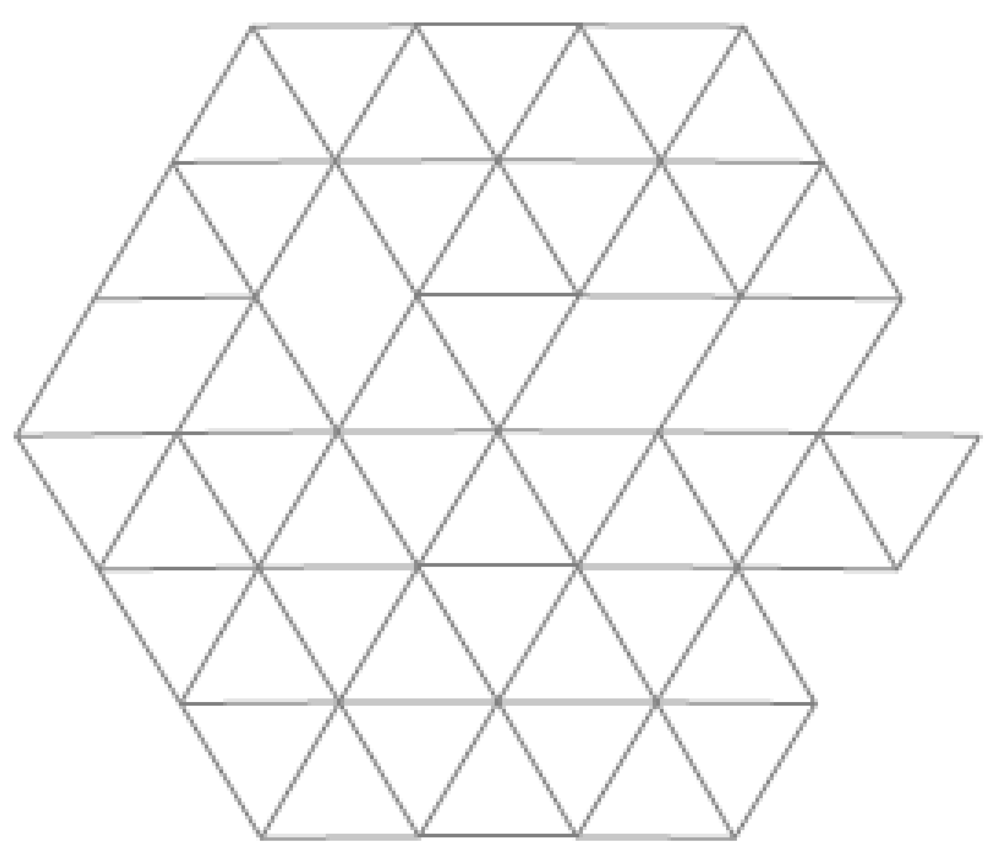
The University of Vermont
Transportation Research Center

Paper No. 09-2644: Investigation of Link Capacity-Disruption in the Calculation of a Transportation Network Robustness Index

Jim Sullivan[a], Lisa Aultman-Hall[a], David Novak[a], and Darren M. Scott[b]
[a]University of Vermont [b]McMaster University



Well-Connected Network with No Isolated Sub-Networks

Abstract
This study examines the use of severe capacity-disruptions in lieu of link-removal, in assessing transportation network robustness. A procedure that utilizes capacity-disruption will be immune to the effects of poor connectivity and isolating links in real-world transportation networks. This study builds on the recently proposed Network Robustness Index (NRI), which used complete link removal, to find the best capacity-disruption level to use. A range of optimal capacity-disruption levels was found by analyzing the NRI values for each of 3 hypothetical networks with varying levels of connectivity and capacity loss between 30% and 100%. A new form of the metric, Network Trip Robustness (NTR), was introduced to provide scalability for inter-network comparisons. The primary conclusion from this investigation is that the use of complete link-removal to model network robustness is not only infeasible for networks with isolating links, but also does not yield unique results due to the influence of Braess' Paradox involved with this computational procedure. Analysis of the NTR values suggests an upper limit of 99% on disruption levels to be used in robustness analysis. However, this upper limit may fall as low as 95%, depending on network connectivity. A link rank-order analysis of the NRI values indicates that the most stable range for capacity loss will vary with the level of connectivity of the network, but is likely to fall between 75% and 99%. Therefore, the optimal range of capacity disruption levels falls between 75% and 99%.

