic signal to break
31 from synchrony and allow an eastbound (or northbound) car to pass through; nine such scenarios are shown in Figure
32 3. The effect of each is investigated on 5000 randomly chosen initial conditions of $\rho = 0.4$ on a 100x100 cell array.
33 We simulate each realization either until it jams or for 200,000 time steps, whichever comes first. The cells update
34 exactly as in the synchronous model, unless the local neighborhood is in the specific case under consideration. Note
35 that only the instances for northbound updating are depicted in Figure 3, but the analogous breaking from synchrony
36 occurs during eastbound updating as well. The results of these investigations are summarized in Table 1.
37
38
39

40 The two scenarios that cause the biggest decline in flow, namely 6 and 7, fail to ensure that the car being passed
41 through will move out of the way of the car that is being forced to wait. Three other scenarios which show a decline
42 in performance, namely 3-5, are the ones in which the cell north of the traffic signal is occupied by an eastbound
43 car, i.e. the northbound star car is blocked by an eastbound car at time=3. It is interesting to note that scenarios 8
44 and 9 are identical to scenarios 1 and 2, except for the state of the bottom left corner. In scenarios 1 and 2, if the
45 northbound car that would normally advance is forced to wait, it might block an eastbound car that would otherwise
46 be free to advance at the next time step. To avoid this problem in scenarios 8 and 9, the lower left corner cell may
47
48
49
50
51
52

1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31
32
33
34
35
36
37
38
39
40
41
42
43
44
45
46
47
48
49
50
51
52

