Investigation of Link Capacity-Disruption in the Calculation of a Transportation Network Robustness Index

Submitted July 2008 for review to the Transportation Research Board

7,380 words (including tables and figures, but not the abstract)

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ABSTRACT

This study examines the use of severe capacity-disruptions in lieu of link-removal, in assessing transportation network robustness in the presence of isolated sub-networks. A procedure that utilizes capacity-disruption will be immune to the effects of poor connectivity and isolating links in the network being studied. One of the goals here was to build on the recently proposed NRI measure to find the best capacity-disruption level to use by analyzing the NRI values for each of 3 hypothetical networks with varying levels of connectivity. A new form of the metric, Network Trip Robustness (NTR), provides scalability for the network robustness measure. The primary conclusion from this investigation is that the use of complete link-removal to model network robustness is not only infeasible for networks with isolating links, but also does not yield unique results due to the influence of Braess' Paradox involved with this computational procedure. Analysis of the NTR values suggests an upper limit of 99% on disruption levels to be used in robustness analysis. However, this upper limit may fall as low as 95%, depending on network connectivity. The rank-order analysis indicates that the most stable range for the rank-orders will vary with the level of connectivity of the network, but is likely to fall between 75% and 99%.

INTRODUCTION

Recent major catastrophic events, both natural and anthropogenic, have proven to be numerous enough, and serious enough, to warrant increased consideration of robustness in network design and optimization for critical infrastructures (1, 2, 3, and 4). For transportation networks, the dynamics of disruption models are somewhat different than those intended for other infrastructure networks, like telecommunications and electrical power. The most critical difference in these networks is that transportation networks incorporate independent, critical-thinking users, whose decisions and independent behaviors affect system performance and the accuracy of the model significantly. And in most cases (air transport possibly excluded), the effectiveness of strategies intended to alter user-behavior is limited. So a critical step in the analysis of a transportation network becomes the *traffic assignment*. Traffic assignment includes models to predict how users select routes. Traffic assignment models are driven from an *origin-destination (OD) flow matrix* representing demand and most models seek to minimize travel time.

Modeling disruptions on a transportation network began as an effort to quantify the adverse effects of compromised capacity on network links, and this type of study continues to be prevalent in the field (5, 6, 7, 8, 9, 10, and 11). It became apparent in the wake of more serious threats that more severe scenarios should be considered (1, 2, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, and 33). Studies which consider *element-removal* scenarios are now equally as numerous. From a basic standpoint, these studies were considered investigations of *connectivity degradation*, whereas the capacity-related studies were considered investigations of *capacity degradation* (5 and 9). Additional studies expanded the analysis of connectivity that occurred when a network element was removed, was the specific traffic reassignment that resulted after the element was removed, and the network re-stabilized. So traffic assignment and traffic re-assignment became the critical aspects of these element-removal studies.

The Network Robustness Index (NRI)

The focus of this paper is the further development of a system-wide measure, which considers the affect that the removal of a given link has on the overall travel time for users of the network. The significant aspects to this approach are that we sequentially consider each and every link in the network, in an effort to find those that are most and least critical, and then develop a single measure to quantify the overall robustness of the network in the face of a variety of serious disruptive scenarios.

This research builds on the Network Robustness Index, or NRI, proposed by Scott et al. (34). The NRI provides a comprehensive, system-wide approach to identifying critical links and evaluating transportation network performance. The NRI was proposed as an alternative to the link-based volume to capacity (V/C) ratio for identifying critical links

in a highway network and incorporates the spatially-based gamma index for measuring connectivity. The NRI accounts for both network-wide demand and traffic reassignment.

The link-specific NRI is calculated in two steps. First, the system-wide, travel-time cost when all links are present and operational in the network is calculated for the base-case scenario. This system-wide, travel time cost, c, is found by taking the sum of the products of link-travel-time and link-flow across each link in the network:

$$\mathbf{c} = \sum_{i=1}^{n} \mathbf{t}_i \mathbf{x}_i \tag{1}$$

Where t_i is the travel time across link *i*, in minutes per trip, and x_i is the flow on link *i* at user equilibrium. *I* is the set of all links in the network. The travel time factor, $t_i x_i$, is the number of minutes of travel per day on link *i*. Note that this factor can be calculated for analysis time-intervals other than one day, using the appropriate values for travel time and demand.

Second, the system-wide, travel-time cost, c_a , after link *a* is disrupted and system traffic has been re-assigned to a new equilibrium, is found by repeating the calculation:

$$c_{a} = \sum_{i \ I/a} t_{i}^{(a)} x_{i}^{(a)}$$
[2]

Where $t_i^{(a)}$ is the new travel time across link *i* when link *a* has been removed, and $x_i^{(a)}$ Finally, the NRI of link *a* is calculated as the increase in system-wide travel-time over the base case and is written as:

$$NRI_a = c_a - c$$
[3]

The time interval selected for the analysis is defined up-front as a global variable, and the units of the NRI can be shortened to minutes.