Calculation of the NRI

The link-specific NRI is calculated in two steps. First, the system-wide, travel-time cost when all links are present and operational in the network is calculated for the base-case scenario:

$$c = \Sigma_{i\in I}\, t_i x_i$$

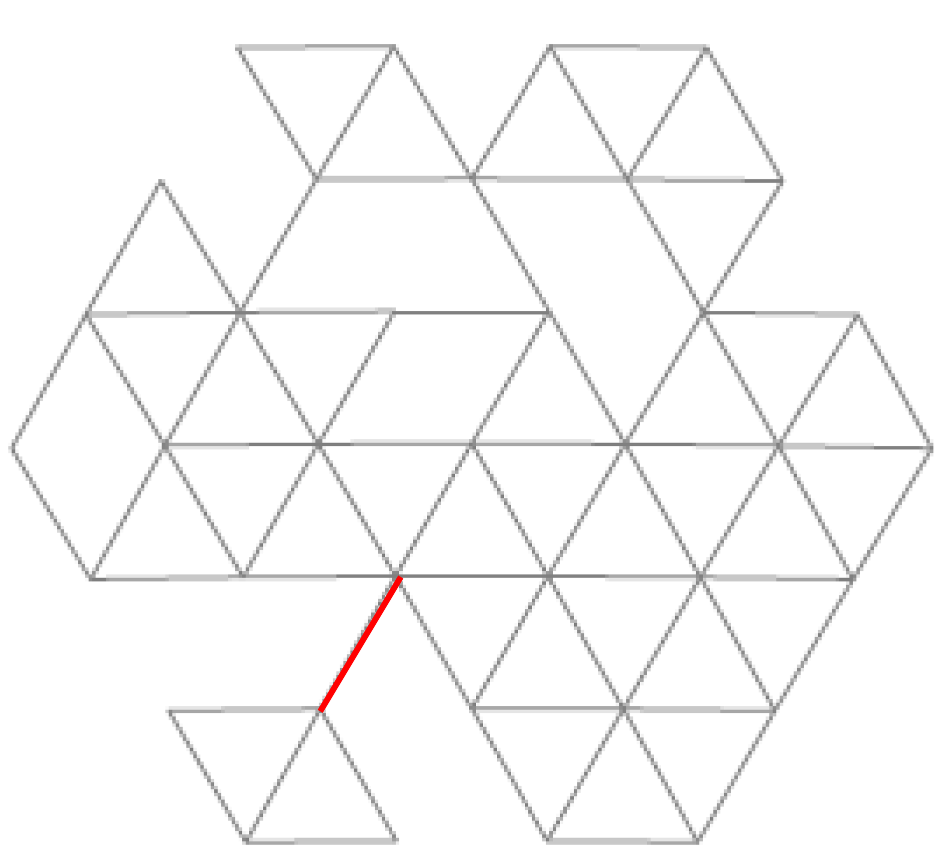
where t_i is the travel time and x_i is the flow on link *i* at user equilibrium. *I* is the set of all links in the network.

Second, the system-wide, travel-time cost, c_a, after link *a* is disrupted and system traffic has been re-assigned to a new equilibrium, is found by repeating the calculation:

$$c_a = \Sigma_{i\in I/a}\, t_i^{(a)} x_i^{(a)}$$

Where $t_i^{(a)}$ is the new travel time, and $x_i^{(a)}$ is the new flow on link *i* when link *a* has been removed. Finally, the NRI of link *a* is calculated as the increase in system-wide travel-time over the base case and is written as:

$$NRI_a = c_a - c$$



Moderately-Connected Network with One Isolated Sub-Network

The Network Robustness Index (NRI)

The focus of this project is a system-wide measure which considers, in turn, the affect that the removal of a given link has on the system-wide travel cost for users in a transportation network, after the traffic is re-assigned (also known as the total vehicle-hours of travel, or VHTs). The significant aspect to this approach is that it considers each and every link in the network, in an effort to find those that are most/least critical. The Network Robustness Index, or NRI, was developed in 2006 to provide these metrics. The NRI was proposed as an alternative to the volume-to-capacity (v/c) ratio for identifying critical links in a highway network. Transportation planning efforts, especially those involving highway capacity expansions, have traditionally relied on the v/c ratio to identify highly congested, or critical links, resulting in localized solutions that do not consider benefits to all users of the network. The NRI represents, in both cases, a more comprehensive measure since it accounts for network-wide demand and traffic re-assignment. As currently formulated, calculation of the NRI requires sequential removal of individual network links and iterative application of a user equilibrium traffic assignment model (middle left).

