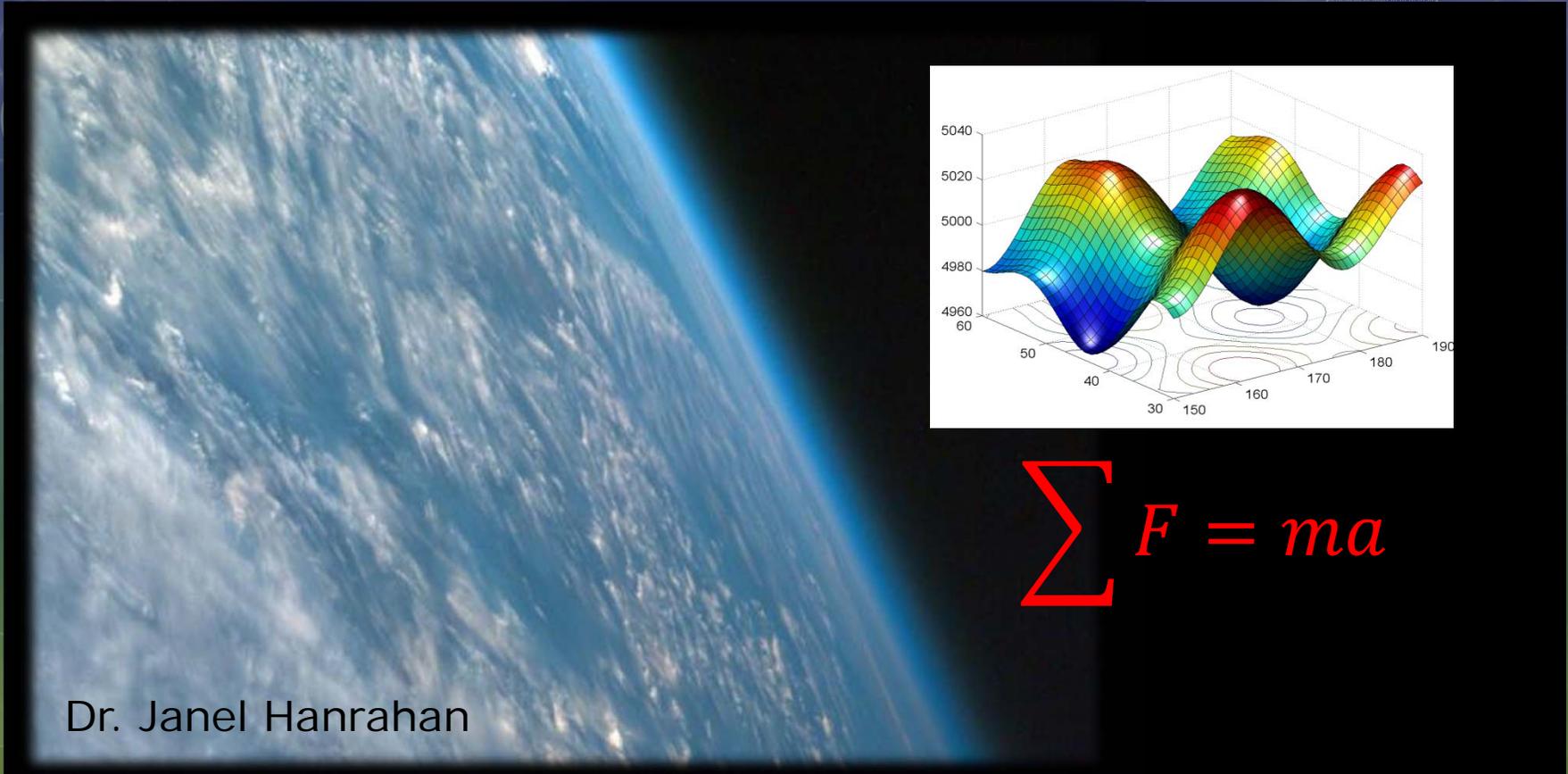


Satellites, Weather and Climate Module 33: *Atmospheric sciences and the mathematics common core standards*



Dr. Janel Hanrahan

$$\sum F = ma$$



Satellites, Weather and Climate Module 33:

Atmospheric sciences and the mathematics common core standards

High School

CCSS.Math.Content.HSF-BF.A.1c Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.

The Atmosphere

- 99% is within 20 miles of surface
- All water

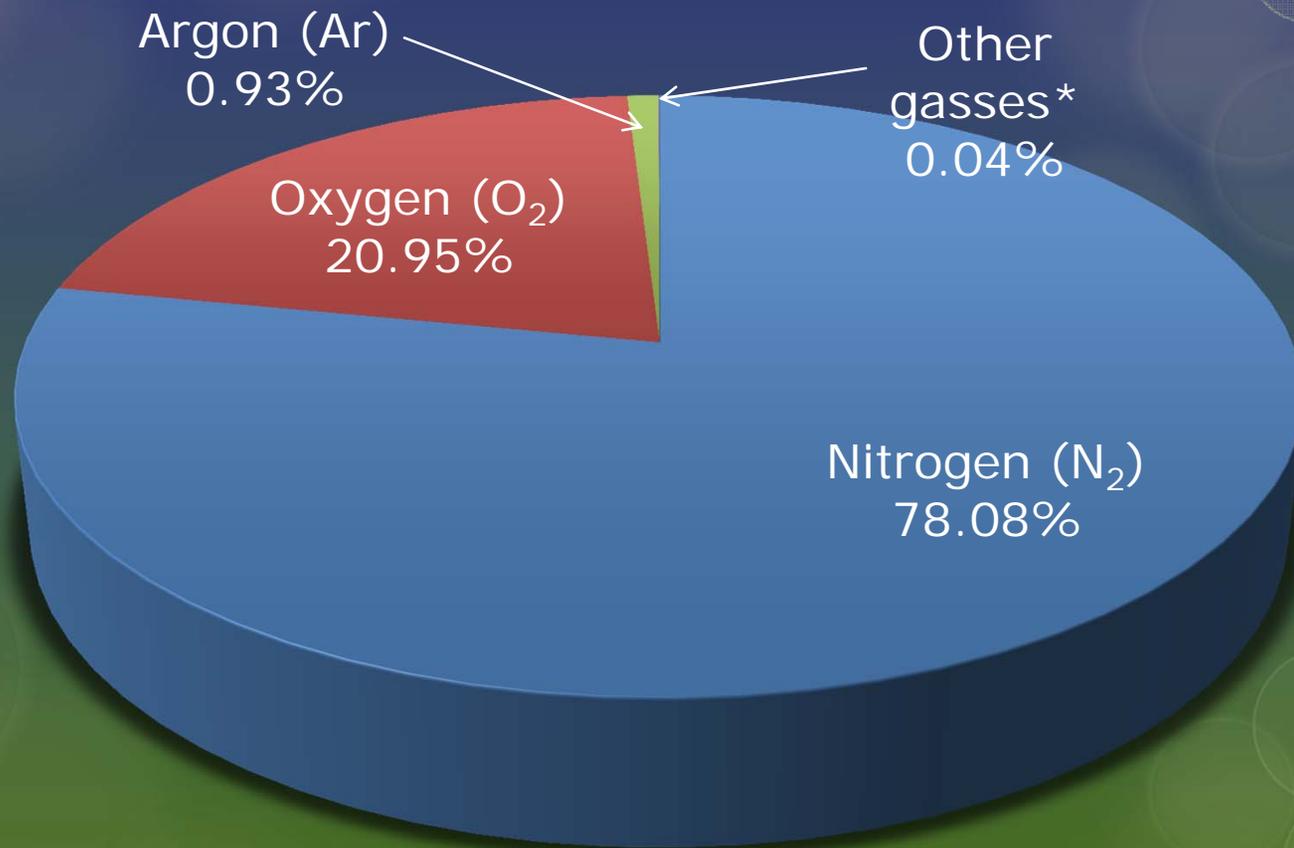
But what is it????