The key to implementing the NRI is computing realistic network flows and travel times for input to equations [1] and [2]. The user equilibrium assignment model first proposed by Wardrop is ideally suited to this task and in this case is implemented in TransCAD®, a geographic information system (GIS) for transportation demand model applications.

The NRI is intended to be a network-wide index. At the same time, the methodology allows us to examine the network internally to determine which link(s) are detracting most significantly from its overall robustness. The ultimate goal is identify networks that will best allow business to be conducted after any adverse disruption.

A limitation of the NRI, and a limitation generally seen in methods that remove links from a network, is its inability to quantify the effects of the removal of an isolating link. An isolating link is one that is the sole connection for a subset of the network to the rest of the network (see Figure 1). Therefore, it becomes



Figure 1. Illustration of an isolated sub-network.

impossible to determine the

effect of this removal on travel time for the trips originating from, or destined for, locations within the *isolated sub-network*. It also becomes impossible to quantify the robustness of the entire network system because total traffic is reduced.

Objectives

Resolving the Problem of Isolated Sub-Networks

Matisziw et al. (22) propose to address the issue of isolated sub-networks in developing measures of performance for disrupted networks by selectively defining the boundaries of the network under study. We see two primary limitations to this approach. The first is that we are seeking to develop a system-wide performance measure that is readily adaptable to a state's or a municipality's existing transportationplanning network. A performance metric is not generalizable or even particularly useful if system boundaries need to be continually redefined on a scenario-by-scenario basis before the model will run successfully. Second, we realize that, for networks with one or more isolated sub-networks, the level of traffic that needs to use the isolating links in the network will impact true network robustness. Traffic on an isolating link has no alternative travel path when the isolating link is disrupted, but realistically the impact of the disruption should be measured in common units and included in the overall network performance measure.

An obvious alternative to addressing the problem caused by isolated sub-networks is to use a capacity-disruption lower than 100% to calculate the index, which essentially allows the capacity to be reduced on each isolating link instead of removing the link altogether. However, the use of an arbitrary capacity-disruption, like 99%, does not have a sound methodological basis. This study seeks to initiate the formation of a methodological basis for the use of a specific capacity-disruption in lieu of link-removal in determining network robustness. Therefore, the results of this study are to allow the NRI to be adapted to situations in the real-world where isolated sub-networks are common.

Development of a System-Wide Performance Measure

The second objective of this paper is to derive an overall network performance measure that is more useful for inter-network comparisons than the average NRI used in the paper by Scott et al. (34). Inter-network indices can be used to compare the robustness of a network to other similar networks, or to itself in a future expansion or disruption scenario. These types of measures can be useful when large-scale budgeting decisions need to be made amongst a number of separate networks within, for example, a state. The results of this study are expected to contribute to the continuing refinement of a truly scalable measure of network robustness.

To derive a single, system-wide measure of the overall network robustness that can be normalized for inter-network comparisons, it is necessary to standardize the individual link-specific NRI values. Scott et al. (34) accomplished this standardization by averaging these values - dividing the sum of link-based NRIs by the total number of links in the network. The number of links in the network, however, impacts travel times and link flows in the traffic assignment procedure. Simply put, a network with fewer links will tend to have higher travel costs than a comparable network that attempts to satisfy the same level of demand with more links. Therefore, dividing by the number of links is, in effect, "double-counting" the contribution of the number of links to the overall performance measure. We propose a more appropriate procedure which divides the total of all the link-specific NRI values by the total demand in the network, resulting in a network-specific measure of Network Trip Robustness (NTR) formulated below:

$$NTR_{n} = \left(\sum_{a \ I} NRI_{a}\right) / D_{n}$$
[4]

Where D_n is the total trips between all origins and all destinations in network n. As D_n will be a total number of trips, the units for the resulting NTR will be minutes per day per trip. Using this formulation avoids double counting any of the aspects of network structure designed to satisfy the demand, and "standardizes" the NRI values to facilitate inter-network comparisons. In addition, the NTR will provide scalability for the robustness measure, since larger networks will typically have more demand to satisfy, so the resulting NTR value for a much larger network will be comparable to that for a smaller network, with less demand to satisfy.