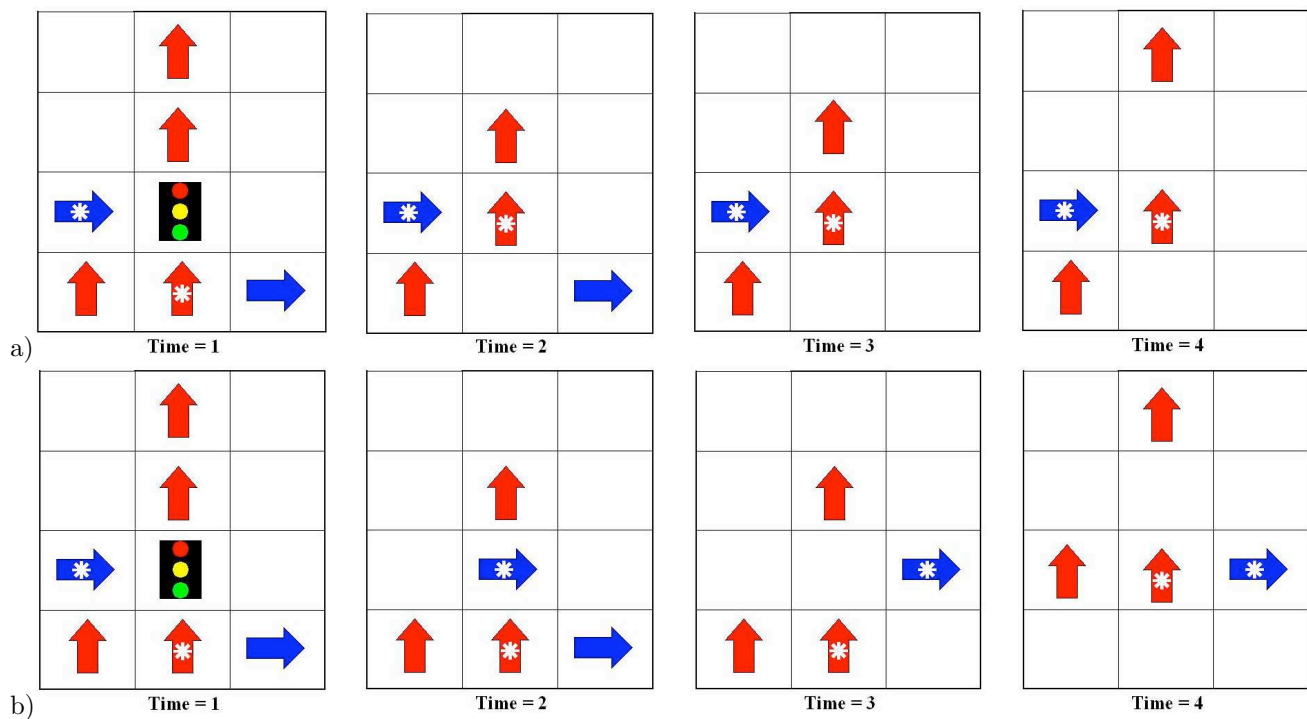


Figure 2: a) Given the local conditions at time=1 with the northbound (red) cars set to advance, synchronous application of the BML rule results at time=4 in the northbound star car having advanced one cell north, while the eastbound star car has not advanced at all. b) The local conditions are the same as Figure 2a at time=1, with the northbound cars set to advance, but this time the traffic signal breaks from synchrony at time=1. Instead of letting the northbound star car advance one cell north, it allows the eastbound star car to advance one cell east before resynchronizing with the rest of the traffic signals at time=2. At time=4 the northbound star car has still advanced one cell north, but the eastbound star car has advanced 2 cells east, safely to the right of the northbound cars.

Gating Scenario	Percent Gating Lasted Longer	Mean Steps to End	Mean Steps Gained	Relative Gain over BML	Number of Realizations out of 5000 Where Both Didn't Jam	BML Jam, Gating Didn't	Gating Jam, BML Didn't
BML	-	21,705.8	-	-	-	-	64
1	54.0	23,320.3	1,614.6	0.07	0	58	64
2	53.7	22,313.6	607.8	0.03	1	47	63
3	48.6	21,413.6	-292.2	-0.01	1	59	63
4	46.6	17,095.7	-4,610.1	-0.21	2	40	62
5	32.0	4,132.7	-17,573.2	-0.81	0	3	64
1 & 2	62.5	32,129.2	10,423.4	0.48	4	86	60
1 & 2 & 3 & 4	59.9	25,694.6	3,988.8	0.18	3	57	61
6	4.7	864.1	-20,841.7	-0.96	0	0	64
7	5.7	932.4	-20,773.4	-0.96	0	0	64
8 & 9	48.6	12,536.5	-4,584.7	-0.21	1	32	63

Table 1: The results from 5000 randomly chosen initial conditions of $\rho = 0.4$ on a 100x100 array. Each realization was simulated until jamming or 200,000 time steps. The biggest improvement is given by the combination of scenarios 1 & 2, when an eastbound(northbound) car is passed through a line of northbound(eastbound) traffic, increasing the time to jam by 48%. The biggest decline in flow, scenarios 6 & 7, results from the traffic signal failing to ensure that the car being let through will move out of the way of the car that is being forced to wait.