Relative thickness:

- Earth diameter: 7,900 miles
- Atmosphere is 7/7900 or 0.1%



Composition of Earth's atmosphere



*including water vapor, carbon dioxide, neon, helium, methane, krypton, hydrogen, nitrous oxide, carbon monoxide, xenon, ozone, nitrogen dioxide, iodine, and ammonia

Question

**How much does the air in
our room weigh???**

Question

How much does the air in
our room weigh???



The person with the closest
guess without going over
gets a **NEW CAR!!!***

***Well, not really. But it is fun to guess right?**

The weight of our atmosphere

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Many people do not consider that the Earth's atmosphere has weight. Indeed, our atmosphere is made up of various gasses, the most abundant of which are Nitrogen (78.08%), Oxygen (20.95%) and Argon (0.93%). These and the remaining 0.04% of atmospheric gases and particles, all have mass, and therefore have weight. For this activity, we will compute the weight of the air in this room.

Question: How much does the air in our room weigh?

Required values	Equations	Conversions and constants
$W = \underline{\hspace{2cm}}$ lbs	$F = m \cdot a$	$1 \text{ lb} = 4.448 \text{ N}$
$W = \underline{\hspace{2cm}}$ N		$T_K = 273 + T_C$
		$1 \text{ mb} = 100 \text{ Pa}$
		$g = 9.8 \text{ m s}^{-2}$
		$R_d = 287 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$

Putting it all together

Solution:

Common Core Standards addressed:

Newton's second law of motion

$$F = ma$$

force

mass

acceleration

The diagram shows the equation $F = ma$ in white serif font. Three yellow arrows point from labels to the variables: one from 'force' to 'F', one from 'mass' to 'm', and one from 'acceleration' to 'a'. The labels are in yellow sans-serif font. The background is a dark blue-to-green gradient with faint circular patterns.

Newton's second law of motion

$$W = mg$$

weight

(force due to gravity)

gravitational
acceleration

mass

The weight of our atmosphere

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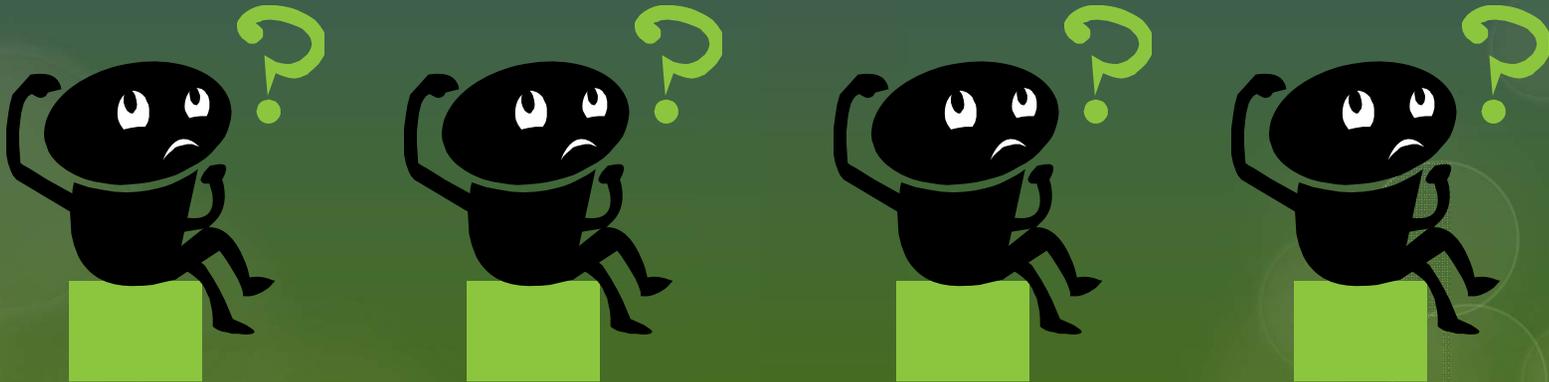
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		$R_d = 287 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$
		1 ft = 0.3048 m

Question

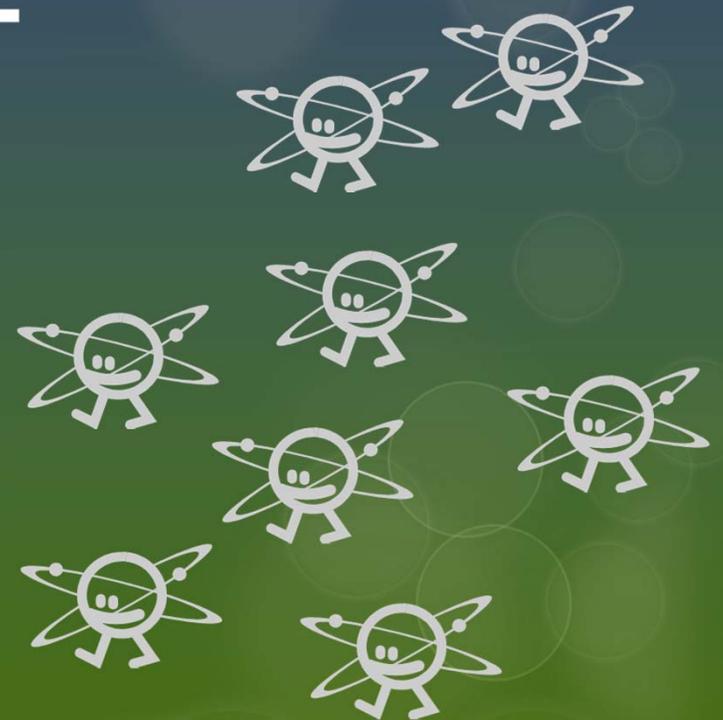
What is the mass of the air
in our room?