BACKGROUND LITERATURE

Definitions of Disruption: Vulnerability, Reliability and Robustness

The goals of previous studies in network disruption can be categorized in three ways:

- (1) to maximize the network's robustness (capability of adapting to and recovering from disruption) (6, 7, 33, and 34),
- (2) to maximize the network's reliability (resistance to disruption) (1, 3, 5, 6, 7, 8, 9, 14, 17, 22, 26, 28, 29, and 30), or

(3) to minimize the network's vulnerability (potential for disruption) (2, 3, 4, 12, 13, 16, 17, 18, 19, 20, 22, 23, 25, 30, 31, and 35).

The end results of these studies of robustness and vulnerability often fall into one of two general categories – those that seek to develop a network performance index (for potential inter-network comparisons) (33 and 34) and those that seek to compare individual elements of a given network to each another (intra-network comparisons). Examples of inter-network performance indices for disruption scenarios are the NRI (34) and the network reliability (6). Examples of intra-network comparison tools are NRI (34), ratings of criticality (2) and importance (18).

Disruption Measures and Indices

It is also valuable to consider how previous studies have expressed their final output and how useful their measures will be for policy purposes. The most common network assessment is a ranking of the network elements based on their relative impact on the network if removed (30,12, 10, 1, 22, 28, 6, 7, 13, 33, 32, 34, 17, 2, 18, 19, 20, 23, 4, 25, and 14). The following algorithms or models fall into this category:

- The Most Vital Arcs Problem (MVAP)
- The Node Removal Impact Problem (NRIP)
- The p-Cutset Problem (PCUP)
- The Flow Interdiction Model (FIM)

Other similar studies seek only to identify the one link that is most in need of improvement (12, 6, 9, 16, and 29), presumably as a resource-allocation measure. Other studies, particularly those relying on probability-based measures, have developed *reliability envelopes* to demonstrate the likelihood of a network's continued functioning under various states of degradation or element-removal (8, 26, 22, 5, 7, and 9). A last group of studies has also gone beyond these efforts to attempt a more global measure of network performance. A few have attempted to identify regions within the network, which are most vulnerable to diminished functionality under element-removal scenarios (29, 18, 19, and 20). Other common measures of network performance developed in the context of disruption-analysis are robustness, (33, 31, 34, 1, and 28), importance (or criticality) (30, 12, 6, 7, 13, 32, 17, 2, 18, 19, 20, 23, 4, 25, and 29), adaptability or resilience (35 and 27), capacity reliability (7, 8, and 9), connectivity reliability (26, 14, 1, and 22), and travel-time reliability (5 and 10).

Accounting for Isolated Sub-Networks

A sub-set of the previous studies of network disruption have acknowledged the inherent problem encountered when modeling the effects of link-removal on a network with one or more isolated sub-networks (18, 19, 20, 1, 15, 2, 23, 4, 29, and 22). Studies that are seeking to quantify only the instantaneous impact of element removal do not have a

problem with isolating links, but usually acknowledge that its removal creates a more serious situation than the removal of a non-isolating link. The problem occurs when rerouting after the instantaneous impact is included in the model. Since the trips that had previously used the isolating link can no longer be completed, their travel time is not quantified by most models. Consequently, the effects of these trips are not included in a model which uses flow and travel-time to develop its performance measure when, in fact, these lost or delayed trips may be the most significant contributors to the adverse effect of a link-removal. Therefore, it is imperative that these trips be included in the models.

Jenelius (18, 19, and 20) has attempted to account for trips across isolating links in two ways. One way (18 and 20) is to count them separately from the primary performance measure and attach a secondary measure of "unsatisfied demand" to the study results. However, the two measures cannot be combined into a single performance measure. Nor can the relative importance of the one measure be quantified against the other. Jenelius also attempts to account for these trips by assuming that none of the cut off trips are lost (19), but are delayed according to the length of time that the disruption lasts. This method works well for relatively short-duration disruptions, but is not as useful when the disruptions lasts a day or longer. These longer disruptions seem to lend an inordinate bias to the isolated sub-networks, skewing the resulting index considerably. Most of the network disruption studies are concerned with long-term disruptive events which require that the network return to a "business-as-usual" functioning without the missing link. Therefore, this approach does not specifically consider the value or the importance of the missing link(s).

Several other studies (15 and 36) account for the presence of isolated sub-networks inadvertently with an alternate formulation of the performance measure. The notions of *social efficiency* and *point closeness*, although not traditional in the transportation realm, utilize a cost factor with the primary cost variable (in our case, time) in the denominator.

Summary of Literature Review

The literature studied in the area of network disruption-analysis covered a wide variety of numerical approaches which shared some common themes. All of the studies are aimed at developing effective, inclusive methods of quantifying the ability of a network to function under varying states of disruption. The most common mode of disruption is the loss of a single link in the network. From this point, the studies diverge in how the details of the analysis are handled. None of the studies reviewed has completely addressed the problem of isolated sub-networks in the development of a scalable network-robustness performance measure.