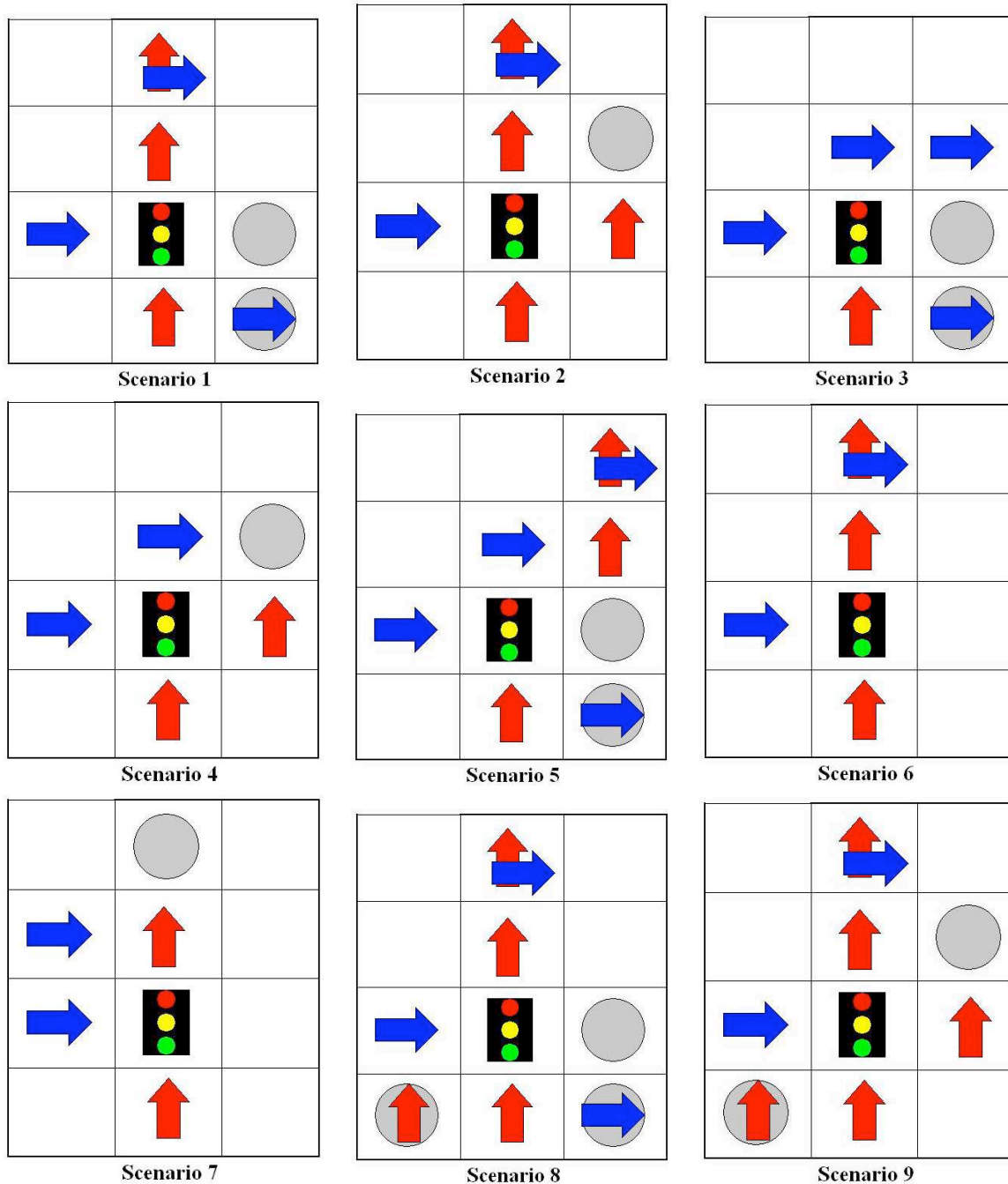


Figure 3: The exact local conditions causing the traffic signals to break synchrony for one time step are shown. These deterministic modifications to the BML model were investigated using 5000 randomly chosen initial conditions for $\rho = 0.4$ on a 100×100 array. The traffic signals under consideration are shown. They are currently scheduled to let the northbound car south of the signal advance, but in the situations shown they break synchrony and let the eastbound car to the west advance instead, before subsequently returning to synchrony. Blank cells above are irrelevant and can be in any state, cells with a gray circle must be empty, and cells containing two symbols can be in either state shown.

not be occupied by an eastbound car. Somewhat counter-intuitively, this modification leads to reduced flow. This may be due to the fact that jams tend to grow southwest(i.e. upstream), so it is more effective to let traffic to the

north (for northbound) or east (for eastbound) pass through, than to advance and block it.

In general, a modification is most effective when it allows an eastbound(northbound) car to pass through a northbound(eastbound) line of traffic without impeding the flow of the northbound(eastbound) cars. Averaged across 5000 randomly chosen initial conditions, the combination of scenarios 1 and 2 (hereafter referred to as the *gating* model) shows improvement of more than 10,000 time steps(48%) over the original BML model. It also remained in the intermediate flow regime for the full 200,000 steps for 90 realizations compared to 64 for the original version. These realizations should be simulated beyond 200,000 time steps to observe whether they eventually jam. Of course, all realizations that don't jam will eventually repeat due to the finite number of states and fully deterministic rules of evolution.

Further investigation into the gating model shows that it extends the density range at which the system self-organizes into free-flowing bands, and that in this extended range the improvement in flow over the original model is significant. The results of 500 realizations run at each of a range of densities from $\rho = 0.34$ through $\rho = 0.405$ on a 100x100 cell array simulated out to jamming or 200,000 time steps are shown in Figure 4. The mean ending average velocity for the BML model is less than 0.9 for $\rho = 0.345$, while the gating model results in $\bar{v} > 0.9$ for densities up to $\rho = 0.365$. In other words, the gating model continues to self-organize into free-flowing bands of slope -1 for the vast majority of realizations, even when more than 200 cars (roughly 6% more cars) have been added to the system. Although the gating model shows an extended range of self-organization and free-flow, it does not appear to extend the range at which global jamming is inevitable, as both models approach a mean ending average velocity of 0 at $\rho = 0.4$ ($C = 4,000$ cars).