Motivation

Transportation planning efforts, especially those involving highway capacity expansions, have traditionally relied on the v/c ratio to identify "highly congested" or critical links, resulting in localized solutions that do not consider system-wide impacts related to congestion, security, and emergency response. The NRI is a new, comprehensive, system-wide approach to identifying critical links and evaluating network performance that relies on readily available sources of data from transportation demand planning models. The research is based on the premise that a fundamental change in highway network design philosophy is needed. Instead of identifying individual congested or critical links based on localized measures, such as level of service, we argue that infrastructure management should focus on maximizing the robustness of the overall transportation system or on minimizing the system vulnerability.

Study Objectives

The objectives of this investigation were to:

1. Solve the problem encountered when we attempt to calculate the NRI for a network with isolating links (bottom left) so that the NRI can be calculated for real-world networks.

2. Develop a network performance measure that is more useful for inter-network comparisons than the average NRI used in the early applications. We propose a more appropriate measure which divides the sum of all the link-specific NRI values by the total demand in the network, resulting in a network-specific measure called the Network Trip Robustness (NTR):

$$NTR_n = (\Sigma_{a\in I} NRI_a) / D_n$$

where D_n is the total trips between all origins and all destinations in network *n*

Methodology and Results

This study introduced a methodological basis for the use of a specific capacity-disruption in lieu of link-removal in determining network robustness. A range of capacity-disruption levels from 30% to 100% was tested to calculate NRIs. The results suggest that using 100% capacity-disruption, or complete link-removal, as a worst-case representation of severe link-capacity loss is not appropriate, as shown in Table 1.

Table 1
Summary of NRI Values

Network	Maximum NRI		Minimum NRI		Average NRI (minutes)
	Value (minutes)	Disruption Level	Value (minutes)	Disruption Level	
1a	12.3 million	100%	-121,360	50%	1.3 million
1b	124.5 million	99%	-156,000	70%	5.8 million
1c	3.3 billion	99%	-6.0 million	80%	159.4 million