METHODOLOGY

This study introduces a methodological basis for the use of a specific capacitydisruption in lieu of link-removal in determining network robustness. First we examine the measures of overall network robustness when varying levels of link capacitydisruption are used in the calculation of the NRI. We use a set of hypothetical transportation networks and a hypothetical origin-destination trip matrix. These networks were derived by Scott et al. (34). The three hypothetical networks each have a different level of connectivity as measured by the gamma index, as shown in Figure 2.



Figure 2 - Three Hypothetical Networks Used in this Study

The gamma index is a relatively simple connectivity index that considers the actual number of links divided by the maximum possible number of links for the network. Therefore, the index varies between 0 (disconnected network) and 1 (completely connected network). It was determined by Scott et al. (34) that, although the NRI is an improved measure of robustness over the volume to capacity ratio, the range of NRIs for a network is related to the gamma index of the network. Therefore, in order to provide an adequate range of values for the NRI, networks with a wide variety of gamma indices are used. The same origin-destination trip matrix is used for each network (34). The matrix was generated by assuming that the central node in the network is a Level 1 population centers (200,000 - 600,000 people), the 6 surrounding nodes are Level 2 population centers (50,000 - 200,000 people)). Each person was assumed to generate 0.6 trips per day, and then a production-constrained gravity model was used to derive the trip matrix. The total number of trips demanded was 3,439,490.

A range of capacity-disruption levels from 30% to 100% for all links, instead of complete link-removals, was used to calculate NRIs.

As individual link-capacities were sequentially reduced, a portion of the traffic normally assigned to the disrupted link re-routes in the same way that it had when the link was completely removed. These higher travel times contribute to larger NRI values in the same way that they did when complete link-removals were used. Note that we are ignoring behavioral effects beyond route selection and the potential loss of origin-destination demand due to unacceptable increases in travel time, particularly across the isolating link(s). We are assuming that the total demand continues to be realized when a disruption occurs, and no trips are lost. This assumption is reasonable for this study since we are attempting to quantify the effect of a disruption, not simulate the precise travel-time increase.

RESULTS AND FINDINGS

The analysis resulted in 2,592 separate NRI values – one for each disrupted link, on each network, under each of 12 capacity-disruption scenarios between 30% and 100%. This section describes the specific NRI values, their rank-order, and the 36 values for the scaleable NTR.

NRI Values

A summary of the NRI values yielded by this analysis can be found in Table 1. The maximum NRI represents the "worst-case" scenario, and the link for which the maximum occurs is the most critical link for that scenario. The maximum NRI for the well-connected Network 1a under the 100% capacity-disruption scenario is fairly intuitive, although the exact location of the critical link may not be. However, finding the minimum NRI at the 50% capacity-disruption, and not the 30% capacity-disruption, is not consistent with expectations – the most minimal network-wide effect would be expected under the most minimal capacity-disruption. This inconsistency suggests that the computation is not stable at the lower capacity-disruptions, or that all of the results for the minimum NRIs, representing the least critical links in the network, are not useful.

Table 1											
Network	Maximu	m	Minimum	1	Average						
1a	12.3 million	100%	-121,360	50%	1.3 million						
1b	124.5 million	99%	-156,000	70%	5.8 million						
1c	3.3 billion	99%	-6 million	80%	159.4 million						
All values are in minutes Disruption levels for the maxima and minima are also given.											

For Network 1b and 1c, there are two important findings in the results. The first is that the maximum occurs not at the 100% capacity-disruption scenario as one might expect, but at 99%. This observation indicates that network-wide disruptions approaching 100% might result in larger overall travel-time increases than those at 100% (or complete link-removal). Considered further, though, this situation is consistent with Braess' Paradox (37), albeit on a larger scale. Braess' Paradox observes that the addition of a link to a network with a relatively small capacity may actually *increase* the total system travel-time, instead of decreasing it as one might expect (37). Under the scenario with the 99% capacity reduction, we are effectively creating the same type of network that Braess' Paradox warns us about - one with a single link whose capacity is considerably smaller than the capacities of all of the other network links. Comparing the total system travel-time for this situation with the total system travel-time without this low-capacity link is parallel to comparing the NRIs for the 99% disruption scenario with the NRIs for the 100% disruption scenario. Therefore, these results suggest that using the 100% capacity-disruption, or complete link-removal, as a worst-case representation of severe link-capacity loss is not appropriate. A capacity-disruption other than 100% may better serve the derivation of robustness indices for networks. In addition, it appears that this property is more significant for networks with less connectivity, or lower gamma indices, since it was not observed for Network 1a.