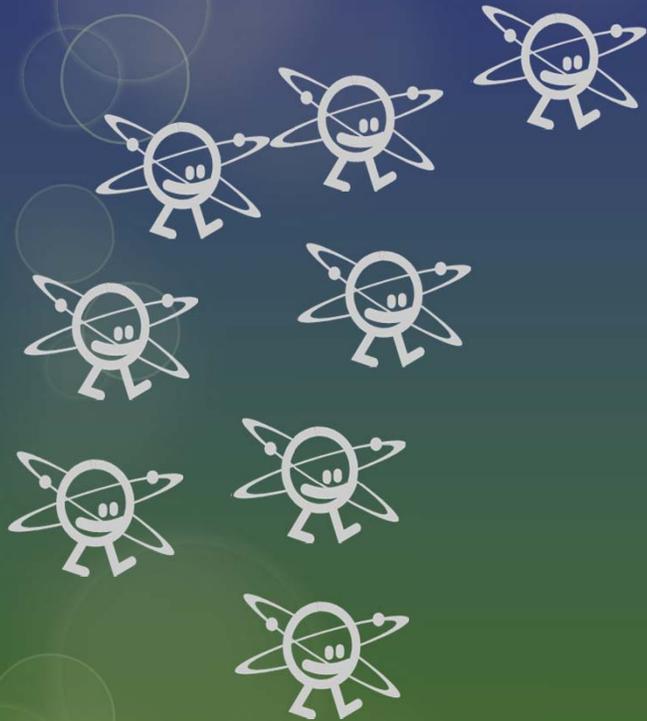
Density

$$\rho = \frac{m}{V}$$

density → ρ ← mass
→ volume



Density



$$\rho = \frac{m}{V}$$

$$m = \rho V$$

mass → density ← volume

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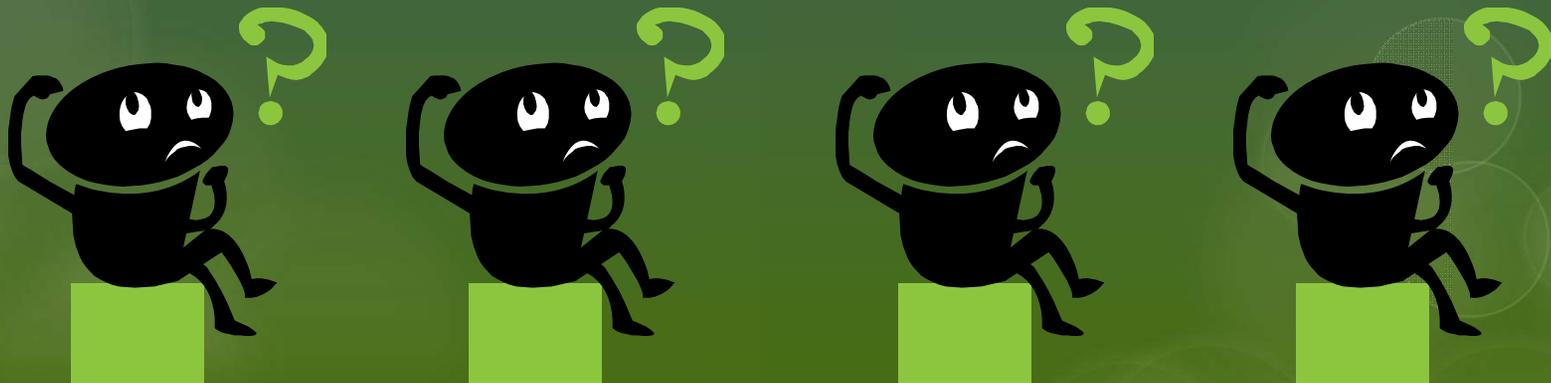
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$\rho = \underline{\hspace{2cm}}$ kg/m ³		$R_d = 287 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$
$V = \underline{\hspace{2cm}}$ m ³		$1 \text{ ft} = 0.3048 \text{ m}$

Questions

What is the **volume** of our room?

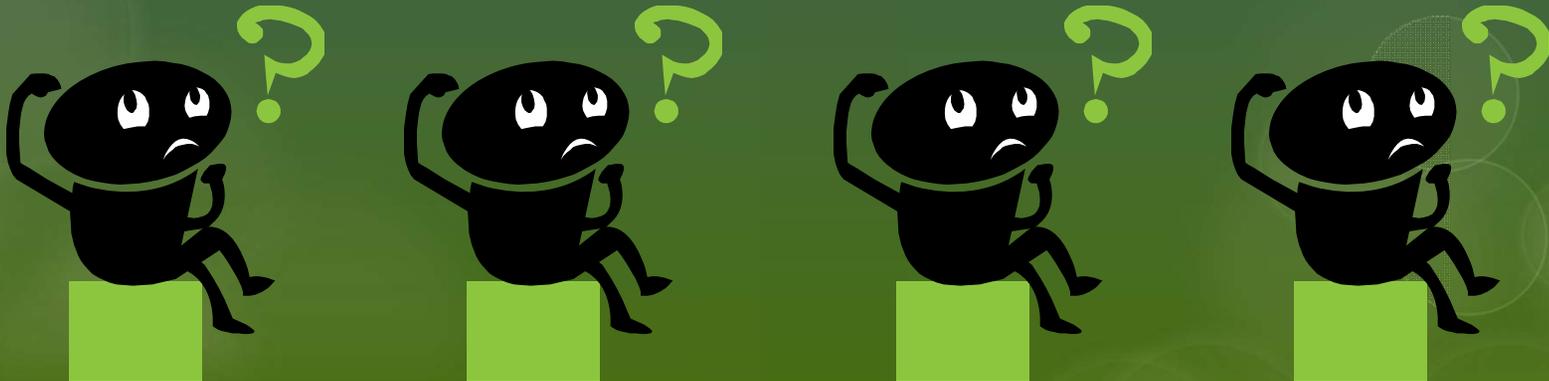
What is the **density** of the air in our room?



Questions

What is the **volume** of our room?

What is the **density** of the air in our room?



Volume

volume

height

$$V^* = lwh$$

length

width

A diagram illustrating the volume formula for a rectangular prism. The word "volume" is written in yellow on the left, with a yellow arrow pointing to the variable V^* in the formula $V^* = lwh$. The word "height" is written in yellow on the right, with a yellow arrow pointing to the variable h . The word "length" is written in yellow below the variable l , with a yellow arrow pointing to it. The word "width" is written in yellow below the variable w , with a yellow arrow pointing to it. The background is a dark blue to green gradient with faint circular patterns.