A more detailed look at the results for $\rho = 0.365$ is shown in Figure 5. For the gating model, the average velocity quickly rises above 0.9, as the vast majority of realizations produce self-organized, free-flowing bands. The BML model shows a constant decrease of average velocity per time step, down to about $\bar{v} = 0.25$. Figure 5 b) shows histograms of ending velocities, and confirms that the vast majority of realizations lead to free-flow in the gating model. The BML model shows no ability to self-organize into the free-flowing state at this density, and most states lead to global jams.

An animation showing the gating model self-organizing into free-flowing bands while the BML model enters the intermediate state for $\rho = 0.365$ on a 100x100 array can be found at:

<http://www.uvm.edu/~dbrown1/?Page=research.html>

Conclusion

The BML traffic model has thus far shown 3 important regimes: the lower density regime of self-organization into free-flowing bands, the slightly higher density regime where free-flow, global jam and the recently discovered

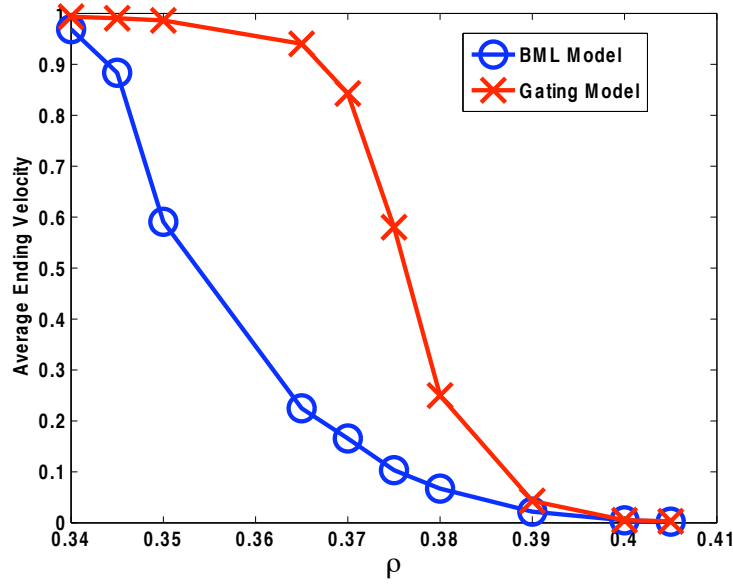


Figure 4: 500 realizations were simulated at each of a range of densities from $\rho = 0.34$ through $\rho = 0.405$ on a 100×100 cell array simulated until jamming or 200,000 time steps. The mean ending average velocity for the BML model is less than 0.9 for $\rho = 0.345$, while the gating model results in $\bar{v} > 0.9$ for densities up to $\rho = 0.365$. In other words, the gating model continues to self-organize into free-flowing bands of slope -1 for the vast majority of realizations, even when more than 200 cars have been added to the system. Although the gating model shows an extended range of self-organization and free-flow, it does not appear to extend the range at which global jamming is inevitable, as both models approach a mean ending average velocity of 0 at $\rho = 0.4$ ($C = 4,000$ cars).