The second important observation to be made from these results is that, again, the least critical link (the one with the lowest NRI value) does not occur at the lowest capacity-disruption (30%). In some instances a capacity disruption can actually improve the network performance. For Network 1b, the link with the lowest NRI occurs at the 70% disruption level, again indicating a possible instability in the NRI values at this level. For Network 1c again the least critical link does not occur at the lowest capacity-disruption level, but at 80%. In addition, further observation of the data indicates that the presence of negative values of the NRI, an indication of the situation alluded to by Braess' Paradox except on a smaller scale, vanishes within a certain region of the capacity-disruption levels lower than 80%. The region of capacity-disruption levels that are free of negative NRI values expands as the gamma index decreases, from >95% for Network 1a, to *between* 85% and 100% for Network 1c. This observation is important because we have observed that the well-connected state of Network 1a represents an optimal level of connectivity, but not a realistic one (34).

NRI Rank-Order

For intra-network comparisons, the most important evaluation of this method comes from consideration of the rank-orders of links for each disruption level on each network. With 12 separate disruption-levels on 3 different networks, we obtained a total of 36 different rank orders. For each network, the change in the rank of each NRI value for each link was recorded as the difference between the rank at the previous disruption level and the rank at the current disruption level. These results were evaluated to find a consistent and stable rank-order as it is hypothesized that this will points to an optimal capacity-disruption level for intranetwork robustness analyses.

An example of the data evaluated in the rank-order analysis for Network 1a is provided in Table 2. At the bottom of the table, the average change in the rank of each NRI value for a given disruption level is given. The location of the minimum value for the average change in rank corresponds with the range of disruption where the rank-order is most stable. For Network 1a, this range is between the 99% and the 95% disruptions levels. For Network 1b, the most stable range of disruption-levels is between 95% and 90%. For Network 1c, the most stable range turns out to be between 80% and 75%. A summary of this analysis for all networks is provided in Table 3.

Network Trip Robustness (NTR) Values

The use of a network performance measure like the NTR facilitates inter-network comparisons. The NTR was calculated for each network at each disruption level to find the disruption-level that results in the most effective inter-network comparisons. The 36 NTRs are provided in Table 4, along with the maximum NRI values for each of these conditions. The NTR, normalized measure of the overall network robustness also exhibits the same tendency to peak before the 100% disruption-level, confirming the influence of Braess' Paradox on a large scale. However, the influence of this condition is even more pronounced when the network performance measure is used. Note in Figure 3, the NTR values for disruption levels below 100% exceed the 100% value. For Network 1c, both the 95% and 99% NTRs exceed the 100% NTR. For Network 1b, the 99% NTR exceeds the 100% NTR.