*Assuming that the room is a rectangular prism

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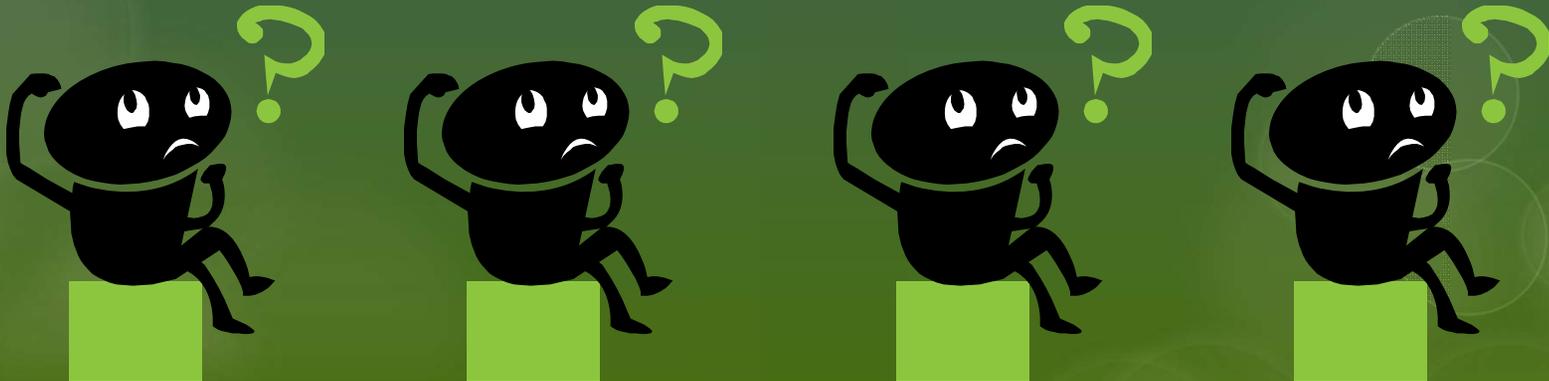
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$L = \underline{\hspace{2cm}}$ m		
$w = \underline{\hspace{2cm}}$ m		
$h = \underline{\hspace{2cm}}$ m		

Questions

What is the **volume** of our room?

What is the **density** of the air in our room?



The Ideal Gas Law

$$\rho = \frac{P}{RT}$$

density ρ pressure P gas constant R temperature T

The diagram shows the equation $\rho = \frac{P}{RT}$ centered on a dark blue background that transitions to green at the bottom. The variable ρ is labeled 'density' with a yellow arrow pointing to it. The variable P is labeled 'pressure' with a yellow arrow pointing to it. The variables R and T are both labeled 'gas constant' and 'temperature' respectively, with yellow arrows pointing to each. The entire equation is rendered in white text.

The weight of our atmosphere

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$\rho = \underline{\hspace{2cm}}$ kg/m ³	$\rho = P/(R \cdot T)$	$R_d = 287 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$
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$L = \underline{\hspace{2cm}}$ m		
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$P = \underline{\hspace{2cm}}$ Pa		
$T = \underline{\hspace{2cm}}$ K		

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$h = \underline{\hspace{2cm}}$ m		
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$P = \underline{\hspace{2cm}}$ Pa		
$T = \underline{\hspace{2cm}}$ K		

**Now we just need to
measure volume, pressure
and temperature.**

The rest can be computed!

The weight of our atmosphere

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$\rho = $ <input type="text"/> $ \text{kg/m}^3$	$\rho = P/(R \cdot T)$	$R_d = 287 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$
$V = $ <input type="text"/> $ \text{m}^3$		$1 \text{ ft} = 0.3048 \text{ m}$
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$R_d = 287 \text{ J/kgK}$		
$P = $ <input type="text"/> $ \text{Pa}$		
$T = $ <input type="text"/> $ \text{K}$		

Question

How much does the air in our room weigh???

Answer:



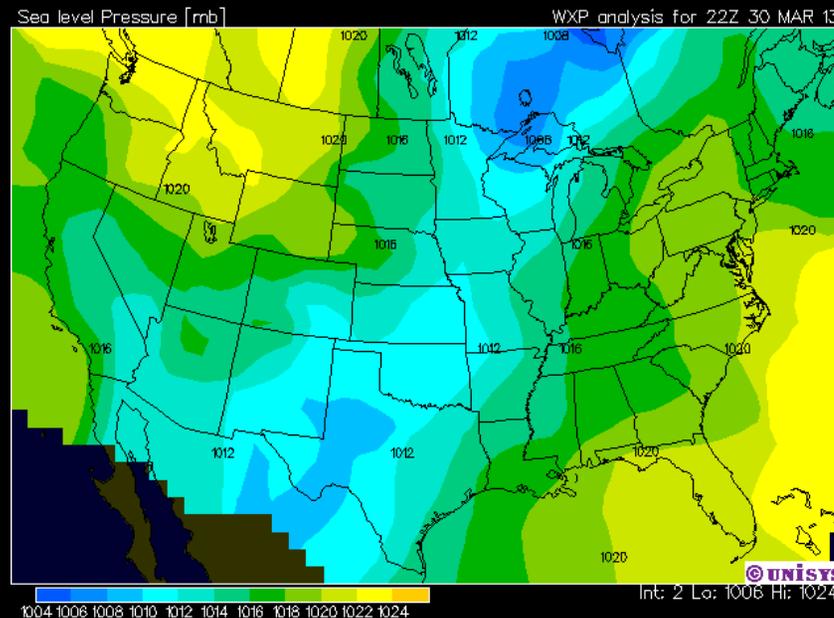
Question

How much does the air in
your classroom
weigh???



What if I don't have fancy instruments?

- ▣ Typical sea-level pressure = 100,000 Pa (1,000 mb)
- ▣ Unisys Weather plotter: weather.unisys.com

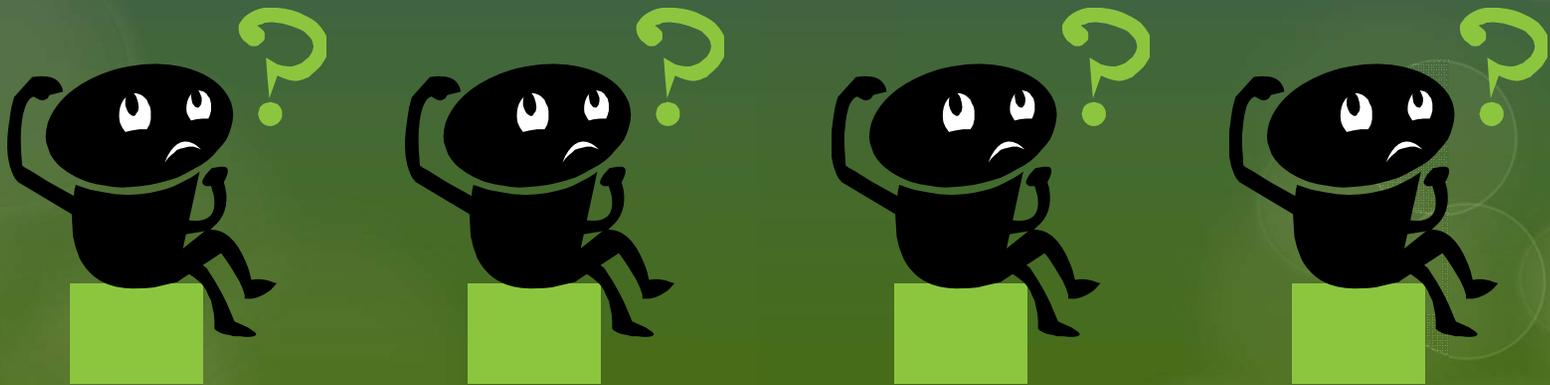


$$\rho = \frac{P}{RT}$$

- ▣ Typical sea-level density = 1.2 kg/m³

Question

What math common core standards are addressed with this activity?



