Atmospheric sciences and the mathematics common core standards

\[ \sum F = ma \]

Dr. Janel Hanrahan
Atmospheric sciences and the mathematics common core standards

High School

CCSS.Math.Content.HSF-BF.A.1c  Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
The Atmosphere

- 99% is within 20 miles of surface
- All weather happens in bottom 7 miles
- Relative thickness:
  - Earth diameter: 7,900 miles
  - Atmosphere is 7/7900 or 0.1%

But what is it????
Composition of Earth’s atmosphere

- Nitrogen (N$_2$) 78.08%
- Oxygen (O$_2$) 20.95%
- Argon (Ar) 0.93%
- Other gases* 0.04%

*including water vapor, carbon dioxide, neon, helium, methane, krypton, hydrogen, nitrous oxide, carbon monoxide, xenon, ozone, nitrogen dioxide, iodine, and ammonia
Question

How much does the air in our room weigh???
Question

How much does the air in our room weigh???

The person with the closest guess without going over gets a NEW CAR!!!*

*Well, not really. But it is fun to guess right?
Many people do not consider that the Earth’s atmosphere has weight. Indeed, our atmosphere is made up of various gasses, the most abundant of which are Nitrogen (78.08%), Oxygen (20.95%) and Argon (0.93%). These and the remaining 0.04% of atmospheric gases and particles, all have mass, and therefore have weight. For this activity, we will compute the weight of the air in this room.

Question: How much does the air in our room weigh?

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<p>| | | |
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| | | 1 mb = 100 Pa |
| | | $g = 9.8 \text{ m s}^{-2}$ |
| | | $R_d = 287 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$ |</p>
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**Solution:**

**Common Core Standards addressed:**
Newton’s second law of motion

\[ F = ma \]

- **force**
- **acceleration**
- **mass**
Newton's second law of motion

\[ W = mg \]

- **weight** (force due to gravity)
- **gravitational acceleration**
- **mass**
The weight of our atmosphere
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Many people do not consider that the Earth’s atmosphere has weight. Indeed, our atmosphere is made up of various gases, the most abundant of which are Nitrogen (78.08%), Oxygen (20.95%) and Argon (0.93%). These and the remaining 0.04% of atmospheric gases and particles, all have mass, and therefore have weight. For this activity, we will compute the weight of the air in this room.

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What is the mass of the air in our room?
Density

\[ \rho = \frac{m}{V} \]

density, mass, volume
Density

\[ \rho = \frac{m}{V} \]

\[ m = \rho V \]
The weight of our atmosphere

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What is the **volume** of our room?

What is the **density** of the air in our room?
Questions

What is the **volume** of our room?

What is the **density** of the air in our room?
Volume

\[ V^* = lwh \]

*Assuming that the room is a rectangular prism*
**The weight of our atmosphere**

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What is the **volume** of our room?

What is the **density** of the air in our room?
The Ideal Gas Law

\[ \rho = \frac{P}{RT} \]

density  \quad \text{pressure}

\text{gas constant}  \quad \text{temperature}
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Now we just need to measure volume, pressure and temperature.

The rest can be computed!
The weight of our atmosphere

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How much does the air in our room weigh???

Answer:
Question

How much does the air in your classroom weigh???
What if I don’t have fancy instruments?

- Typical sea-level pressure = 100,000 Pa (1,000 mb)
- Unisys Weather plotter: weather.unisys.com

Typical sea-level density = 1.2 kg/m³
Question

What math common core standards are addressed with this activity?
CCSS.Math.Content.6.RP.A.3d Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

\[ m = \rho \cdot V \]

If density is in kg/m³, and mass is in kg, what units must be used when we measure the volume of the room?
Question

What other math common core standards can be addressed with this activity?

www.corestandards.org/Math
Email me:
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Take a break!
The atmosphere is a fluid...
Really!

**fluid**

flu·id *n.*
A continuous, amorphous substance whose molecules move freely past one another and that has the tendency to assume the shape of its container; a liquid or gas.

What’s the difference?
Decreasing density
Decreasing overhead mass
Constant density
Decreasing density
Decreasing overhead mass

Liquid: not easily compressed
Gas: easily compressed
**Pressure**
Net force per unit area

**Weight**
Net force due to gravity

**Atmospheric Pressure**
Weight per unit area

Where we live

**GRAVITY**
How does pressure change with height?

**Liquid:** not easily compressed

**Gas:** easily compressed

- **Height** vs. **Pressure**
  - **Liquid:** linear relationship
  - **Gas:** exponential relationship
Pressure decreases exponentially with height

\[ P \approx P_0 e^{-z/H} \]

- pressure
- e-folding depth (scale height)
- height
- sea-level pressure \((z = 0)\)
Pressure decreases exponentially with height.

\[ P \approx P_0 e^{-\frac{z}{H}} \]
Balloon data activity

\[ P \approx P_0 e^{-Z/H} \]

- Plot pressure as a function of height
- Estimate \( P_0 \)
- Compute \( H \)

let \( P_0 = 100,000 \text{ Pa} \)

\[ H \approx \frac{-Z}{\ln(P) - \ln(P_0)} \]
Actual data
Actual data

Pressure equation

\( P_0 = 100,000 \text{ Pa and } H = 8,000 \text{ m} \)
High School

- **CCSS.Math.Content.HSF-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

- **CCSS.Math.Content.HSF-LE.A.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where $a, c,$ and $d$ are numbers and the base $b$ is 2, 10, or $e$; evaluate the logarithm using technology.

HSF-LE: High School Functions – Linear, Quadratic, & Exponential models
Other activities with balloon data

High School

- CCSS.Math.Content.HSF-BF.A.1c Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
Question

What other math common core standards can be addressed with this activity?

www.corestandards.org/Math
Balloon data

- Time
- Height
- Pressure
- Temperature
- Dewpoint temperature
- Relative humidity
- Wind speed
  (1 knot = 0.51 m/s = 1.15 mph)
- Wind direction
  (0 degrees: from the north)
  (180 degrees: from the south)
Thank you!