Table 2																							
Network 1a	Rank-O	rder An	alysis																				
Disrupted																							
Link ID	NRI*00%	⊽rank	NRIPPA	⊽rank	NRISSS	⊽rank	NRI ^{90%}	⊽rank	NRI ^{ses}	⊽rank	NRI	⊽rank	NRI ^{75X}	⊽rank	NRI ^{70%}	⊽rank	NRISON	⊽rank	NRISON	⊽rank	NRI ^{40%}	⊽rank	NRISON
1	18	-	18	1	17	-	17	1	16	1	15	2	13	1	14	1	15	2	17	7	10	5	5
2	15	1	16	3	19	-	19	3	22	4	26	15	41	14	27	10	37	24	61	15	76	13	63
3	7	-	7	-	7	-	7	1	6	1	7	5	12	1	11	-	11	2	13	1	12	2	14
4	2	-	2	-	2	-	2	-	2	-	2	-	2	-	2	2	4	2	6	3	9	2	11
5	3	-	3	1	4	1	5	2	7	1	8	3	11	4	15	5	20	1	19	13	32	6	38
6	22	1	21	-	21	7	28	4	24	1	25	1	24	-	24	2	26	17	43	16	59	23	36
7	53	10	43	4	47	2	45	5	50	4	54	-	54	8	62	13	49	16	33	3	36	20	16
8	67	14	81	7	74	4	70	1	71	5	66	11	55	14	69	6	63	16	79	44	35	6	29
9	72	3	69	3	72	6	66	4	62	5	67	5	72	6	78	11	67	8	75	3	72	-	72
10	51	11	62	6	56	15	71	25	46	15	61	5	56		56	5	61	10	51	13	64	3	67
11	16	4	12	1	13	3	10	1	11	1	10	1	9	2	7	4	3	5	8	-	8	7	15
12	55	8	47	6	53	1	54	5	59	21	38	7	45	11	34	1	35	9	26	8	34	-	34
13	- 34	10	44	8	36	1	37	18	55	9	46	12	34	2	32	6	- 38	1	37	18	55	6	49
14	30	4	34	7	27	1	26	1	25	9	34	5	29	4	25	3	22	2	20	9	29	5	24
15	21	2	23	1	24	2	22	5	17	2	19	4	15	1	16		16	6	10	7	17	9	8
16	54	5	59	2	57	2	59	1	60	8	52	7	59	6	53	10	43	5	48	17	65	17	48
17	56	7	49	-	49	2	47	9	38	17	55	8	47	16	31	15	46	16	30	19	49	1	50
18	52	14	66	2	68	5	63	6	57	6	63	2	61	3	64	9	73	7	80	-	80	3	77
19	31	1	32	2	34	-	34	7	41	4	37	5	42	7	35	19	54	9	45	7	38	4	42
20	9	1	8	-	8	1	9	-	9	3	12	2	10	3	13	1	12	9	21	1	20	7	13
21	47	5	52	4	48	10	38	7	45	14	31	8	23	7	30	1	31	6	25	2	27	24	3
22	1	-	1	-	1	-	1	-	1	-	1	-	1	-	1	1	2	1	3	1	2	5	7
23	14	1	15	-	15	1	14	4	10	1	11	3	8	1	9	4	5	3	2	1	3	1	2
24	10	1	11	1	10	2	12	4	8	2	6	1	5	2	3	2	1	-	1	-	1	-	1
26	44	1	45	12	33	8	41	3	44	8	36	-	36	14	22	5	17	12	5	1	4	-	4
27	5	-	5	-	5	2	3	-	3	-	3	-	3	1	4	3	7	-	7	1	6	-	6
31	27	1	26	4	22	3	25	3	28	5	23	1	22	2	20	3	23	-	23	2	21	4	25
32	24	6	30	2	28	11	- 39	9	30	- 3	33	5	38	9	29	4	33	7	40	6	46	1	45
33	81	5	76	4	80	-	80	1	81	-	81	-	81	1	82	16	66	1	67	-	67	12	55
34	4	-	4	1	3	1	4	-	4	-	4	-	4	1	5	1	6	2	4	3	7	11	18
35	63	12	51	3	54	10	64	10	54	10	64	3	67	28	39	3	36	8	44	21	23	42	65
36	19	1	20	2	18	•	18	•	18	2	16	-	16	1	17	4	13	1	12	2	14	2	12
37	36	3	33	2	31	2	33	2	31	4	27	5	32	6	26	1	27	-	27	9	18	1	19
38	74	5	79	12	67	5	72	4	76	-	76	1	75	6	81	7	74	1	73	5	68	4	64
39	11	1	10	1	11	3	8	6	14	3	17	1	18	1	19	1	18	2	16	-	16	7	9
40	71	4	67	1	66	3	69	2	67	1	68	5	63	9	72	3	75	34	41	•	41	20	21
41	25	4	29	3	32	3	29	3	26	21	47	1	48	6	42	15	57	3	54	9	63	8	71
42	80	4	84	2	82	- 47	82	- 1	83	1	84	-	84	1	83	-	83	•	83	- 1	84	2	82
43	37	1	- 30	2	30	17	50	26	29	1	30	1	31	17	48	10	58	4	62	9		28	43
44	33	5	38	1	3/	5	32	17	33	11	44	9	53	16	3/	4	41	9	32	1	33	6	39
45	/8	1	- 11	4	81	3	/8	17	61	1/	/8	4	82	- 11			64	14	50	2	52	8	60
