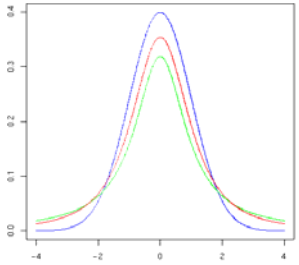
Four Key Probability Distributions

1. Normal Distribution (assuming σ is a known *population parameter*)
   - CLT → Sampling Distribution for $\bar{X}$ is approximately $N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$
   - $z_s = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ has a ***N(0,1)*** distribution

2. Student's T Distribution (assuming σ is unknown)
   - we estimate σ with $s = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}}$
   - $t_s = \frac{\bar{X}-\mu}{s/\sqrt{n}}$ has a *Student's T distribution* with *n-1* degrees of freedom
   - Standard Deviation (of the mean) vs. Standard Error (of the mean)

     $SD_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$ $\qquad$ $SE_{\bar{X}} = \frac{s}{\sqrt{n}}$



Normal Distribution in blue

T Distribution df = 5 in green

T Distribution df = 10 in red

3. Chi-Square Distribution $\left(\chi^2\right)$
   - Variances follow a scaled Chi-Square Distribution
   - $z_i = \frac{Y_i-\mu}{\sigma} \overset{iid}{\sim} N(0,1)$ → $z_i^2 = \left(\frac{Y_i-\mu}{\sigma}\right)^2 \sim \chi_1^2$
   - $z_1^2 + z_2^2 \sim \chi_2^2$ → $\sum_{i=1}^{n} z_i^2 \sim \chi_n^2$ → $\sum_{i=1}^{n} \frac{(Y_i-\mu)^2}{\sigma^2} \sim \chi_n^2$
   - Estimating μ with $\hat{\mu} = \bar{Y}$ → $\sum_{i=1}^{n} \frac{(Y_i-\bar{Y})^2}{\sigma^2} \sim \chi_{n-1}^2$

     $S^2 = \frac{\sum(Y_i-\bar{Y})^2}{n-1}$ → $\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$

4. F Distribution
   - A ratio of two scaled Chi-square random variables
   - $\frac{(n-1)S^2}{\sigma^2} \sim \chi_{n-1}^2$ → $S_1^2 = \frac{\sigma_1^2 \chi_{n_1-1}^2}{n_1-1}$ $\qquad$ $S_2^2 = \frac{\sigma_2^2 \chi_{n_2-1}^2}{n_2-1}$
   - $\frac{S_1^2}{S_2^2} = \frac{\sigma_1^2 \chi_{n_1-1}^2 / n_1-1}{\sigma_2^2 \chi_{n_2-1}^2 / n_2-1} \overset{*}{=} \frac{\chi_{n_1-1}^2 / n_1-1}{\chi_{n_2-1}^2 / n_2-1} = F_{n_1-1, n_2-1}$ $\qquad$ $*under\ H_o: \sigma_1^2 = \sigma_2^2$
   - If T is *Student's T* random variable with *m* df, then $T^2 \sim F_{1,m}$