Common Core Standards

Grade 6

- **CCSS.Math.Content.6.RP.A.3d** Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

$$m = \rho \cdot V$$

If **density** is in kg/m^3 , and **mass** is in **kg**, what units must be used when we measure the **volume** of the room?

Question

What other math common core standards can be addressed with this activity?

www.corestandards.org/Math

Email me:
janel.hanrahan@lyndonstate.edu

Take a break!



The atmosphere is a fluid... Really!

fluid

flu·id *n.*

A continuous, amorphous substance whose molecules move freely past one another and that has the tendency to assume the shape of its container; a liquid or gas.

What's the difference?

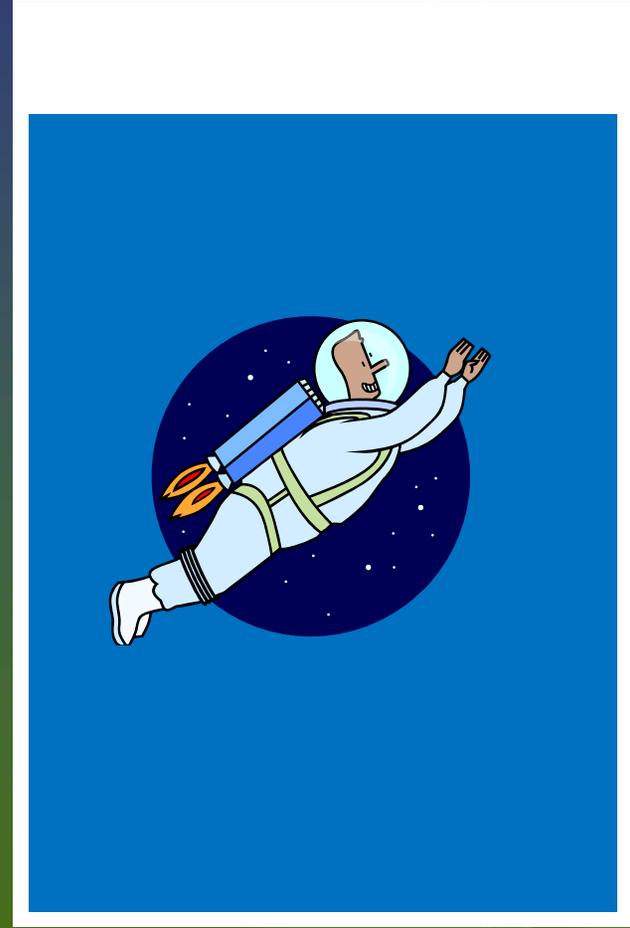
Liquid: not easily compressed