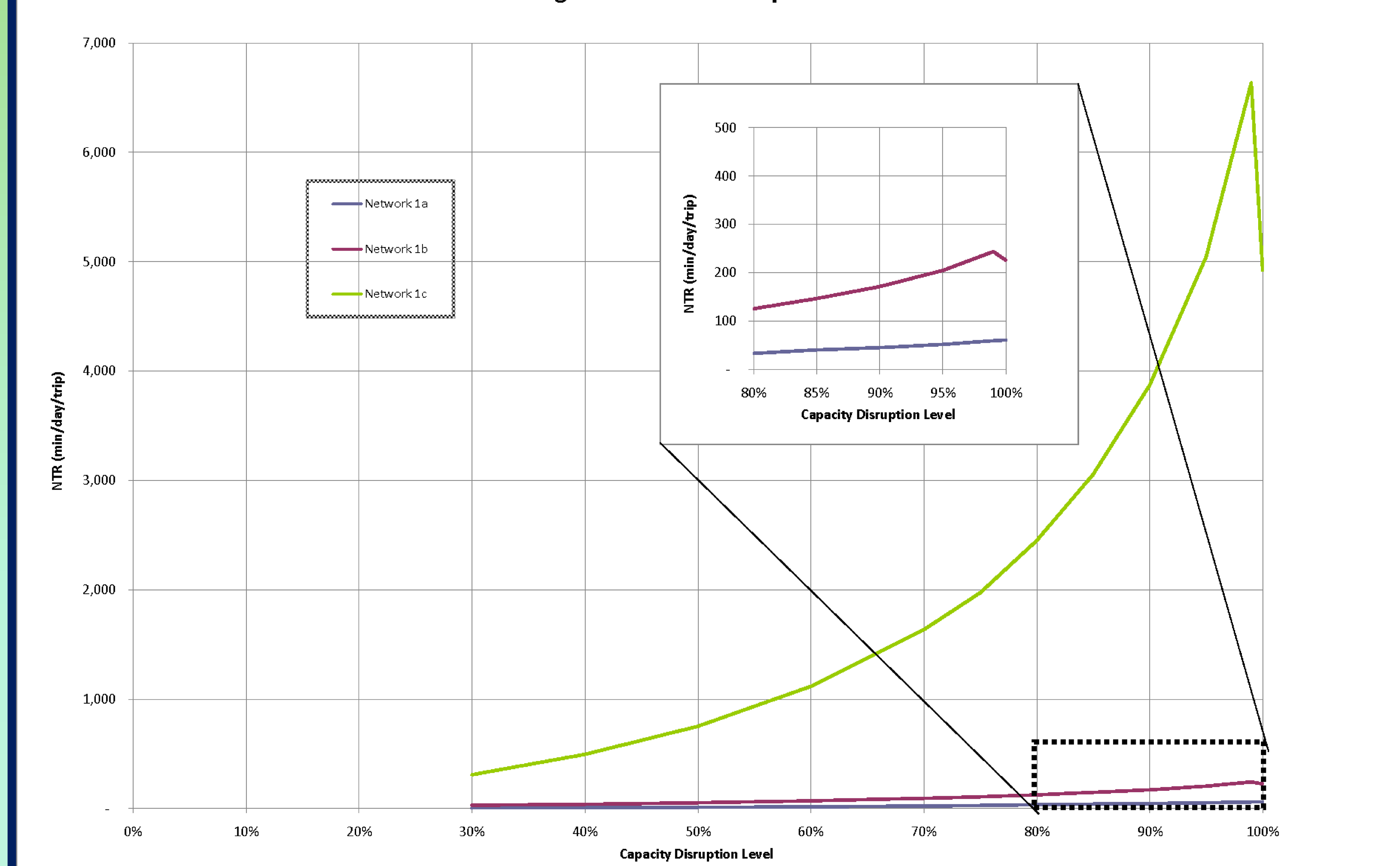
In addition, the use of 100% capacity-disruption is infeasible for networks with isolating links and does not yield unique results due to the influence of Braess' Paradox in the successive traffic assignments involved with this computational procedure. The capacity-disruption level where the rank-order is most stable was found to fall between 75% and 99%, and to depend on the level of network connectivity, as shown in Table 2. The NTR, a normalized measure of the overall network robustness, exhibits the same tendency to peak before the 100% disruption-level, confirming the influence of Braess' Paradox at high capacity disruptions, as seen in Figure 3.