40	40	1	9	-	8	2	11	1	12		14	3	1/	4	21	3	24		22	21	45	1	44
4/	13	17	14	- 42	14	1	13	- 24	13	4	9	47	40	1	8	•	8	- 1	9	6	15	5	10
40	41	17	20	13	45		44	24	26	- 10	5/	17	40	10	55	5	60	4	64	14	50	42	33
49	23		22	3	20		24		35	10	45			0	44	0	52	40	00	3	70	13	70
50	75	0	63	4	19	6	73	9	- 64 22	16	20	4	76	2	74	- 11	20	10	53	15	21	1	79
51	20	7	64		66	4	20 E1		20 52		<u>22</u> 60	4	20	15	20	17	39	0	56	•	51	5	51
50	42	4	20		42		49	40	36				20	24	50		40		70		40	44	20
54	43	10	38	12	43	10	40	12	30	- 6	28	12	20	31	39	6	46	12	50	20	42	22	20
56	46	10	40	13	46	10	36	- 4	- 47	8		10	30	16	40	12	40	10	24	- 6	20	32	22
57	40	1	50	1	51	9	42	-	42	7	35	2	37	1	38	2	40	6	34	5	30	2	41
68	60	4	56	- 1	60	8	68	- 3	85	Á	60	18	51	- 2	40	7	40	23	65	12	53	6	47
50	50	13	37	2	39	4	43	11	32	A	24	3	27	Ā	23	2	21	48	69	8	77	50	27
60	62	1	61	0	70	4	7/	1	73		72	6	79		73	7	80	2	82	1	83	2	81
61	40	5	35	ğ	44	A	52	1	51	7	58	9	49	2	51	1	50	15	35	5	40	â	32
62	58	7	65	11	76	14	62	10	72	19	53	24	77	14	63	6	69	1	68	2	70	3	73
63	65	6	71	2	69	6	75		66		65		68	2	66	5	71	3	74		74	6	68
64	29	Ă	25	ĩ	26	Ă	30	7	37	ģ	28	7	35	17	52	27	25	11	36	10	26	Ă	22
65	26	1 i	27	4	23	-	23	3	20	2	18	1	19	9	10	-	10	4	14	9	5	30	35
66	66	6	60	3	63	7	56	2	58	17	75	11	64	4	60	2	62	4	66	•	66	5	61
67	79	Ť	78	3	75	4	79	3	82	1	83	12	71	1	70	-	70	2	72	3	75	Ĩ	74
68	73	10	63	1	64	12	76	2	78	7	71	9	62	1	61	23	84	-	84	65	19	50	69
69	70	-	70	8	62	5	67	2	69	9	60	-	60	10	50	3	53	7	46	14	60	7	53
70	38	17	55	5	50	1	49	6	43	-	43	3	46	1	47	19	28	10	38	10	48	2	46
71	69	6	75	2	77	-	77	3	74	3	77	3	74	6	68	-	68	3	71	2	69	7	62
72	28	4	24	5	29	8	21	6	27	7	20	-	20	2	18	1	19	4	15	4	11	15	26
73	32	1	31	4	35	-	35	4	39	-	39	13	52	5	57	13	44	5	49	9	58	1	59
74	39	3	42	1	41	10	31	18	49	2	51	7	58	9	67	15	82	22	60	13	47	11	58
75	68	4	72	1	71	14	57	18	75	2	73	4	69	6	75	3	72	15	57	-	57	5	52
76	48	2	46	9	55	5	50	3	53	5	48	9	57	1	58	3	55	2	53	9	44	10	54
77	45	5	40	2	42	2	40	8	48	6	42	9	33	-	33	3	30	12	42	18	24	16	40
78	12	1	13	1	12	4	16	1	15	2	13	8	21	20	41	12	29	1	28	9	37	-	37
79	6	-	6	-	6	-	6	1	5	-	5	1	6	-	6	3	9	9	18	4	22	5	17
80	77	4	73	-	73	15	58	19	77	3	74	4	70	9	79	32	47	8	55	6	61	4	57
81	84	2	82	4	78	5	83	4	79	9	70	10	80	4	84	7	77	1	76	3	79	9	70
82	17	-	17	3	20	5	15	4	19	13	32	7	25	11	36	15	51	4	47	2	45	21	66
83	59	5	54	2	52	6	46	10	56	6	50	23	73	7	80	2	78	49	29	4	25	53	78
84	83	9	74	9	83	1	84	-	84	2	82	1	83	7	76	3	79	2	81	1	82	2	80
85	61	4	57	2	59	6	65	5	70	8	62	4	66	12	54	5	59	20	39	15	54	2	56
86	42	11	53	13	40	13	53	13	40	1	41	2	43	-	43	13	56	4	52	21	73	11	84
	82	2	80	4	84	3	81	1	80	1	79	-	79	2	77	1	76	2	78	3	81	5	76
89	35	7	28	2	30	3	27	6	21		21	7	14	2	12	2	14	3	11	2	13	7	20
90	76	8	68	10	58	2	60	3	63	14	49	16	65	20	45	13	32	45	77	49	28	3	31
		4.43		3.55		4.45		a.52		5.60		5.36		0.36		6.45		0.31		6.31		9.33	