Constant density —
Decreasing overhead mass —



Gas: easily compressed

Decreasing density —
Decreasing overhead mass —



Pressure

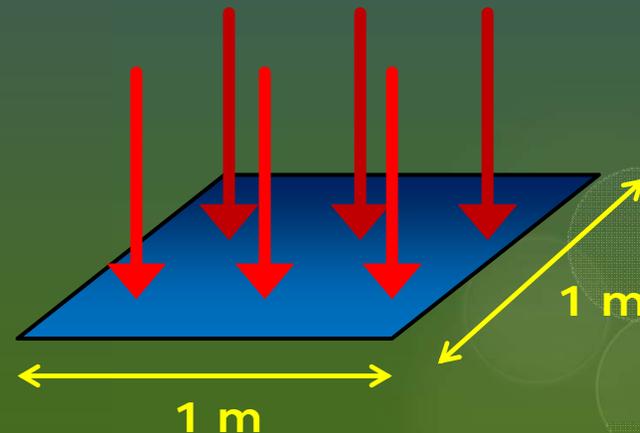
Net force per unit area

Weight

Net force due to gravity

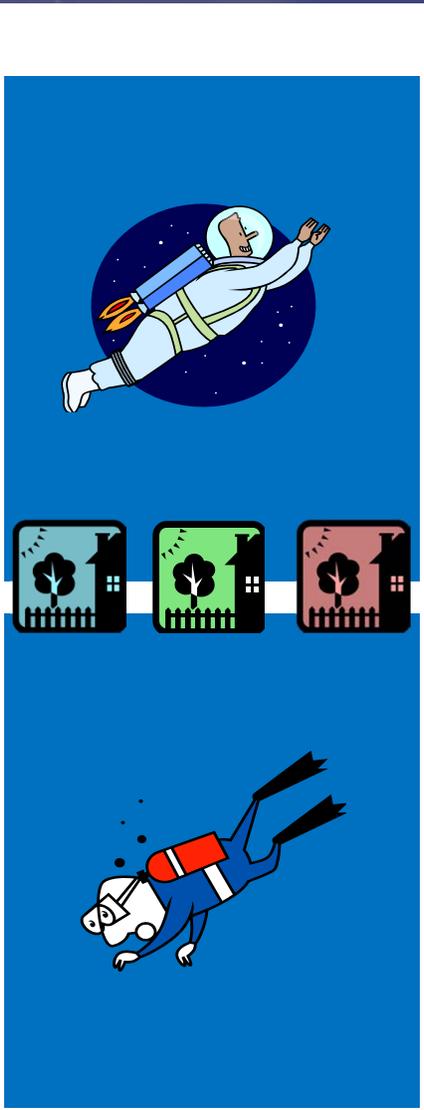
Atmospheric Pressure

Weight per unit area



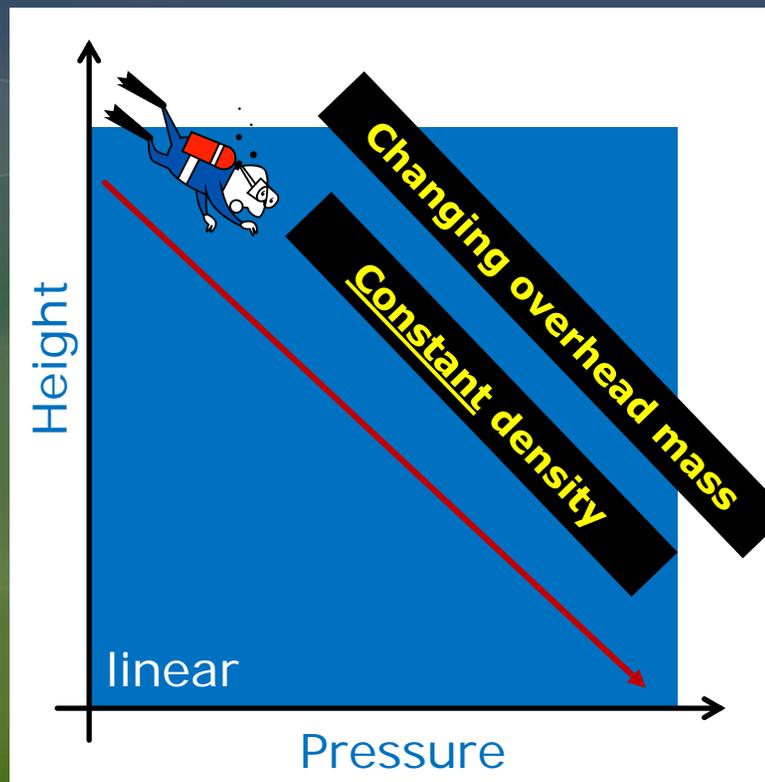
GRAVITY

Where
we live

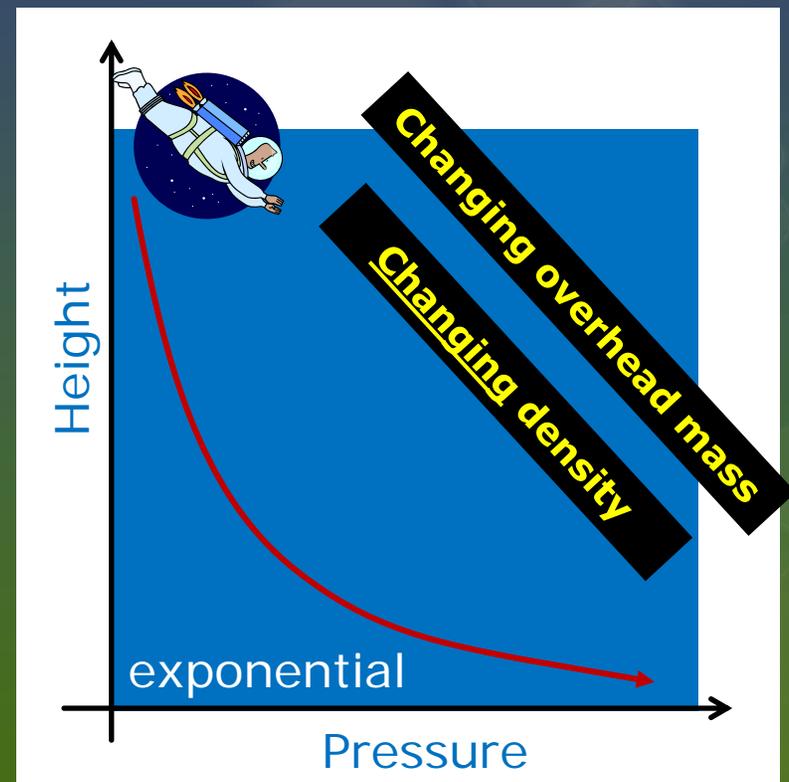


How does pressure change with height?

Liquid: not easily compressed



Gas: easily compressed



Pressure decreases exponentially with height

$$P \approx P_0 e^{-z/H}$$

pressure

e -folding depth
(scale height)

height

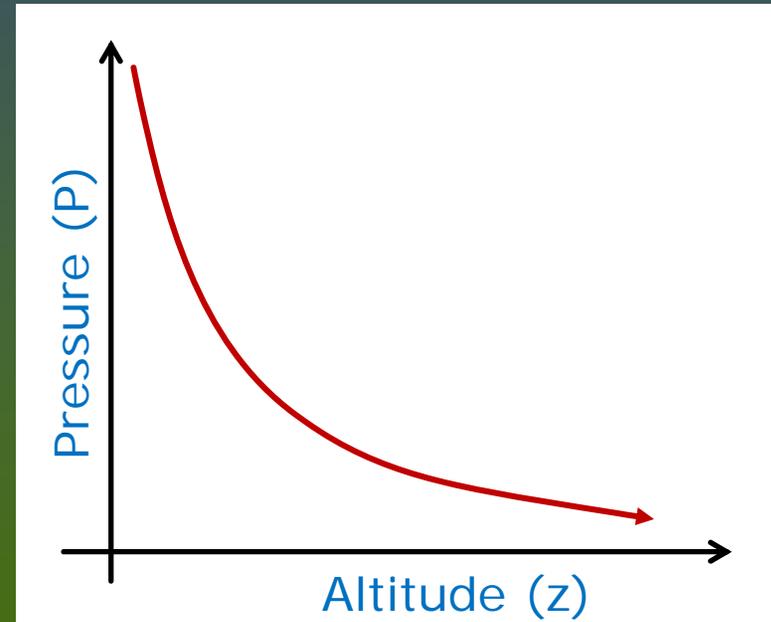
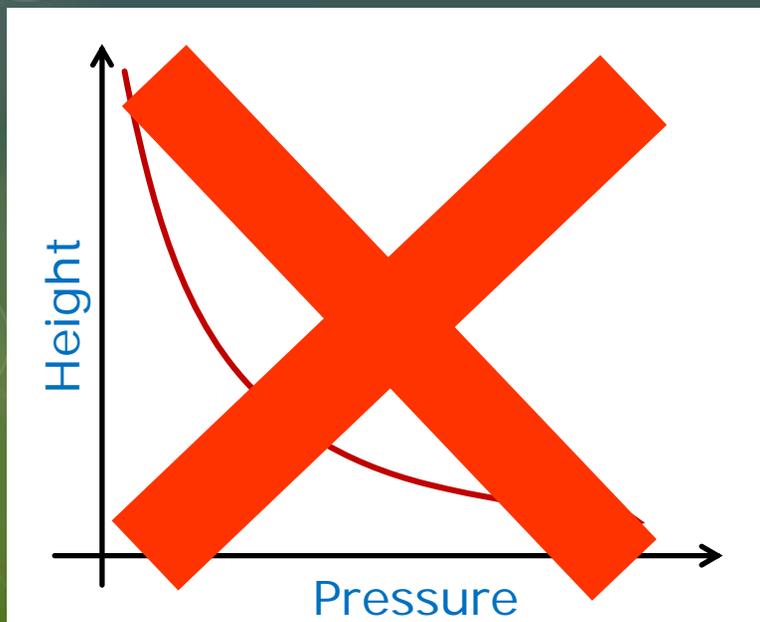
sea-level pressure
($z = 0$)