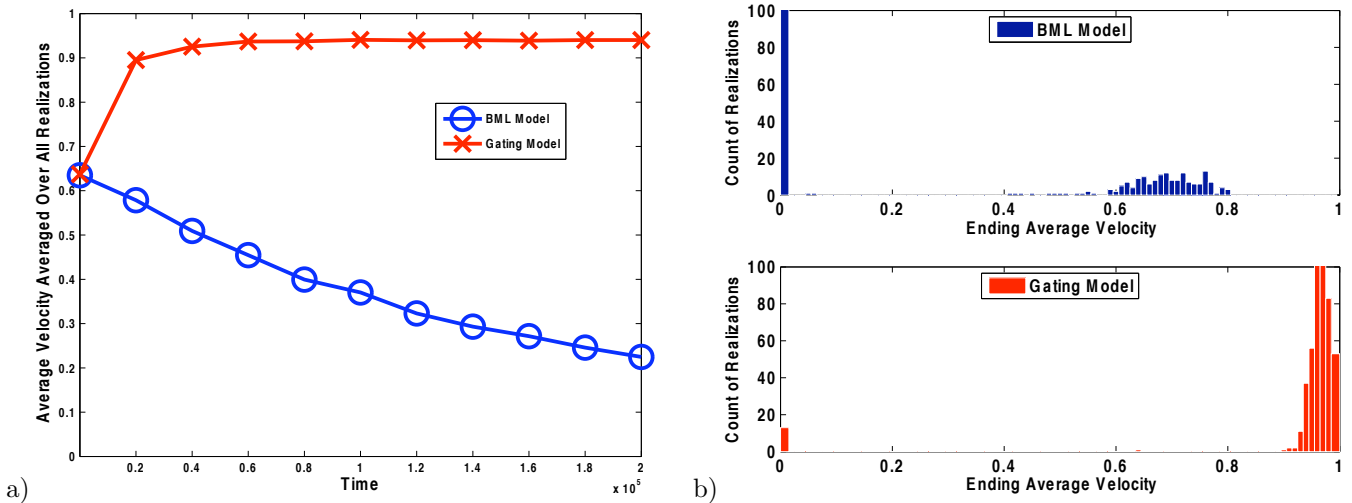


Figure 5: (Color online) Results for 500 realizations for $\rho = 0.365$ on a 100×100 cell array. a) Time series plot of the system for both the original BML model as well as the gating model averaged over all realizations. The gating model shows improved flow for all time as the average velocity stays above 0.9. The BML model shows significant jamming and a steadily decreasing average velocity. b) The ending velocities from all realizations. The gating model self-organizes into free-flowing bands of slope -1 for the overwhelming majority of realizations. The BML model shows no ability to self-organize into free-flow at this density, with about 3/4 of realizations ending in global jam by 200,000 time steps.

intermediate states coexist, and the higher density regime of inevitable global jamming. The addition of local traffic signal intelligence that preserves the deterministic dynamics of the system has been shown to extend the range for which the model self-organizes into free-flowing bands. This *gating* model shows significant improvement in average

1
2
3 velocity for all time over the original BML model in this extended range, and effectively increases the free-flow
4 capacity of the system by over 200 cars on a 100x100 array. This study shows that it may be possible to improve
5 the flow of traffic in cities, without additional capacity, through the use of intelligent, adaptive traffic controls.
6
7

8 **Acknowledgements**

9

10
11 The authors would like to thank Lisa Aultman-Hall for helpful discussions. This work was funded in part by the
12 United States Department of Transportation through the University of Vermont Transportation Research Center.

13 <http://www.uvm.edu/~transctr/>
14

15
16 Computational resources provided by the Vermont Advanced Computing Center.

17 <http://www.uvm.edu/~vacc/>
18
19
20

21 **References**

- 22
23 [1] U.S. Department of Transportation Research and Innovative Technology Administration. RITA 01-07.
24 <http://www.rita.dot.gov/>
25
26 [2] R. Schiesel, and M. J. Demetsky, "Evaluation of Traveler Diversion Due To En-Route Information", University
27 of Virginia Center for Transportation Studies.
28 <http://cts.virginia.edu/>
29
30
31 [3] O. Biham, A. A. Middleton, and D. Levine, Phys. Rev. A **46** R6124 (1992).
32
33 [4] D. Chowdhury, L. Santen, and A. Schadschneider, Phys. Rep. **329** 199 (2000).
34
35 [5] T. Nagatani, Rep. Prog. Phys. **65** 1331 (2002).
36
37 [6] C. Burstedde, K. Klauck, A. Schadschneider and J. Zittartz, Physica A **295** 507 (2001).
38
39 [7] B. S. Kerner, and H. Rehborn, Phys. Rev. E **53** (1996).
40
41 [8] K. Nagel, and H. J. Herrmann, Physica A, **199** 254 (1993).
42
43 [9] R. M. D'Souza, Phys. Rev. E **71** (2005).
44
45 [10] R. M. D'Souza, Complexity **12** 2 (2006).
46
47 [11] J. Trk and J. Kertsz, Physica A **231** 4 (1996).
48
49 [12] K. Wood, Transactions on the Built Environment **16** (1995).
50
51
52