Table 3 Rank-Order Analysis Summary

Capacity	Network 1a	Network 1b	Network 1c		
Disruption Level	Average	Average	Average		
Interval	∆Rank	^A Rank	∆Rank		
100% - 99%	4.43	2.54	2.55		
99% - 95%	3.55	2.57	1.93		
95% - 90%	4.45	2.41	1.59		
90% - 85%	5.52	3.05	2.07		
85% - 80%	5.60	3.11	1.48		
80% - 75%	5.38	3.73	1.28		
75% - 70%	6.36	4.59	1.79		
70% - 60%	6.45	5.32	2.48		
60% - 50%	8.31	5.73	2.62		
50% - 40%	8.31	7.38	3.41		
40% - 30%	9.33	11.46	4.72		

Table 4 Summary of NTR Values

	Net	work 1a	Net	work 1b	Network 1c			
Capacity								
Disruption	NTR	Maximum NRI	NTR	Maximum NRI	NTR	Maximum NRI		
Level	(min./trip)	(minutes)	(min./trip)	(minutes)	(min./trip)	(minutes)		
100%	61	12,258,62	7 226	119,521,85	0 4,924	2,826,177,67		
99%	60	11,884,00	5 244	124,476,94	9 6,638	3,315,887,07		
95%	52	10,641,01) 205	106,937,05	5 5,046	2,734,572,97		
90%	45	9,382,142	. 171	89,881,85) 3,871	2,282,395,43		
85%	41	8,040,937	' 147	76,604,12	3,056 3	1,863,523,70		
80%	33	6,337,728	8 126	64,298,99	3 2,449	1,511,012,62		
75%	28	5,523,977	' 108	54,836,02	0 1,973	1,238,071,33		
70%	24	4,808,724	93	45,556,00	3 1,635	1,046,298,42		
60%	17	3,176,254	70	33,029,76	1,116	711,845,74		
50%	11	2,297,517	' 51	23,149,40) 751	494,953,02		
40%	9	1,908,975	39	14,998,67	9 495	311,570,26		
30%	5	1,031,295	29	9,626,896	308	201,368,59		



CONCLUSIONS AND SUMMARY

Measuring robustness of links within a network and comparing the robustness of different networks are both important. It is reasonable that researchers have used complete link removal in several methods to measure network robustness. Use of reduction below 100% allows networks with isolating links to be evaluated using the existing NRI metric. However, this new investigation has shown that capacity-disruption levels lower than 50% are not likely to result in stable, accurate NRI values and should not be considered for analyses of this type. A more realistic lower limit of about 70% is recommended.

Analysis of the normalized network wide NTR measure for the 36 capacity-disruption scenarios in this study indicates that, due to the presence of Braess' Paradox, a realistic upper limit of 99% on the disruption levels should be used. However, it should be acknowledged that, as the level of connectivity of the network increases, the upper limit on the range of desirable disruption levels decreases, and may fall as low as 95%. Therefore, a practical capacity-disruption range for robustness analysis is between 70% and 99%.

Within this practical range of disruption levels, the stability of the rank-orders of the NRI values is important for the determination of the most critical individual link(s) in the network. Those links whose disruption results in the highest NRI values are the ones that are most critical in improving the robustness of the overall network. Therefore, we desire to find a capacity disruption level where the rank ordering is

stable. The analysis conducted indicates that the most stable range for the rank-orders is network and demand dependent and will vary with the level of connectivity of the network, but is likely to fall between 75% and 99%.

The primary conclusion from this investigation is that the use of complete link-removal, or 100% capacity-disruption, to model network robustness, is not only infeasible for networks with isolating links, but also does not yield unique results due to the influence of Braess' paradox in the successive traffic assignments involved with this computational procedure. Therefore, an alternative capacity disruption level should be sought for determining network robustness when this sequential link-disruption procedure is being used. This capacity-disruption level is likely to fall between 75% and 99%, and it depends on the level of network connectivity and the presence of isolating links. Further research should be conducted to narrow the range of appropriate capacity-disruption levels to an optimum.

Future Direction

Future work is necessary to refine the range of appropriate capacity disruption level. Once an optimal capacity-disruption level is determined, link-specific NRI values can be determined using a modified procedure that incorporates capacity-disruption instead of complete link-removal. A procedure that utilizes capacity-disruption will be immune to the effects of poor connectivity and isolating links in the network being studied. Therefore, the modified procedure will facilitate calculation of NRIs for real-world networks. The link-specific NRI values can then be used to make intra-network comparisons, primarily identifying the most critical links in the network to fortify, augment, or protect.

The Network Trip Robustness, or NTR, is a network performance measure that is effective for inter-network comparisons. Derived from the collection of NRI values for the network, this measure is standardized by the total demand in the network, making it scalable as well. However, this study utilized a constant demand level and before this measure can be used to compare the robustness of a network to other similar networks, or to itself in a future expansion or disruption scenario the utility and stability of the measure should be considered for a variety of demand levels.

Both NRI and NTR measures can be useful when large-scale budgeting decisions need to be made amongst a number of separate networks within, for example, a state. If disruption scenarios are a concern for the state, then inter-network indices should be used to allocate funds more aggressively to those networks with measures of compromised robustness. Application to real world networks would be a prudent next step in this research now that the problem of isolated sub-networks has been addressed.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the USDOT for funding this work through the Transportation Centers at the University of Vermont and MIT (NEUTC). We would also like to thank Peter Dodds at the University of Vermont for his support in this study.

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