Pressure decreases exponentially with height

dependent

independent

$$P \approx P_0 e^{-z/H}$$



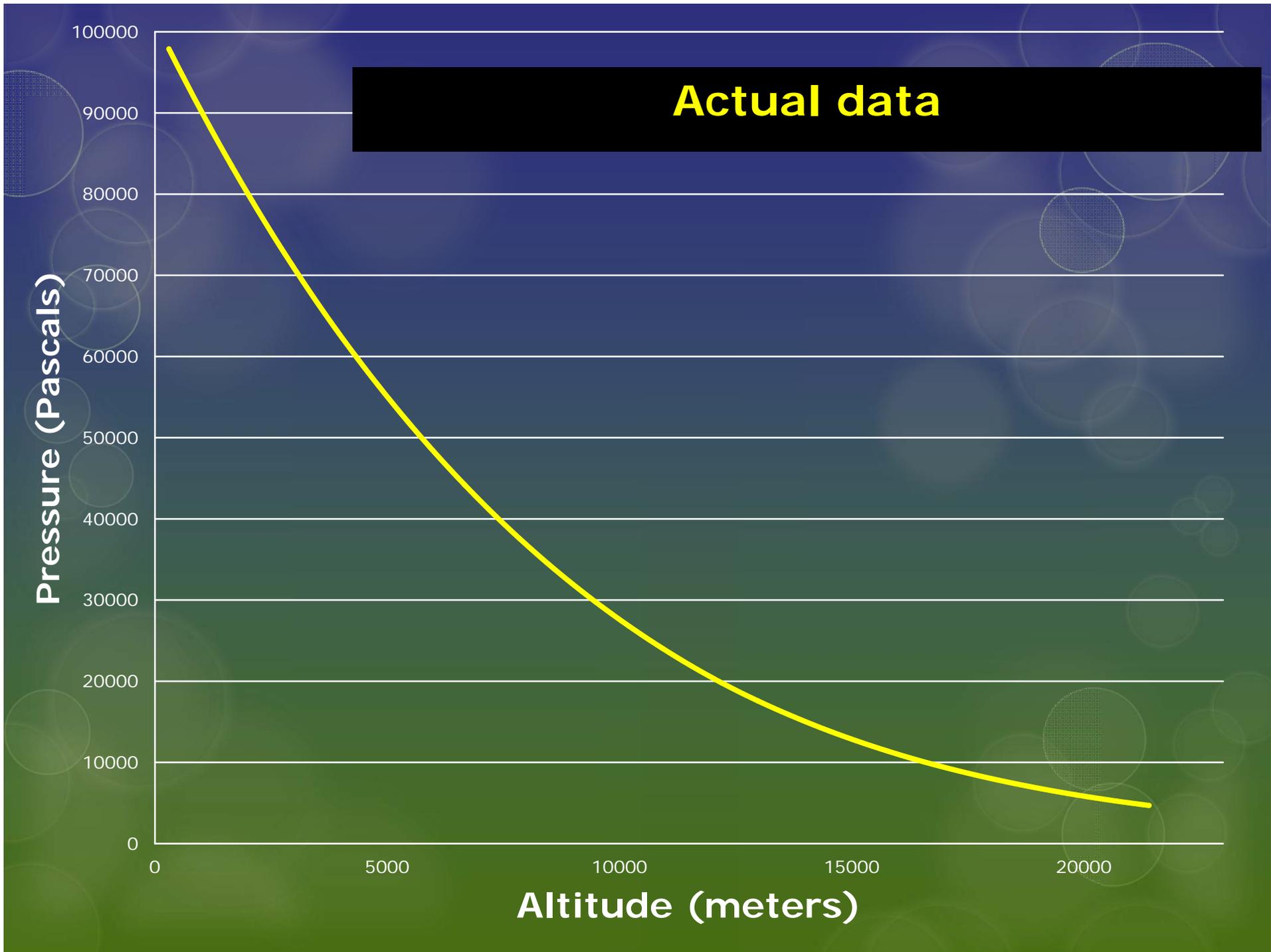
Balloon data activity

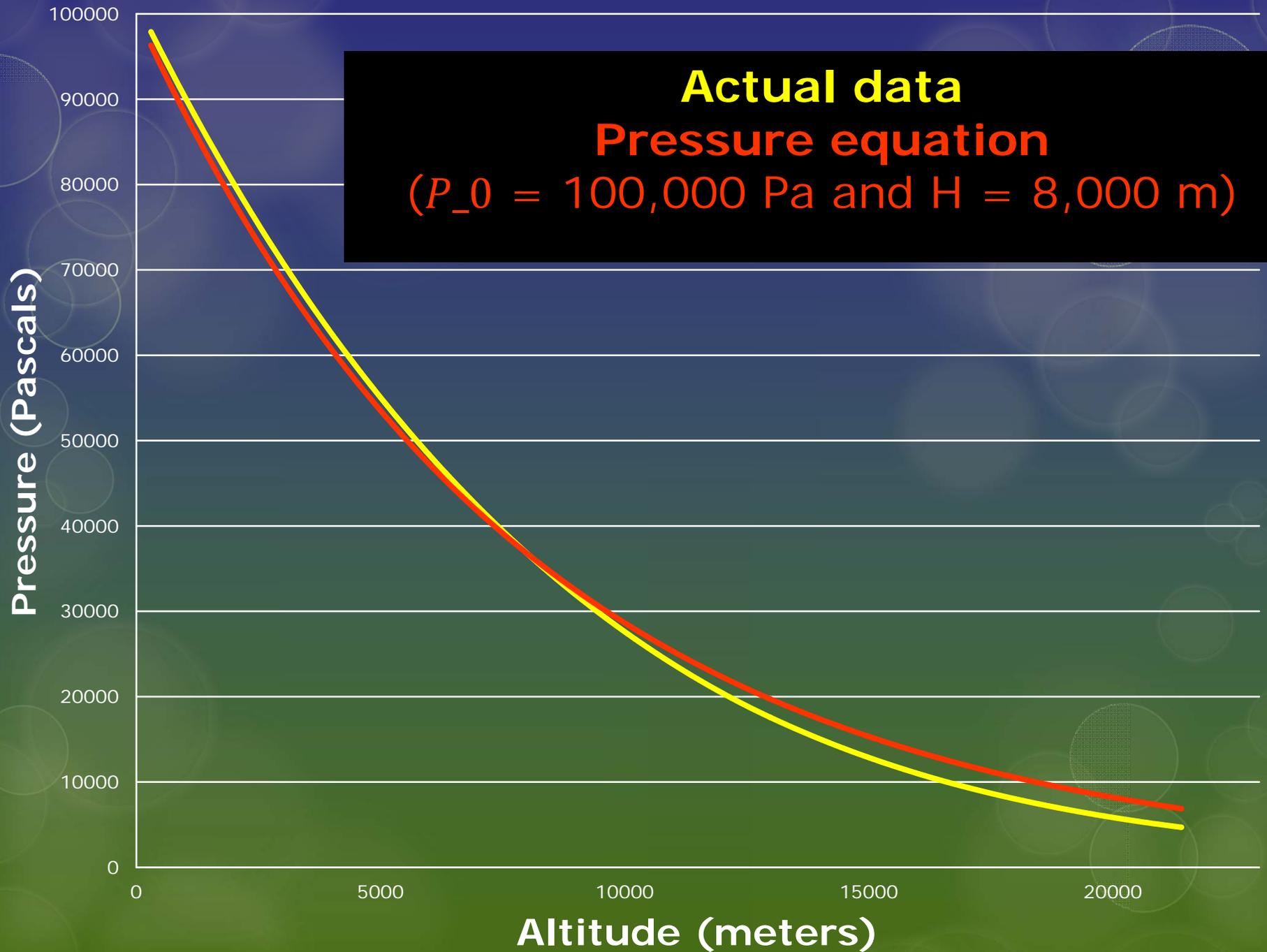
$$P \approx P_0 e^{-Z/H}$$

- Plot pressure as a function of height
- Estimate P_0
- Compute H

$$\text{let } P_0 = 100,000 \text{ Pa}$$

$$H \approx \frac{-Z}{\ln(P) - \ln(P_0)}$$





Common core standards

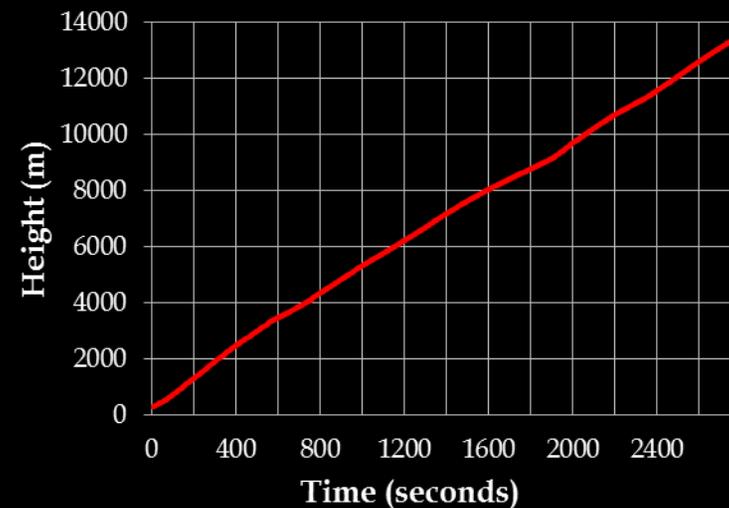
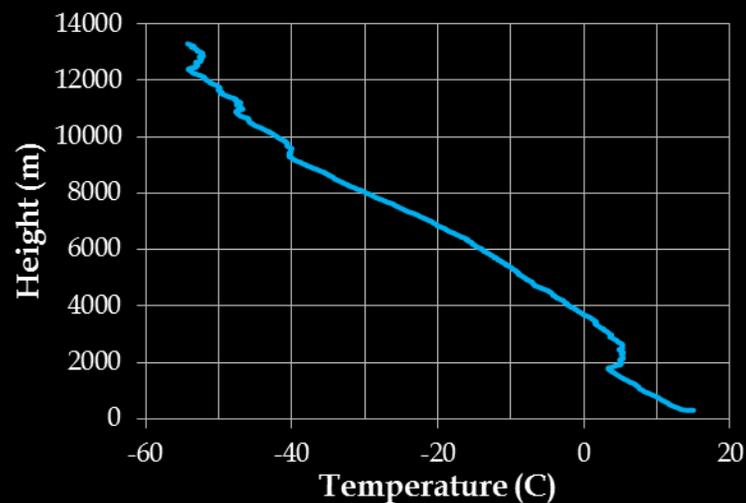
High School

- **CCSS.Math.Content.HSF-LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **CCSS.Math.Content.HSF-LE.A.4** For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.