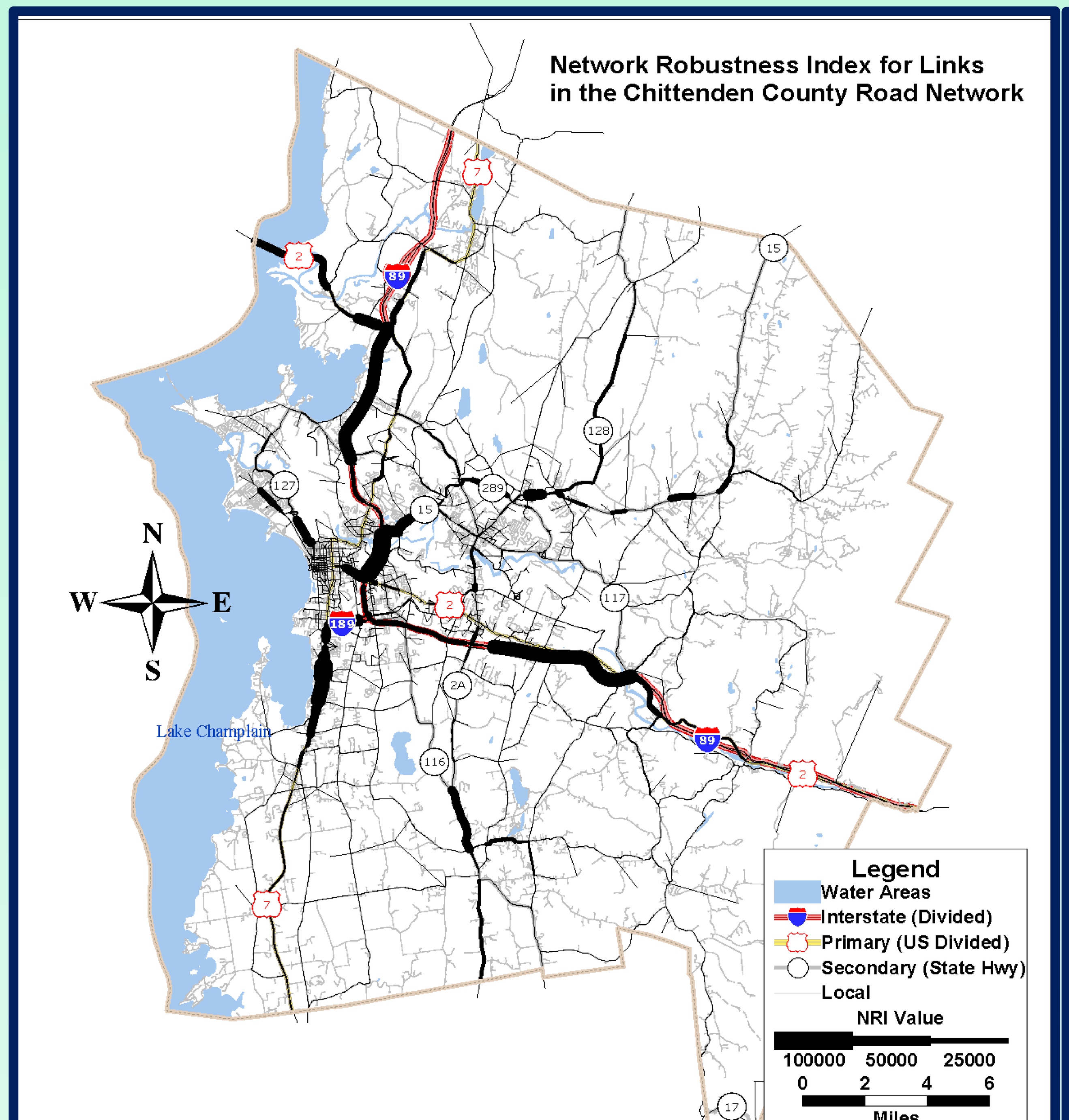
Table 2
Rank-Order Analysis Summary

Capacity Disruption Level Interval	Network 1a Average ΔRank	Network 1b Average ΔRank	Network 1c Average ΔRank
100% - 99%	4.43	2.54	2.55
99% - 95%	3.55	2.57	1.93
95% - 90%	4.45	2.41	1.59
90% - 85%	5.52	3.05	2.07
85% - 80%	5.60	3.11	1.48
80% - 75%	5.38	3.73	1.28
75% - 70%	6.36	4.59	1.79
70% - 60%	6.45	5.32	2.48
60% - 50%	8.31	5.73	2.62
50% - 40%	8.31	7.38	3.41
40% - 30%	9.33	11.46	4.72

Additional analyses were performed on eight other hypothetical networks with varying numbers of isolating links and connectivities. These additional analyses found that the number of isolating links in the network could also be significant. These analyses allowed the range of optimal disruption levels to be narrowed using a combination of "lines of evidence". These "lines of evidence" were (1) the stability of the NRI values, (2) the stability of the NRI rank-orders, (3) the trends in the NTR values (and clustering), (4) the isolating-link shares of the NTR, and (5) the ranks of the isolating links. It now appears that the 75-80% range is the optimal range for disruption levels to be used in assessment of transportation-network robustness.

Figure 3 - Network Trip Robustness





Map of the CCMPO produced in TransCAD 5.0

Future Research

Future research for this line of study includes the following:

1. Application of the modified NRI procedure to a real-world network for national freight, a state, or an MPO, like Chittenden County, Vermont (figure at left)

2. Assessment of the scalability of the NTR, using the modified procedure on networks of higher demand

3. Assessment of the relative importance, or value, of trips in the network relative to one another as links are ranked by the NRI

4. Application of the NRI to supply-chain networks, including weighting of specific commodity movements within traffic flow, and valuation of NRI in $ instead of minutes