Other activities with balloon data

High School

- **CCSS.Math.Content.HSF-BF.A.1c** Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.



Question

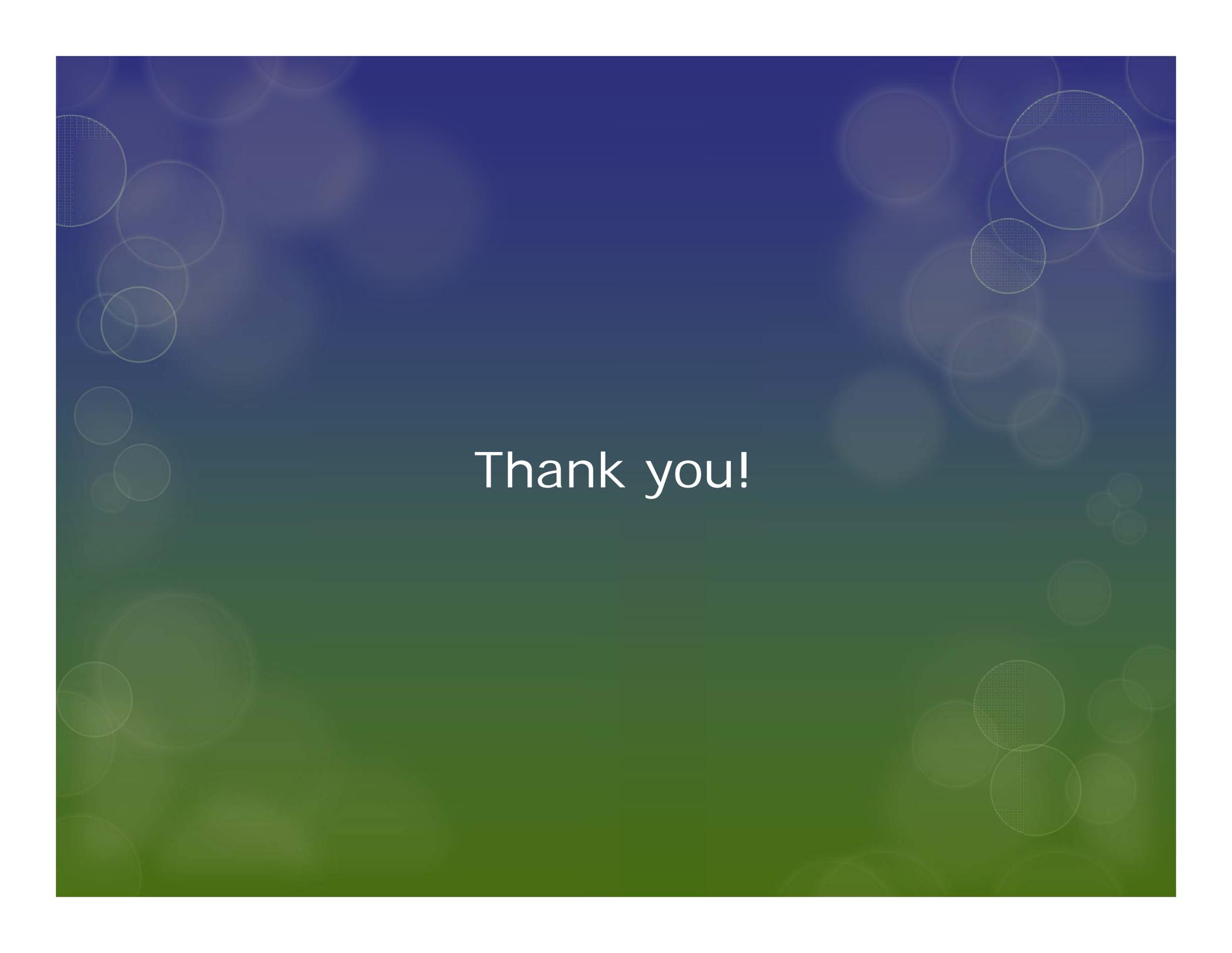
What other math common core standards can be addressed with this activity?

www.corestandards.org/Math

Balloon data

- Time
- Height
- Pressure
- Temperature
- Dewpoint temperature
- Relative humidity
- Wind speed
(1 knot = 0.51 m/s = 1.15 mph)
- Wind direction
(0 degrees: from the north)
(180 degrees: from the south)





Thank you!